ROBUST REGULATION OF A CLASS OF NONLINEAR SINGULARLY PERTURBED SYSTEMS

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Abstract: In this paper, robust regulation of a class of nonlinear singularly perturbed systems, via nonlinear H_∞ approach is considered. Under appropriate assumptions, it is shown through two new theorems that the existence of a positive definite solution for the Hamilton-Jacobi-Isaacs inequality related to the problem of disturbance attenuation for the main singularly perturbed system, can be related to the existence of a solution of a (simpler) Hamilton-Jacobi-Isaacs inequality arising in the problem of disturbance attenuation for the reduced-order system.

Keyword: Nonlinear systems, Singular perturbation, Robust performance, H-infinity control.

1. INTRODUCTION

One drawback of H_∞ design is that the order of the controller is at least equal to the order of the plant, and larger if, as is common, weights are included in the design (Amjadifard, *et al*., 2005). An approach to reduced-order controller design for a class of nonlinear composite systems is introduced by Isidori and Tarn (1995) and Isidori (1999). In these references, the problem of robust disturbance attenuation with internal stability via H_∞ controller for a class of nonlinear composite systems has been investigated. In particular, it has been shown by Isidori and Tarn (1995) that the existence of a positive definite solution for the Hamilton-Jacobi-Isaacs (HJI) inequality arising in the problem of disturbance attenuation for a nonlinear composite system, can be related to the existence of a solution of a (simpler) HJI inequality arising in the problem of disturbance attenuation for a plant which is a part of the main plant.

The problem of disturbance attenuation with local internal stability, via state feedback is to find, if possible, a feedback law such that the corresponding closed loop system has a locally asymptotically stable equilibrium and a L_2 gain from the input disturbance to the regulated output, less than or equal to a prescribed value γ (Isidori and Tarn, 1995). For more details about L_2 gain refer to (Ball, *et al*., 1993; Van der Schaft, 1992; Van der Schaft, 1991; and Isidori, 1991).

Problem of disturbance attenuation via H_∞ approach for linear and nonlinear singularly perturbed systems has been considered in many references. In (Fridman, 2001), the mentioned problem has been solved by considering the related HJI inequality, defining reduced Hamiltonian system, fast ε-independent PDE, and then constructing the H_∞ composite controller. On the other hand, Yazdanpanah, *et al*. (1997) introduced a new algorithm for the problem of robust regulation for linear singularly perturbed systems via treating the fast modes of system as uncertainty using the small gain theorem. Then Amjadifard, et al. (2003) and Amjadifard, et al. (2004) extended the method introduced in (Yazdanpanah, *et al.*, 1997) to a class of nonlinear affine systems.

In this paper, the method introduced in (Isidori, 1999) is extended to a class of nonlinear singularly perturbed systems through two new theorems. In fact, robust regulation of a class of singularly perturbed systems that are nonlinear in slow states, via nonlinear H_∞ technique is considered. Under appropriate assumptions, we show that the existence of a positive definite solution for the HJI inequality related to the problem of disturbance attenuation for the main singularly perturbed system, can be related to the existence of a solution of a (simpler) HJI inequality arising in the problem of disturbance attenuation for the reduced-order system. We will prove theorems related to the problem of disturbance attenuation, with global asymptotic internal stability for the nonlinear singularly perturbed system.

The proposed method is applied to an example to show the desired behavior of the designed composite controller.

2. PROBLEM FORMULATION

Consider a class of nonlinear singularly perturbed systems as

$$\dot{x}_1 = a_1(x_1) + A_1 x_2 + d_1 w = f_1(x_1, x_2, w) \qquad \text{(1-a)}$$

$$\varepsilon\dot{x}_2 = a_2(x_1) + A_2 x_2 + b_2 u = f_2(x_1, x_2, u) \qquad \text{(1-b)}$$

$$z = h_1(x_1) \qquad \text{(1-c)}$$

where $x_1 \in R^{n_1}$ is the slow state, $x_2 \in R^{n_2}$ is the fast state, $u \in R^m$ is the control input, w is an L_2 norm-bounded disturbance, and z is the controlled output. A_1, A_2, d_1 and b_2 are matrices with proper dimensions. $\varepsilon \geq 0$ is the perturbation parameter. It is assumed that $a_1(x_1)$ and $a_2(x_1)$ are smooth vector fields with $a_1(0) = 0$ and $a_2(0) = 0$, and the equilibrium point of the closed-loop system is at $(x_1, x_2) = (0,0)$.

The objective is to find a control law

$$u = \varphi(x_1, x_2) \qquad (2)$$

such that the closed loop system (1),(2) has an asymptotically stable equilibrium point at $(x_1, x_2) = (0,0)$ and a disturbance attenuation level of γ.

2.1 The reduced-order system

If A_2^{-1} exists, replacing $\varepsilon = 0$ in equation (1-b), we have

$$x_2 = H(x_1) = -A_2^{-1}(a_2(x_1) + b_2 u)\ . \qquad (3)$$

The composite control method seeks the control u as the sum of slow and fast controls [Kokotovic, *et al.*, 1986),

$$u = u_s + u_f$$

where u_s is a feedback function of x_1,

$$u_s = u(x_1, H(x_1)) = \Gamma_s(x_1)$$

and u_f is a feedback function of x_1 , x_2.

Replacing the quasi steady state x_2 (defined in equation (3)) in equation (1-a), the reduced-order system (or for simplicity, the reduced system) is obtained as:

$$\dot{x}_1 = a_0(x_1) + b_0 u_s + d_1 w = f_0(x_1, u_s, w) \qquad (4)$$

where

$$a_0(x_1) = a_1(x_1) - A_1 A_2^{-1} a_2(x_1)\ ,$$
$$b_0 = -A_1 A_2^{-1} b_2\ .$$

2.2 The composite system

Changing x_2 coordinate to

$$\mu = x_2 - H(x_1)$$

we obtain the new equations of system (1) as follows:

$$\dot{x}_1 = a_0(x_1) + A_1\mu + b_0\Gamma_s(x_1) + d_1 w \qquad \text{(5-a)}$$
$$\dot{\mu} = \tfrac{1}{\varepsilon} A_2\mu + \tfrac{1}{\varepsilon} b_2 u_f\ . \qquad \text{(5-b)}$$

Theorem 1. Consider system of equations (5) with $u_s \equiv 0$, $u_f \equiv 0$ and $w \equiv 0$. Suppose that the equilibrium $x_1 = 0$ of the reduced system is locally asymptotically stable, and the equilibrium $\mu = 0$ of the system $\dot{\mu} = \frac{1}{\varepsilon} A_2\mu$ is also locally asymptotically stable. Then the equilibrium $(x_1, \mu) = (0,0)$ of system (5) will be locally asymptotically stable.

Proof: Proof of this theorem in accordance to the Tikhonov theorem (Kokotovic, et al., 1986) and theorem 10.3.1 of (Isidori, 1999) will be clear.

3. NONLINEAR H_∞ CONTROLLER DESIGN

As we know from (Van der Schaft, 1991), the problem of disturbance attenuation with internal stability for a nonlinear system of form (5) is related to the problem of finding a positive definite solution of a special Hamilton-Jacobi-Isaacs inequality. If we set

$$F(x_1,x_2)=\begin{bmatrix} a_0(x_1)+b_0\Gamma_s(x_1)+A_1(x_2-H(x_1)) \\ \frac{1}{\varepsilon}A_2(x_2-H(x_1)) \end{bmatrix},$$

$$G_1=\begin{bmatrix} d_1 \\ 0 \end{bmatrix},\ G_2=\begin{bmatrix} 0 \\ \frac{1}{\varepsilon}b_2 \end{bmatrix}$$

then the HJI inequality will be as follows

$$\begin{aligned} &W_{(x_1,x_2)}F(x_1,x_2)+h_1^T(x_1)h_1(x_1) \\ &\quad+\tfrac{1}{4}W_{(x_1,x_2)}(\tfrac{1}{\gamma^2}G_1G_1^T-G_2G_2^T)W_{(x_1,x_2)}^T\le 0 \end{aligned}. \tag{6}$$

Where

$$W_{(x_1,x_2)}=\begin{pmatrix} \dfrac{\partial W(x_1,x_2)}{\partial x_1} & \dfrac{\partial W(x_1,x_2)}{\partial x_2} \end{pmatrix}.$$

If we can find a positive definite solution $W(x_1,x_2)$ for this inequality, then the control law

$$u_f=-\tfrac{1}{2}G_2^TW_{(x_1,x_2)}^T \tag{7}$$

causes the closed loop system (5)-(7), to have an L_2 gain less than or equal to a prescribed value γ. Furthermore, if some additional conditions are satisfied, the control law (7) also asymptotically stabilizes the equilibrium point $(x_1,x_2)=(0,0)$ of the system (Isidori, 1991).

Suppose that there exists a positive definite solution $V(x_1)$ for the HJI inequality related to the problem of disturbance attenuation for the reduced system, i.e.,

$$\begin{aligned} &V_{x_1}a_0(x_1)+h_1^T(x_1)h_1(x_1) \\ &\quad+\tfrac{1}{4}V_{x_1}(\tfrac{1}{\gamma^2}d_1d_1^T-b_0b_0^T)V_{x_1}^T\le 0 \end{aligned}. \tag{8}$$

We set

$$\begin{aligned} \alpha_1(x_1)&=\tfrac{1}{2\gamma^2}d_1^TV_{x_1}^T, \\ \alpha_2(x_1)&=\tfrac{-1}{2}b_0^TV_{x_1}^T. \end{aligned} \tag{9}$$

As shown in (Basar, and Bernhard, 1990), the structure of these two functions is the same as the equilibrium solution of the two-player zero-sum differential game associated with the problem of disturbance attenuation for plant (4). Using the function $H(x_1)$ (Equation (3)), and defining

$$\begin{aligned} \delta_1(x_1)&=\left(\frac{\partial H(x_1)}{\partial x_1}-I\right)G_1, \\ \delta_2(x_1)&=\left(\frac{\partial H(x_1)}{\partial x_1}-I\right)G_2. \end{aligned} \tag{10}$$

we come up with the following result.

Theorem 2. Suppose:

i) There exists a solution $V(x_1)>0$ for the HJI inequality (8),
ii) The matrix M defined as follows

$$M=\begin{bmatrix} \frac{1}{\gamma^2}\delta_1(x_1)\delta_1^T(x_1)-\delta_2(x_1)\delta_2^T(x_1)-\frac{2}{\varepsilon}A_2-\frac{2}{\varepsilon}b_2\delta_2^T(x_1) & 0 \\ 0 & \frac{1}{4}b_0b_0^T \end{bmatrix} \tag{11}$$

is positive semi-definite for all x_1.

Then the positive definite function

$$W(x_1,x_2)=V(x_1)+\|x_2-H(x_1)\|^2$$

is a solution of the HJI inequality (6).

Proof: By defining $\alpha_1(x_1)$ and $\alpha_2(x_1)$ as in (9), we have (Isidori and Astolfi, 1992):

$$\begin{aligned} &V_{x_1}\dot{x}_1+h_1^T(x_1)h_1(x_1)+u^Tu-\gamma^2w^Tw \\ &\quad=HJ_V(x_1)+\|u-\alpha_2(x_1)\|^2-\gamma^2\|w-\alpha_1(x_1)\|^2 \end{aligned} \tag{12}$$

where $HJ_V(x_1)$ denotes the left hand side of inequality (8). Let $HJ_W(x_1,x_2)$ denote the left-hand side of inequality (6) in which we set

$$W(x_1,x_2)=V(x_1)+\|x_2-H(x_1)\|^2.$$

Therefore,

$$\begin{aligned} \frac{\partial W(x_1,x_2)}{\partial x_1}&=V_{x_1}-2\mu^T\frac{\partial H(x_1)}{\partial x_1}, \\ \frac{\partial W(x_1,x_2)}{\partial x_2}&=2\mu^T, \end{aligned}$$

and

$$\begin{aligned} &\dot{W}+h_1^Th_1+u_f^Tu_f-\gamma^2w^Tw \\ &\quad=\frac{\partial W}{\partial x_1}\dot{x}_1+\frac{\partial W}{\partial x_2}\dot{x}_2+h_1^Th_1+u_f^Tu_f-\gamma^2w^Tw \\ &\quad=V_{x_1}\dot{x}_1+2\mu^T\dot{\mu}+h_1^Th_1+u_f^Tu_f-\gamma^2w^Tw \end{aligned} \tag{13}$$

where the relation $\dot{\mu} = \dot{x}_2 - \frac{\partial H}{\partial x_1}\dot{x}_1$ is used.

Also, we have

$$\begin{aligned}&\dot{W} + h_1^T h_1 + u_f^T u_f - \gamma^2 w^T w \\ &\quad = HJ_W(x_1, x_2) + \left\|u_f - \overline{\alpha}_2\right\|^2 - \gamma^2 \left\|w - \overline{\alpha}_1\right\|^2\end{aligned} \tag{14}$$

where $\overline{\alpha}_1 = \frac{1}{2\gamma^2} G_1^T W_{(x_1,x_2)}^T$, $\quad \overline{\alpha}_2 = \frac{-1}{2} G_2^T W_{(x_1,x_2)}^T$.

Using (9) and (10), $\overline{\alpha}_1$ and $\overline{\alpha}_2$ will be in the form

$$\begin{aligned}\overline{\alpha}_1 &= \alpha_1 - \frac{1}{\gamma^2}\delta_1^T(x_1)\mu, \\ \overline{\alpha}_2 &= \delta_2^T(x_1)\mu.\end{aligned}$$

Noting to (13) and (14), we have:

$$\begin{aligned}&V_{x_1}\dot{x}_1 + 2\mu^T\dot{\mu} + h_1^T(x_1)h_1(x_1) + u_f^T u_f - \gamma^2 w^T w \\ &\quad = HJ_W(x_1, \mu + H(x_1)) + \left\|u_f - \delta_2^T(x_1)\mu\right\|^2 \\ &\qquad - \gamma^2\left\|w - \alpha_1(x_1) + \tfrac{1}{\gamma^2}\delta_1^T(x_1)\mu\right\|^2\end{aligned} \tag{15}$$

Comparison of (12) and (15) yields:

$$\begin{aligned}&HJ_W(x_1, \mu + H(x_1)) + \left\|u_f - \delta_2^T(x_1)\mu\right\|^2 \\ &\quad - \gamma^2\left\|w - \alpha_1(x_1) + \tfrac{1}{\gamma^2}\delta_1^T(x_1)\mu\right\|^2 \\ &= HJ_V(x_1) + \left\|u_s - \alpha_2(x_1)\right\|^2 - \gamma^2\left\|w - \alpha_1(x_1)\right\|^2 \\ &\quad - u_s^T u_s + u_f^T u_f + 2\mu^T(\tfrac{1}{\varepsilon}A_2\mu + \tfrac{1}{\varepsilon}b_2 u_f)\end{aligned}.$$

Choosing

$$w = \alpha_1(x_1) - \tfrac{1}{\gamma^2}\delta_1^T(x_1)\mu \tag{16}$$

$$u_f = \delta_2^T(x_1)\mu \tag{17}$$

and using assumption i), we obtain

$$\begin{aligned}HJ_W(x_1, \mu + H(x_1)) &= HJ_V(x_1) \\ &+ \mu^{\mathrm{T}}(-\tfrac{1}{\gamma^2}\delta_1(x_1)\delta_1^T(x_1) + \delta_2(x_1)\delta_2^T(x_1) \\ &+ \tfrac{2}{\varepsilon}b_2\delta_2^T(x_1) + \tfrac{2}{\varepsilon}A_2)\mu - \tfrac{1}{4}V_{x_1}b_0 b_0^T V_{x_1}^T\end{aligned}$$

which can be written as

$$\begin{aligned}HJ_W(x_1, \mu + H(x_1)) &= HJ_V(x_1) \\ &\quad - \begin{bmatrix}\mu^T & V_{x_1}\end{bmatrix} M \begin{bmatrix}\mu \\ V_{x_1}^T\end{bmatrix}\end{aligned}$$

Where M is defined in (11). At this point, the assumption ii) completes the proof.

Note that the right hand side of (15) has a saddle point at the points defined in equations (16) and (17).

Now, in the case where the full state (x_1, x_2) of the system (1) is available for feedback, it will be shown that under additional conditions the feedback law

$$u_f = \delta_2^T(x_1)(x_2 - H(x_1)) \tag{18}$$

which yields disturbance attenuation, will also asymptotically stabilizes the system (5) (Isidori and Tarn, 1995).

As we know from (Isidori, 1991), a feedback law associated with a positive definite solution of an HJI inequality is always a stabilizing (in the sense of Lyapunov) law. Thus, it should be shown that the trajectories of the closed-loop system asymptotically converge to origin as time goes to infinity.

Assumption 1 (Isidori and Astolfi, 1992). Any bounded trajectory $x_1(t)$ of the system $\dot{x}_1 = a_0(x_1)$ satisfying $h_1(x_1(t)) = 0$ for all $t \geq 0$, is such that $\lim_{t\to\infty} x_1(t) = 0$.

Theorem 3. Assume Assumption 1. Also assume:

i) There exist a proper solution $V(x_1) > 0$ for the HJI inequality (8),
ii) The matrix M defined in (11) is positive definite.

Then the feedback law (18) solves the problem of disturbance attenuation, with global asymptotic internal stability for system (1).

Proof: Consider the closed loop system (1)-(18) in the (x_1, μ) coordinates. Set $w = 0$. Along any trajectory, the positive definite function $W(x_1, \mu + H(x_1))$ satisfies (Isidori, 1999; Isidori and Tarn, 1995):

$$\begin{aligned}\dot{W} &= -\left\|h_1(x_1)\right\|^2 - \left\|\delta_2^T(x_1)\mu\right\|^2 \\ &\quad + HJ_W(x_1, \mu + H(x_1)) - \gamma^2\left\|-\overline{\alpha}_1\right\|^2 \\ &\leq -\begin{bmatrix}\mu^T & V_{x_1}\end{bmatrix} M \begin{bmatrix}\mu \\ V_{x_1}^T\end{bmatrix} \leq 0,\end{aligned}$$

that shows the stability (in the sense of Lyapunov) of the equilibrium $(x_1, x_2) = (0,0)$. If $\dot{W} = 0$, then necessarily $\begin{bmatrix}\mu^T & V_{x_1}\end{bmatrix} M \begin{bmatrix}\mu \\ V_{x_1}^T\end{bmatrix} = 0$ and $h_1(x_1) = 0$.

The first equality, in view of condition ii), yields $\mu = 0$ and $V_{x_1} = 0$, which in turn yields $\alpha_2(x_1) = 0$. Thus the trajectories of the closed loop system with constraint $\dot{W} = 0$ are trajectories in which $\mu(t) = 0$ while $x_1(t)$ is a solution of $\dot{x}_1 = a_0(x_1)$ which $h_1(x_1(t)) = 0$. Noting to Assumption 1, these trajectories converge to $(x_1, \mu) = (0,0)$. The global asymptotic stability of the system may be shown by considering the properness of $V(x_1)$ (which means $V(x_1)$ is radially unbounded) and then the properness of $W(x_1, x_2)$ in view of Theorem 3-3 of (Jacques, *et al*., 1991).

Thus, the condition for disturbance attenuation with internal stability for singularly perturbed system was derived.

Remark: It is worth noting that a region for ε (the perturbation parameter) can be determined via Condition ii) of theorem 2.

Example: Consider a nonlinear singularly perturbed system in the form:

$$\begin{aligned} \dot{x}_1 &= x_1 - x_1^3 + x_2 + w \\ \varepsilon \dot{x}_2 &= x_1 - x_2 + u \end{aligned}$$

Fig. 1 shows the behavior of open loop system with a disturbance input $w = \sin 2t$ and initial condition $x_{10} = -1$, $x_{20} = 0.01$. The quasi steady state x_2 is obtained as $H(x_1) = x_1 + u_s$; and therefore, the reduced-order system will be:

$$\dot{x}_1 = 2x_1 - x_1^3 + u_s + w.$$

We design a robust H_∞ controller in order to disturbance attenuation (to a prescribed value γ) and to stabilize the reduced-order system through obtaining a positive definite function $V(x_1)$ such that the HJI inequality

$$V_{x_1}(2x_1 - x_1^3) + x_1{}^2 + \tfrac{1}{4} V_{x_1}(\tfrac{1}{\gamma^2} - 1) V_{x_1}^T \le 0$$

is satisfied. We choose $V(x_1) = \dfrac{6\gamma^2}{\gamma^2 - 1}(x_1^2 - \tfrac{1}{4}x_1^4)$ (with conditions $-\sqrt{2} < x_1 < 0$ and $\gamma > 1$) as the positive definite solution of HJI inequality. The slow input control based on equation (9) will be

$$u_s = -\tfrac{1}{2} b_0^T V_{x_1}^T = \frac{-3\gamma^2}{\gamma^2 - 1}(2x_1 - x_1^3). \qquad (19)$$

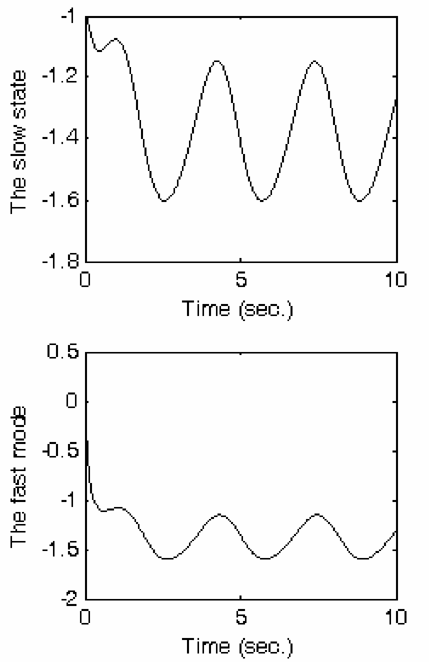

Fig. 1. Open-loop response of system.

Using change of variable $\mu = x_2 - H(x_1)$, the composite system will be obtained as:

$$\begin{aligned} \dot{x}_1 &= 2x_1 - x_1^3 + \mu + u_s(x_1) + w \\ \dot{\mu} &= \tfrac{-1}{\varepsilon}\mu + \tfrac{1}{\varepsilon} u_f \end{aligned} \qquad (20)$$

The matrix *M* defined in (11) will be:

$$M = \begin{bmatrix} \frac{1}{\gamma^2}(1 - \frac{3\gamma^2}{1-\gamma^2}(-2 + 3x_1^3))^2 + \frac{1}{\varepsilon^2} + \frac{2}{\varepsilon} & 0 \\ 0 & \frac{1}{4} \end{bmatrix}$$

Noting to M_{11} and M_{22}, the elements of *M*, they are always positive definite. Therefore, Condition ii) of theorem 2 has been satisfied and using equation (18), the controller u_f will provide disturbance attenuation to a level γ and also stabilize the composite system (20). Considering (19), application of the composite controller in the form

$$u = u_s + u_f = \frac{-3\gamma^2}{\gamma^2 - 1}(2x_1 - x_1^3) + -\tfrac{1}{\varepsilon}(x_2 - H(x_1))$$

results in a closed loop system response with input disturbance $w = \sin 2t$ and initial conditions $x_{10} = -1$, $x_{20} = 0.01$ as depicted in Fig. 2. The disturbance attenuation level is obtained 0.11.

4. CONCLUSION

In this paper, we have discussed the existence of a feedback law that solves the problem of disturbance attenuation for a class of nonlinear singularly perturbed systems.

Contribution of the paper is in providing two theorems, which present sufficient conditions for the solution (global solution) of a disturbance attenuation problem for a class of nonlinear singularly perturbed systems by solving an appropriate disturbance attenuation problem for the reduced-order system.

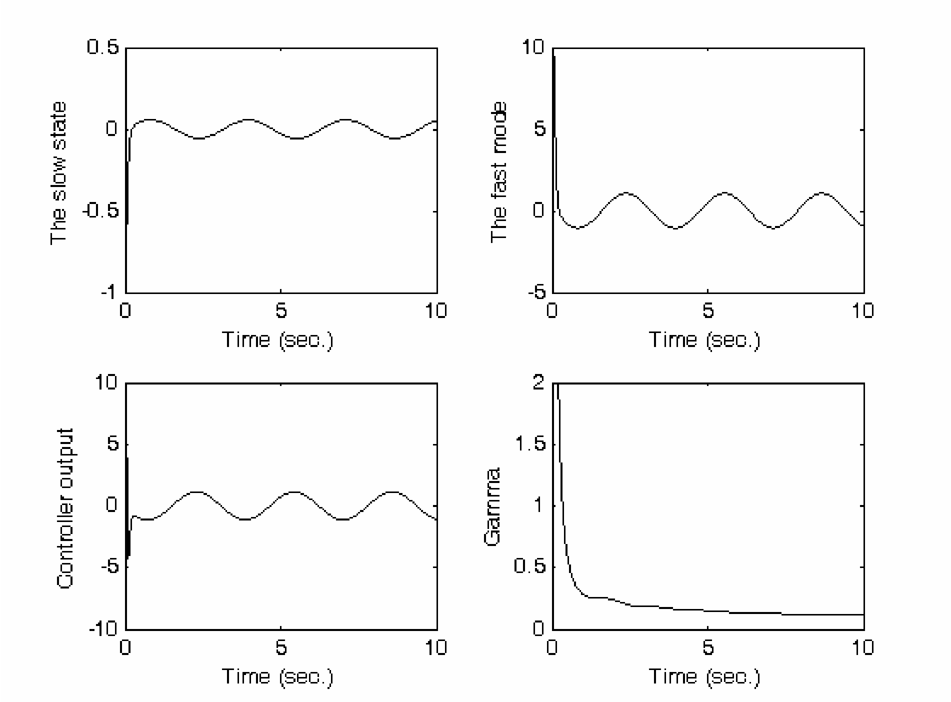

Fig. 2. Closed loop response with input disturbance $w = \sin 2t$.

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