UPPER BOUND $\mathcal{H}_\infty$ AND $\mathcal{H}_2$ CONTROL FOR SYMMETRIC MECHANICAL SYSTEMS

Kazuhiko Hiramoto * **Yuanqiang Bai** **
Karolos M. Grigoriadis **

* *Department of Mechanical Engineering, Akita University, 1-1 Tegata-gakuen-cho, Akita 010-8502 Japan*
E-mail: hira@ipc.akita-u.ac.jp
** *Department of Mechanical Engineering, University of Houston, Houston, TX 77204 U. S. A.*

Abstract: A static rate feedback problem for symmetric mechanical systems is considered in this paper. The feedback gain matrix is obtained by solving LMIs to minimise an upper bound on the closed-loop $\mathcal{H}_\infty$ or $\mathcal{H}_2$ norm. If the coefficient matrices of the system's equations of motion are linear functions of structural design parameters the obtained result for the controller synthesis can be easily extended to solve an integrated design problem of structural and control systems without loss of the LMI structure with respect to the feedback gain matrix but also structural design parameters.

Keywords: Symmetric Systems, LMIs, Integrated Design of Structural and Control Systems

1. INTRODUCTION

Symmetric systems are defined as dynamic systems whose transfer function matrices become symmetric and appear in various fields of applications. Some control analysis and synthesis methodologies which exploit the system's symmetric property have been proposed (Ikeda, 1995; Yang, *et al.* 2002). Precisely symmetric systems are classified into internally and externally symmetric systems. Internally symmetric systems are defined as the systems whose all coefficient matrices of the state-space realization can be given as symmetric matrices. Meanwhile externally symmetric systems are defined as the systems only when the corresponding transfer function matrices are symmetric.

Internally symmetric systems have been studied from a theoretical aspect in Liu, *et. al* (1998) and Tan and Grigoriadis (2001). Especially, in Tan and Grigoriadis (2001), a static output feedback synthesis problem for minimising the closed-loop $\mathcal{H}_\infty$ norm is shown to result in an LMI optimisation problem.

Mechanical systems with collocated sensors and actuators, which will be dealt with in the present paper, are externally symmetric systems. The externally symmetric mechanical systems frequently appear in a vibration control of large space structures. In Ikeda *et. al* (1993) a DVDFB control for an externally symmetric system is shown to correspond to the optimal feedback gain matrix minimising a quadratic performance index of certain weighting matrices. An extension of the result in Tan and Grigoriadis (2001) to the $\mathcal{H}_\infty$ control problem for externally symmetric systems has been studied in Bai *et. al* (2004). In Bai *et. al* (2004) a static output feedback gain matrix satisfying a constraint on an *upper bound* of the closed-loop $\mathcal{H}_\infty$ norm can be obtained with a simple algebraic operation.

In this paper we deal with an $\mathcal{H}_\infty$ or an $\mathcal{H}_2$ control for externally symmetric systems using static output feedback control law as an extension of Bai *et. al* (2004). The main objective of this paper is to show two results given as follows:

(1) For externally symmetric systems a control design problem minimising an *upper bound* of the closed-loop $\mathcal{H}_\infty$ or $\mathcal{H}_2$ norm with static rate feedback control law results in an LMI problem. Using this fact we can obtain the global optimal static rate feedback gain matrices for such systems in the sense of the upper bound of the closed-loop norm effectively.

(2) From the structure of the above LMI, we can easily extend the result in (1) to integrated design of structural and control systems (Onoda and Haftka, 1987). The *global optimal* rate feedback gain matrix and structural design parameters, e.g., the mass, the damping coefficient and the stiffness minimising the upper bound of the closed-loop $\mathcal{H}_\infty$ or $\mathcal{H}_2$ norm can be obtained by solving a single LMI.

The rest of the paper is organised as follows: In section 2 the mathematical model of the control object, i.e., mechanical systems with collocated sensors and actuators, is described. The LMI conditions on upper bounds of the closed-loop $\mathcal{H}_\infty$ and $\mathcal{H}_2$ norm are shown in section 3. Based on the result in section 3, section 4 is devoted to present the LMI conditions for the static rate feedback design of such mechanical systems. A design example is presented. In section 5 we extend the result in section 4 to integrated design. A design example of the integrated design is shown to demonstrate the effectiveness of proposed approach. Finally the conclusion is given in section 6.

2. PLANT DESCRIPTION

A symmetric mechanical system is defined as the following:

$$\begin{cases} M\ddot{q}(t) + D\dot{q}(t) + Kq(t) = F(u(t) + w(t)) \\ y(t) = F^T\dot{q}(t) \end{cases} \tag{1}$$

where $q(t) \in \mathbf{R}^n$, $u(t) \in \mathbf{R}^{nu}$, $w(t) \in \mathbf{R}^{nu}$ and $y(t) \in \mathbf{R}^{nu}$ are the displacement, the control force, the disturbance and the measured output vector, respectively. The matrices $M = M^T > 0 \in \mathbf{R}^{n\times n}$, $D = D^T > 0 \in \mathbf{R}^{n\times n}$ and $K = K^T > 0 \in \mathbf{R}^{n\times n}$ are the mass, the damping and the stiffness matrices, respectively. The matrix $F \in \mathbf{R}^{n\times nu}$ is the control influence matrix. Because of the symmetry of the system the matrix F also defines the property of the measured output, i.e., the sensor placement. In this paper the matrix F is assumed to have full column rank, i.e., $\mathrm{rank}(F) = nu$ [1].

By taking the state vector $x(t) := [q(t)^T\ \dot{q}(t)^T]^T$ the state-space representation of the mechanical system is given by

$$\begin{cases} \dot{x}(t) = Ax(t) + B(u(t) + w(t)) \\ y(t) = Cx(t) \end{cases}, \tag{2}$$

where

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}D \end{bmatrix},\ B = \begin{bmatrix} 0 \\ M^{-1}F \end{bmatrix},$$
$$C = \begin{bmatrix} 0 & F^T \end{bmatrix}.$$

The transfer function matrix from $u(s)$ to $y(s)$ is given as $G(s) := sF^T(Ms^2 + Ds + K)^{-1}F$ which is obviously symmetric transfer matrix whereas no state-space realizations exist such that all coefficient matrices of the state-space form become symmetric matrices. Therefore the mechanical system given in Eq. (2) is an externally symmetric system.

3. UPPER BOUND OF THE CLOSED-LOOP $\mathcal{H}_\infty$ AND $\mathcal{H}_2$ NORM

For the plant in Eq. (1) we assume the following static rate output feedback control law given as

$$u(t) = -Ry(t), \tag{3}$$

where $R = R^T \in \mathbf{R}^{nu\times nu}$. The closed-loop system is given as

$$\begin{cases} \dot{x}(t) = A_{cl}x(t) + Bw(t) \\ y(t) = Cx(t) \end{cases}, \tag{4}$$

where

$$\begin{aligned} A_{cl} &= A - BRC \\ &= \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}(D + FRF^T) \end{bmatrix}. \end{aligned}$$

Note that the stability of the closed-loop system is always guaranteed because of the symmetric property of the plant. Define the transfer function matrix of the closed-loop system in Eq. (4) as $G_{cl}(s) = C(sI - A_{cl})^{-1}B$. Then the bounded real lemma guarantees that $\|G_{cl}(s)\|_\infty \le \gamma$ ($\gamma > 0$) if and only if the following LMI is satisfied with $P_\infty = P_\infty^T \in \mathbf{R}^{2n\times 2n}$

$$\begin{bmatrix} A_{cl}^T P_\infty + P_\infty A_{cl} & P_\infty B & C^T \\ B^T P_\infty & -\gamma I & 0 \\ C & 0 & -\gamma I \end{bmatrix} \le 0. \tag{5}$$

In the $\mathcal{H}_2$ norm case the condition $\|G_{cl}(s)\|_2^2 < \nu$ ($\nu > 0$) is satisfied if and only if there exists

[1] This assumptions is not restrictive in control of mechanical systems.

matrices $P_2 = P_2^T \in \mathbf{R}^{2n\times 2n}$ and $Q = Q^T \in \mathbf{R}^{ny\times ny}$ satisfying the following LMIs:

$$\begin{bmatrix} A_{cl}^T P_2 + P_2 A_{cl} & P_2 B \\ B^T P_2 & -I \end{bmatrix} < 0,$$

$$\begin{bmatrix} P_2 & C^T \\ C & Q \end{bmatrix} > 0, \ \mathrm{Tr}(Q) \leq \nu \tag{6}$$

where Tr denotes the trace of a matrix. Using those LMI conditions we obtain a following theorems.

Theorem 1. An upper bound on the $\mathcal{H}_\infty$ norm of the closed-loop system $G_{cl}(s)$ is given by the minimum $\gamma_u > 0$ satisfying the following LMIs:

$$\begin{bmatrix} -2\sigma(D + FRF^T) & \sigma F & F \\ \sigma F^T & -\gamma_u I & 0 \\ F^T & 0 & -\gamma_u I \end{bmatrix} \leq 0, \tag{7}$$

$$\sigma > 0 \tag{8}$$

Proof. Consider a matrix P_∞ as

$$P_\infty := \sigma \begin{bmatrix} K & 0 \\ 0 & M \end{bmatrix}. \tag{9}$$

Note that P_∞ in Eq. (9) is positive definite iff $\sigma > 0$. By substituting P_∞ in Eq. (9) and $\gamma_u > 0$ into Eq. (5) we obtain the LMI conditions Eqs. (7) and (8). ■

Theorem 2. An upper bound on the square of the closed-loop $\mathcal{H}_2$ norm of $G_{cl}(s)$ is given by the minimum $\nu_u > 0$ satisfying the following LMIs:

$$\begin{bmatrix} -2\sigma(D + FRF^T) & \sigma F \\ \sigma F^T & -I \end{bmatrix} \leq 0, \tag{10}$$

$$\begin{bmatrix} \sigma M & F \\ F^T & Q \end{bmatrix} > 0, \ , \tag{11}$$

$$\mathrm{Tr}(Q) \leq \nu_u, \ \sigma > 0 \tag{12}$$

Proof. The proof of this theorem is similar to that of Theorem 1. Let

$$P_2 := \sigma \begin{bmatrix} K & 0 \\ 0 & M \end{bmatrix}. \tag{13}$$

and substitute this P_2 and ν_u into Eq. (6) we can obtain Eqs. (10)-(12). The matrix on the left hand side of Eq. (11) after substituting P_2 given in Eq. (13) becomes

$$\begin{bmatrix} \sigma K & 0 & 0 \\ 0 & \sigma M & F \\ 0 & F^T & Q \end{bmatrix} \tag{14}$$

We can eliminate the first row and column of the matrix because of $K > 0$ and obtain Eq. (11). ■

The results of the above two theorems state that an upper bound on the closed-loop $\mathcal{H}_\infty$ or $\mathcal{H}_2$ norm of the symmetric mechanical systems can be obtained by solving LMIs on σFRF^T which is an affine function on the static output feedback gain matrix R. In general, the static output feedback controller design is a BMI problem even in the simpler case of the stabilisation problem (Syrmos *et. al* 1997). In the following sections we present a controller design method based on the above bounds and an extension to the integrated design.

4. CONTROLLER DESIGN

4.1 Synthesis conditions

By taking a new design parameter matrix $R_s = \sigma R$ in Theorems 1 and 2 we can immediately give two synthesis LMI conditions as following theorems:

Theorem 3. A static output feedback gain matrices R in Eq. (3) yielding $\|G_{cl}(s)\|_\infty \leq \gamma_u$ ($\gamma_u > 0$) exists if there exists a symmetric matrix $S_\infty \in \mathbf{R}^{nu\times nu}$ satisfying the following conditions:

$$\begin{bmatrix} -2(\sigma D + FR_sF^T) & \sigma F & F \\ \sigma F^T & -\gamma_u I & 0 \\ F^T & 0 & -\gamma_u I \end{bmatrix} \leq 0, \tag{15}$$

$$\sigma > 0. \tag{16}$$

Then a feedback gain matrix R in Eq. (3) can be obtained as the following:

$$R = \frac{R_s}{\sigma} \tag{17}$$

Theorem 4. A static output feedback gain matrix R in Eq. (3) yielding $\|G_{cl}(s)\|_2^2 \leq \nu_u$ ($\nu_u > 0$) exists if there exists a symmetric matrix $S_2 \in \mathbf{R}^{nu\times nu}$ satisfying the following conditions:

$$\begin{bmatrix} -2(\sigma D + FR_sF^T) & \sigma F \\ \sigma F^T & -I \end{bmatrix} \leq 0, \ \begin{bmatrix} \sigma M & F \\ F^T & Q \end{bmatrix} > 0, \tag{18}$$

$$\mathrm{Tr}(Q) \leq \nu_u, \ \sigma > 0, \tag{19}$$

Then a feedback gain matrix R in Eq. (3) can be obtained as follows:

$$R = \frac{R_s}{\sigma} \tag{20}$$

Remark 1. In the matrix conditions in each synthesis theorem we cannot deal with the amount of the energy consumption for the control. In the practical situation it is always favourable to suppress (or minimise) the amount of the energy as long as the closed-loop performance specification is met. In this problem, we can impose such kind of the energy constraint by restricting the matrix $\frac{R_s}{\sigma}$ in some senses. For example, following norm constraint may be given:

$$\left\| \frac{R_s}{\sigma} \right\| < \delta R, \ \delta R > 0 \tag{21}$$

The above condition can be transformed to the following LMI on R_s and σ:

$$\begin{bmatrix} \sigma \times \delta R I & R_s \\ R_s & \sigma \times \delta R I \end{bmatrix} > 0 \qquad (22)$$

Note that R_s is symmetric. We can obtain the feedback gain matrix incorporated the energy constraint in the sense of Eq. (22) by solving each synthesis LMIs with (22) simultaneously.

4.2 Design example 1

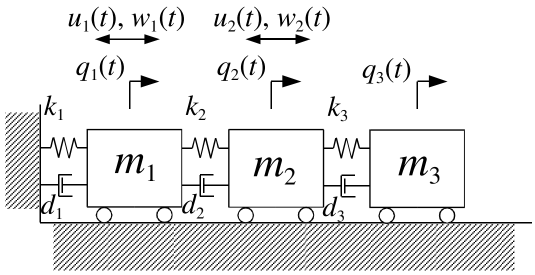

Fig. 1. 3-dof system

Let us consider a 3-dof system considered in Bai *et. al* (2004). The coefficient matrices in Eq. (1) is given as follows:

$$q(t) := \left[\, q_1(t)\ q_2(t)\ q_3(t) \,\right]^T,\ u(t) := \left[\, u_1(t)\ u_2(t) \,\right]^T,$$
$$w(t) := \left[\, w_1(t)\ w_2(t) \,\right]^T,$$
$$M := \mathrm{diag}(m_1, m_2, m_3),$$
$$D := \begin{bmatrix} d_1 + d_2 & -d_2 & 0 \\ -d_2 & d_2 + d_3 & -d_3 \\ 0 & -d_3 & d_3 \end{bmatrix},$$
$$K := \begin{bmatrix} k_1 + k_2 & -k_2 & 0 \\ -k_2 & k_2 + k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix},\ F := \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

We take $m_1 = m_2 = m_3 = 1$, $d_1 = d_2 = d_3 = 10^{-2}$ and $k_1 = k_2 = k_3 = 1$ respectively in this example. For the plant in Eq. (1) we firstly design a feedback control law $u(t) = -Ry(t)$ minimising the upper bound of the closed-loop $\mathcal{H}_\infty$ norm γ_u. We assume a norm constraint on the feedback gain matrix R as Eq. (22). We obtain the feedback gain matrix R for various values of δR with LMIs in Theorem 3 and evaluate the error between the obtained upper bound γ_u and the actual closed-loop $\mathcal{H}_\infty$ norm denoted by γ. The upper bound γ_u and the actual value γ with respect to $\delta R := [10^{-2}, 10^2]$ is shown in Fig. 2.

Secondly we obtain the output feedback gain matrix R minimising the upper bound of the square of the closed-loop $\mathcal{H}_2$ norm ν_u with the LMIs given in Theorem 4 and Eq. (22) for $\delta R = [10^{-2}, 10^2]$. The result is shown in Fig. 3. In Fig. 3 the square of the actual closed-loop $\mathcal{H}_2$ norm is denoted by ν. Although the error between ν_u and ν is large in relatively small δR, the error is quite small in large δR. We can conclude that the proposed upper bound of the $\mathcal{H}_2$ norm achieves quite nice estimate of the actual $\mathcal{H}_2$ norm of the closed-loop system especially in the case of the relatively high control authority.

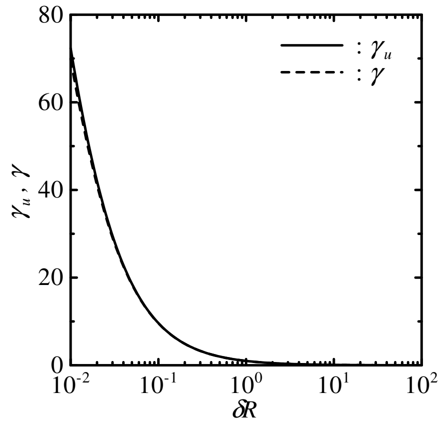

Fig. 2. $\mathcal{H}_\infty$ norm of the closed-loop system (The upper bound γ_u and the actual value γ)

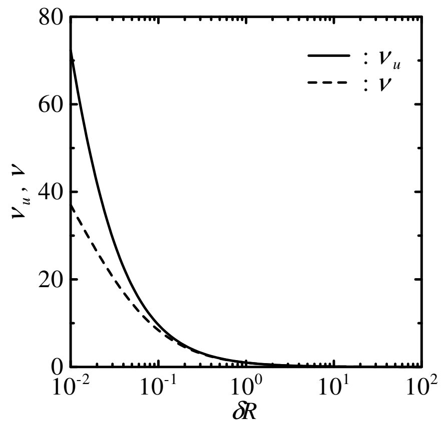

Fig. 3. $\mathcal{H}_2$ norm of the closed-loop system (The upper bound ν_u and the actual value ν)

5. EXTENSION TO INTEGRATED DESIGN PROBLEM

5.1 LMIs for integrated design problem

The synthesis result in the previous section can be easily extended to solve an integrated design of structural and control parameters. Assume that the matrices M and D are linear functions on structural design parameters. Then we can represent each function as a sum of the nominal value and the perturbation matrix caused by the tuning of the structural design parameter(s), that is such matrices are defined as

$$M := M^0 + \Delta M,\ \Delta M \in \mathbf{R}^{n\times n}, \qquad (23)$$
$$D := D^0 + \Delta D,\ \Delta D \in \mathbf{R}^{n\times n}, \qquad (24)$$

where the matrices with superscript ^{0} denote the nominal value matrices and the matrices $\Delta\star$ ($\star$:

M or D) are perturbation matrices. We pose the following inequality constraint on each perturbation matrix:

$$\underline{\Delta\star} \leq \Delta\star \leq \overline{\Delta\star} \quad (25)$$

where the matrices $\underline{\Delta\star}$ and $\overline{\Delta\star}$ are the lower and the upper bounds of the matrix $\Delta\star$ respectively. We formulate the integrated design problem as to obtain the optimal structural perturbation $\Delta\star$ and the feedback gain matrix R to minimise the upper bound of the closed-loop $\mathcal{H}_\infty$ or $\mathcal{H}_2$ norm.

By substituting Eqs. (23) and (24) into the corresponding matrices in Theorem 3 and 4 we have the following results:

Theorem 5. The upper bound of the closed-loop $\mathcal{H}_\infty$ norm in Theorem 1 is less than or equal to $\gamma_u > 0$ if the following conditions are satisfied:

$$\begin{bmatrix} -2(\sigma D^0 + \Delta D_s + FR_sF^T) & \sigma F & F \\ \sigma F^T & -\gamma_u I & 0 \\ F^T & 0 & -\gamma_u I \end{bmatrix} \leq 0, \quad (26)$$

$$\sigma > 0, \sigma\underline{\Delta D} \leq \Delta D_s \leq \sigma\overline{\Delta D} \quad (27)$$

where $\Delta D_s := \sigma \Delta D$. With the solution of the above LMIs the corresponding ΔD and R are obtained as follows:

$$R = \frac{R_s}{\sigma}, \ \Delta D = \frac{\Delta D_s}{\sigma} \quad (28)$$

Theorem 6. The static output feedback gain matrices R in Eq. (3) and the structural perturbation ΔM and ΔD yielding $\|G_{cl}(s)\|_2^2 \leq \nu_u$ ($\nu_u > 0$) exists if there exists a symmetric matrix $S_2 \in \mathbf{R}^{nu \times nu}$ satisfying the following conditions:

$$\begin{bmatrix} -2(\sigma D^0 + \Delta D_s + FR_sF^T) & \sigma F \\ \sigma F^T & -I \end{bmatrix} \leq 0, \quad (29)$$

$$\begin{bmatrix} \sigma M^0 + \Delta M_s & F \\ F^T & Q \end{bmatrix} > 0, \quad (30)$$

$$\mathrm{Tr}(Q) \leq \nu_u, \ \sigma > 0, \quad (31)$$

$$\sigma\underline{\Delta M} \leq \Delta M_s \leq \sigma\overline{\Delta M}, \quad (32)$$

$$\sigma\underline{\Delta D} \leq \Delta D_s \leq \sigma\overline{\Delta D}, \quad (33)$$

where $\Delta M_s := \sigma \Delta M$ and $\Delta D_s := \sigma \Delta D$ respectively. Then the corresponding feedback gain matrix R and structural perturbations ΔM and ΔD are obtained as the following:

$$R = \frac{R_s}{\sigma}, \ \Delta M = \frac{\Delta M_s}{\sigma}, \ \Delta D = \frac{\Delta D_s}{\sigma} \quad (34)$$

The proof of those two theorems are quite simple and is omitted. The given conditions are clearly LMIs on all unknown parameters.

We can incorporate the energy constraint in Remark 1 in the previous section also in this integrated design case.

We can obtain the global optimal structural design parameters and feedback gain matrices simultaneously by minimising γ_u or ν_u in LMIs of Theorem 5 and 6 without any heuristic iterations. This is the advantage of the proposed method because most integrated design methods only guarantee the convergence to a local optimal solution by employing heuristic iterative methods, e.g., coordinate descent method or homotopy method, etc. (Grigoriadis *et. al*, 1996; Hiramoto *et. al*, 2000; Lu and Skelton, 2000). This difficulty comes from the BMI nature of the integrated design problem (Tanaka and Sugie, 1998). On the other hand the proposed method guarantees to obtain the global optimal static rate feedback gain matrix and the structural design parameters. The price for the advantage of the proposed scheme is that we can only optimise the upper bound of the closed-loop $\mathcal{H}_\infty$ or $\mathcal{H}_2$ norm.

5.2 Design example 2

Let us consider the 3-dof system in Fig. 1 again. We assume that the damping coefficients of d_1, d_2 and d_3 can be adjusted. The nominal value of the structural parameters are same as those of the design example 1. Define δd_j ($j = 1, 2, 3$) as the perturbation (design parameter) of each damping coefficient. Then the representation of the damping matrix D is given as

$$D = D^0 + \Delta D, \ \Delta D = \sum_{j=1}^{3} W_j \delta d_j, \quad (35)$$

where

$$D^0 = \begin{bmatrix} d_1 + d_2 & -d_2 & 0 \\ -d_2 & d_2 + d_3 & -d_3 \\ 0 & -d_3 & d_3 \end{bmatrix}, \ W_1 := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$W_2 := \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ W_3 := \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}.$$

In this example we set the lower and the upper bounds of δd_j's as follows:

$$0 < d_j + \delta d_j \leq 10 d_j, \ j = 1, 2, 3 \quad (36)$$

As the energy constraint for the controller we impose a norm constraint given by Eq. (22) on the feedback gain matrix R by taking $\delta S = 1$. The upper bounds of the closed-loop $\mathcal{H}_\infty$ and $\mathcal{H}_2$ norm are optimised with the proposed method. The result is presented in Table 1 ($\mathcal{H}_\infty$ norm case) and Table 2 ($\mathcal{H}_2$ norm case) respectively. The δd_j's ($j = 1, 2, 3$) in each table are the resulted values of structural design parameters respectively. For comparison purpose the result in the case of the fixed structural design parameters with the same norm constraint on the feedback gain matrix (obtained in the method presented in the previous section) is shown in each table. We can conclude from the data of each table that the proposed integrated design accomplishes the better result than that of controller design.

Table 1. The result of the proposed integrated design ($\mathcal{H}_\infty$ case)

	Controller design	Integrated design
γ_u	0.99620	0.96321
γ	0.99571	0.95979
Error [%]	0.048607	0.35667
δd_1	0	8.9995×10^{-2}
δd_2	0	8.9987×10^{-2}
δd_3	0	6.4999×10^{-2}

Table 2. The result of the proposed integrated design ($\mathcal{H}_2$ case)

	Controller design	Integrated design
ν_u	0.99620	0.96321
ν	0.98109	0.86014
Error [%]	1.5393	11.984
δd_1	0	8.9997×10^{-2}
δd_2	0	8.9993×10^{-2}
δd_3	0	6.4898×10^{-2}

6. CONCLUDING REMARKS

The $\mathcal{H}_\infty$ and $\mathcal{H}_2$ static rate feedback controller synthesis problem for externally symmetric systems has been considered. The design problem of static rate feedback controller minimising the upper bound of the closed-loop $\mathcal{H}_\infty$ or $\mathcal{H}_2$ norm can be cast as LMIs on the gain matrix. The result is an extension of Bai *et. al* (2004). The methodology can be easily extended to an integrated design of structural and control systems. We can obtain the global optimal structural design parameters and the feedback gain matrix simultaneously minimising the upper bound of closed-loop $\mathcal{H}_\infty$ or $\mathcal{H}_2$ norm. The effectiveness of the proposed method is presented with design examples.

REFERENCES

Bai, Y., K. M. Grigoriadis and M. Demetriou (2004). H^∞ collocated control of structural systems: An analytical bound approach, *Proc. American Control Conference,* 2801-2806.

Grigoriadis, K. M., G. Zhu, and R. E. Skelton (1996). Optimal redesign of linear systems, *Transactions of the ASME, Journal of Dynamic Systems, Measurement and Control,* **118**, September, 598-605.

Hiramoto, K., H. Doki and G. Obinata (2000). Simultaneous optimal design of structure and control systems based on Youla parameterisation, *JSME International Journal, Series C,* **43,** 2, 326-334.

Ikeda, M., K. Koujitani and T. Kida (1993). , Optimality of direct velocity and displacement feedback for large space structures with collocated sensor and actuators, *Preprints 12th IFAC World Congress,* **VI,** 91-94.

Ikeda, M. (1995). Symmetric controllers for symmetric plants, *Proc. 3rd European Control Conference,* 988-993.

Liu, W. Q., V. Sreeram and K. L. Teo (1998). Model reduction for state-space symmetric systems, *Systems & Control Letters,* **34,** 209-215.

Lu, J. and R. E. Skelton (2000). Integrating structure and control design to achieve mixed H_2/H_∞ performance, *International Journal of Control,* **73,** 16, 1449-1462.

Onoda, J. and R. T. Haftka (1987). An approach to structure/control simultaneous optimisation for large flexible spacecraft, *AIAA Journal,* **25,** 8, 1133-1138.

Skelton, R. E., T. Iwasaki and K. Grigoriadis (1998). *A Unified Algebraic Approach to Linear Control Design,* Taylor & Francis, London.

Syrmos, V. L., C. T. Abdallah, P. Dorato and K. Grigoriadis (1997). Static output feedback–A survey, *Automatica,* **33,** 2, 125-137.

Tan, K. and K. M. Grigoriadis (2001). Stabilisation and H^∞ control of symmetric systems: An explicit solution, *Systems & Control Letters,* **44,** 57-72.

Tanaka, H. and T. Sugie (1998). General framework and BMI formulae for simultaneous design of structure and control systems, *Transactions of the Society of Instrument and Control Engineers,* **34,** 1, 27-33 (in Japanese).

Yang, G.-H. and L. Qiu (2002). Optimal symmetric $\mathcal{H}_2$ controllers for systems with collocated sensors and actuators, *IEEE Trans. Automat. Contr.,* **47**, 12, pp. 2121-2125.