OUTPUT BASED CONTROL FOR AN UNDERACTUATED SYSTEM: EXPERIMENTAL RESULTS

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Abstract: This paper considers output based tracking control for an experimental underactuated H-drive manipulator. This manipulator can be transformed into a so-called second-order chained form by a coordinate- and feedback transformation. An observer is proposed to solve the output feedback problem for systems in second-order chained form. For the designed observer global stability of the closed loop system is proved. Due to friction in the unactuated rotational joint the closed loop system is no longer asymptotically stable, but with a heuristic modification of the observer both the tracking and observer errors are bounded. Experimental results show the validity of this approach.

Keywords: Observers, Nonlinear control, Robotic manipulators, Cascade control, Tracking systems.

1. INTRODUCTION

The control of nonholonomic systems has received a lot of attention in the last decades (Kolmanovsky and McClamroch, 1995; Lefeber *et al.*, 2000; Aneke *et al.*, 2003b; Behal *et al.*, 2002; Ma *et al.*, 2002). Here attention will be drawn to underactuated mechanical systems with acceleration constraints (Reyhanoglu *et al.*, 1999). The tracking problem is addressed in e.g. (Aneke, 2003a; Luca and Oriolo, 2000; Arai *et al.*, 1998). In (Aneke, 2003a) global stability is achieved for systems which can be transformed into the so-called second-order chained form.

In this paper the global stability results of the state feedback controller proposed in (Aneke, 2003a) are extended by solving the corresponding output feedback problem. This is done by making use of properties for systems in cascaded form, as defined in (Panteley and Loria, 1998), which make it possible to divide the nonlinear system into a linear part and a linear time-varying part. In this way, this work forms an extension of (Lefeber *et al.*, 2000), where the same problem is addressed for first order chained form systems. The controller/observer combination is validated on an experimental set-up consisting of an underactuated H-drive manipulator with a freely rotating arm.

The paper is organized as follows. In section 2 some preliminaries and the output based tracking problem are presented. Section 3 deals with the observer design, while in section 4 global uniform asymptotically stability of the total closed loop system is shown. The underactuated H-drive manipulator is described in section 5. Experimental results are also presented and discussed in this section. Finally conclusions are drawn in section 6.

2. PROBLEM FORMULATION

Consider the second-order chained form with 3 degrees of freedom and 2 actuators given by (see e.g. (Imura *et al.*, 1996; Aneke *et al.*, 2003b))

$$\begin{aligned} \ddot{\xi}_1 &= u_1 \\ \ddot{\xi}_2 &= u_2 \\ \ddot{\xi}_3 &= \xi_2 u_1. \end{aligned} \tag{1}$$

For the second-order chained form (1) the error dynamics for the tracking problem can be written in the following form

$$\begin{aligned} &\Delta_1 \begin{cases} \dot{x}_{31} = x_{32} \\ \dot{x}_{32} = x_{21}u_{1d} + (x_{21} + \xi_{2d})(u_1 - u_{1d}) \end{cases} \\ &\Delta_2 \begin{cases} \dot{x}_{21} = x_{22} \\ \dot{x}_{22} = u_2 - u_{2d} \end{cases} \\ &\Delta_3 \begin{cases} \dot{x}_{11} = x_{12} \\ \dot{x}_{12} = u_1 - u_{1d} \end{cases} \end{aligned} \tag{2}$$

where $x_{i1} = \xi_i - \xi_{id}$, $x_{i2} = \dot{\xi}_i - \dot{\xi}_{id}$ and the subscript d indicates desired reference values.

In (Aneke *et al.*, 2003b) a cascaded backstepping approach has been used to stabilize the origin of the error dynamics. In this approach, the stabilization problem for (2) is decoupled into two separate stabilization designs for the subsystems Δ_3 and (Δ_1, Δ_2), respectively. The proposed linear time-varying tracking controller is given by

$$\begin{aligned} u_1 &= u_{1d} - k_1(\xi_1 - \xi_{1d}) - k_2(\dot{\xi}_1 - \dot{\xi}_{1d}) \\ u_2 &= u_{2d} - k_{11}(t)(\xi_2 - \xi_{2d}) - k_{12}(t)(\dot{\xi}_2 - \dot{\xi}_{2d}) \\ &\quad -k_{13}(t)(\xi_3 - \xi_{3d}) - k_{14}(t)(\dot{\xi}_3 - \dot{\xi}_{3d}), \end{aligned} \tag{3}$$

with $\mathbf{K}_2 = [k_1\ k_2]$ a constant feedback matrix and $\mathbf{K}_1(t) = [k_{11}(t)\ k_{12}(t)\ k_{13}(t)\ k_{14}(t)]$ a time-varying feedback matrix presented in (Aneke, 2003a) in which the entries k_{1i} are depending on u_{1d} and derivatives thereof. It can be seen from (3) that the full state is necessary to calculate the controller. In this paper it is assumed that only position measurements ξ_1 and ξ_3 are available for feedback.

Problem. The output based tracking problem consists of finding appropriate continuous time-varying output feedback controllers of the form

$$u_1 = u_1(t, \hat{\mathbf{x}}, \overline{\mathbf{u}}_d), \quad u_2 = u_2(t, \hat{\mathbf{x}}, \overline{\mathbf{u}}_d) \tag{4}$$

which can be designed such that the closed loop system is globally uniformly asymptotically stable. The vector $\hat{\mathbf{x}}$ is an estimate of $\mathbf{x}$ and the vector $\overline{\mathbf{u}}_d$ contains u_{1d}, u_{2d} and higher order derivatives.

Following the lines of the cascaded backstepping approach from (Aneke *et al.*, 2003b), the controller/observer design is also decoupled into a fourth-order and a second-order design for the respective subsystems:

- The (Δ_1, Δ_2) subsystem (2), which can be seen as a linear time-varying system (LTV) as soon as $u_1 - u_{1d} \equiv 0$. With time-varying matrices:

$$\mathbf{A}_1(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & u_{1d}(t) & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \mathbf{B}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix},$$
$$\mathbf{C}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \tag{5}$$

- The Δ_3 subsystem, which is a linear time-invariant system (LTI). The constant matrices are:

$$\mathbf{A}_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \mathbf{B}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \mathbf{C}_2 = \begin{bmatrix} 1 & 0 \end{bmatrix} \tag{6}$$

3. OBSERVER DESIGN

3.1 An observer for the (Δ_1, Δ_2) subsystem

For the LTV subsystem (5) a general observer is given by

$$\begin{aligned} \dot{\hat{\mathbf{x}}}_1(t) &= (\mathbf{A}_1(t) - \mathbf{L}_1(t)\mathbf{C}_1(t))\hat{\mathbf{x}}_1(t) \\ &\quad + \mathbf{B}_1(t)u_2(t) + \mathbf{L}_1(t)y_1(t), \end{aligned} \tag{7}$$

where $\hat{\mathbf{x}}_1 = [\hat{x}_{31}\ \hat{x}_{32}\ \hat{x}_{21}\ \hat{x}_{22}]$ and $y_1 = x_{31} = \xi_3 - \xi_{3d}$. Based on the results of Theorem 15.2 in (Rugh, 1993) the observer problem can be transformed into a controller problem by means of the transformation $\tilde{\mathbf{A}}(t) = \mathbf{A}^T(-t)$ and $\tilde{\mathbf{B}}(t) = \mathbf{C}^T(-t)$:

$$\dot{\mathbf{x}}_1(t) = \tilde{\mathbf{A}}(t)\mathbf{x}_1(t) + \tilde{\mathbf{B}}(t)u_2(t) \tag{8}$$

With the $\mathbf{A}_1(t)$ and $\mathbf{C}_1$ matrices (5), these transformed matrices become:

$$\tilde{\mathbf{A}}(t) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & u_{1d}(-t) & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad \tilde{\mathbf{B}}(t) = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{9}$$

System (9) can be transformed, using $\mathbf{x}_1(t) = \mathbf{P}\mathbf{z}(t)$ (if the matrix $\mathbf{P}$ is invertible), into:

$$\dot{\mathbf{z}}(t) = (\mathbf{P}^{-1}\tilde{\mathbf{A}}(t)\mathbf{P})\mathbf{z}(t) + \mathbf{P}^{-1}\tilde{\mathbf{B}}(t)u_2(t). \tag{10}$$

Choosing $\mathbf{P}$ as follows:

$$\mathbf{P} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} = \mathbf{P}^{-1} \tag{11}$$

and define the time reversed input $\alpha(t) = u_{1d}(-t)$, the system

$$\dot{\mathbf{z}}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha(t) & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}\mathbf{z}(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}u_2(t), \tag{12}$$

is obtained, which formally resembles the control system $(\mathbf{A}_1, \mathbf{B}_1)$ in (5). Hence the dual control system (12) can be stabilized by the linear state feedback $u_2(t) = \tilde{\mathbf{K}}_1(t)\mathbf{z}(t)$ proposed in (Aneke *et al.*, 2003b) where now $\tilde{\mathbf{K}}_1(t)$ is $\mathbf{K}_1(t)$ computed along $\alpha(t) = u_{1d}(-t)$. Basically this amounts to using (3) in the dual setting. Under the assumption that the function $\alpha(t)$ is uniformly bounded in t, continuously differentiable and persistently exciting, system (12) is GUES. The closed loop system

$$\dot{\mathbf{z}}(t) = (\mathbf{P}^{-1}\tilde{\mathbf{A}}(t)\mathbf{P} - \mathbf{P}^{-1}\tilde{\mathbf{B}}\tilde{\mathbf{K}}_1(t))\mathbf{z}(t) \qquad (13)$$

may be transformed back to (9) with the feedback matrix $\tilde{\mathbf{K}}(t) = \tilde{\mathbf{K}}_1(t)\mathbf{P}^{-1}$ yielding the closed loop system

$$\dot{\mathbf{x}}_1(t) = (\tilde{\mathbf{A}}(t) - \tilde{\mathbf{B}}\tilde{\mathbf{K}}(t))\mathbf{x}_1(t). \qquad (14)$$

By duality (Rugh, 1993, Theorem 15.2), it is concluded that the error dynamics

$$\dot{\mathbf{e}}(t) = (\mathbf{A}_1(t) - \mathbf{L}_1(t)\mathbf{C}_1)\mathbf{e}(t) \qquad (15)$$

are GUES if the observer gain $\mathbf{L}_1(t)$ is chosen according to $\mathbf{L}_1(t) = \tilde{\mathbf{K}}^T(-t)$ yielding

$$\begin{aligned} L_{11}(t) &= (l_5 + l_6)u_{1d}^2 + (l_3 + l_4), \\ L_{12}(t) &= l_5 l_6 u_{1d}^4 + (l_3 + l_4)(l_5 + l_6)u_{1d}^2 \\ &\quad -(5l_5 + 3l_6)\dot{u}_{1d}u_{1d} + l_3 l_4, \\ L_{13}(t) &= l_5 l_6 (l_3 + l_4)u_{1d}^3 + (l_5 + l_6)l_3 l_4 u_{1d} \\ &\quad +(5l_5 + l_6)u_{1d}^{(2)} - (6 l_5 l_6 u_{1d}^2 \\ &\quad +(l_3 + l_4)(3l_5 + l_6))\dot{u}_{1d}, \\ L_{14}(t) &= l_5 l_6 l_3 l_4 u_{1d}^3 - 2l_5 u_{1d}^{(3)} \\ &\quad +(3 l_5 l_6 u_{1d}^2 + 2l_5(l_3 + l_4))u_{1d}^{(2)} \\ &\quad -(3 l_5 l_6 (l_3 + l_4)u_{1d}^2 + 2 l_5 l_3 l_4)\dot{u}_{1d} \\ &\quad +6 l_5 l_6 u_{1d}\dot{u}_{1d}^2, \end{aligned} \qquad (16)$$

in which $u_{1d}^{(k)}$ denotes the k-th derivative of u_{1d}. This clearly is dual to the derivation of $\mathbf{K}_1(t)$, see (3).

3.2 An observer for the Δ_3 subsystem

The following full order observer for the LTI system (6) is proposed:

$$\begin{aligned} \dot{\hat{\mathbf{x}}}_2(t) &= (\mathbf{A}_2 - \mathbf{L}_2\mathbf{C}_2)\hat{\mathbf{x}}_2(t) \\ &\quad +\mathbf{B}_2 u_1(t) + \mathbf{L}_2 y_2(t) \end{aligned} \qquad (17)$$

where $\hat{\mathbf{x}}_2 = [\hat{x}_{x11}\ \hat{x}_{x12}]$ and $y_2 = x_{11} = \xi_1 - \xi_{1d}$, with linear error dynamics

$$\dot{\mathbf{e}}(t) = (\mathbf{A}_2 - \mathbf{L}_2\mathbf{C}_2)e(t). \qquad (18)$$

The system is completely observable and the error dynamics (18) can be made exponentially stable by choosing the matrix $\mathbf{L}_2$ such that $(\mathbf{A}_2 - \mathbf{L}_2\mathbf{C}_2)$ is Hurwitz.

4. STABILITY ANALYSIS

4.1 Cascaded systems

Consider the system

$$\begin{aligned} \dot{\mathbf{z}}_1 &= f_1(t, \mathbf{z}_1) + g(t, \mathbf{z}_1, \mathbf{z}_2)\mathbf{z}_2 \\ \dot{\mathbf{z}}_2 &= f_2(t, \mathbf{z}_2) \end{aligned} \qquad (19)$$

where $\mathbf{z}_1 \in \mathbb{R}^n, \mathbf{z}_2 \in \mathbb{R}^m, f_1(t, \mathbf{z}_1)$ is continuously differentiable in $(t, \mathbf{z}_1)$ and $f_2(t, \mathbf{z}_2), g(t, \mathbf{z}_1, \mathbf{z}_2)$ are continuous in their arguments, and locally Lipschitz in $\mathbf{z}_2$ and $(\mathbf{z}_1, \mathbf{z}_2)$, respectively. The system (19) can be viewed as the system

$$\Sigma_1 : \dot{\mathbf{z}}_1 = f_1(t, \mathbf{z}_1) \qquad (20)$$

that is perturbed by the state of the system

$$\Sigma_2 : \dot{\mathbf{z}}_2 = f_2(t, \mathbf{z}_2) \qquad (21)$$

When Σ_2 is asymptotically stable, $\mathbf{z}_2$ tends to zero, which suggests that, eventually, the $\mathbf{z}_1$ dynamics in (19) reduces to Σ_1. Therefore asymptotic stability of both Σ_1 and Σ_2 implies asymptotic stability of (19). This is not true in general. However, global uniform asymptotic stability (GUAS) of (19) is proved in (Lefeber *et al.*, 2000, Theorem 2.7) under three assumptions.

4.2 Stability of the designed system

The closed loop systems (2), consisting of the controllers (3) and the described estimators, can be expressed in the cascaded form (19) by setting

$$\mathbf{z}_1 = [x_{31}, x_{32}, x_{21}, x_{22}, \tilde{x}_{31}, \tilde{x}_{32}, \tilde{x}_{21}, \tilde{x}_{22}]^T \qquad (22)$$

$$\mathbf{z}_2 = [x_{11}, x_{12}, \tilde{x}_{11}, \tilde{x}_{12}]^T \qquad (23)$$

$$f_1(t, \mathbf{z}_1) = \begin{bmatrix} \mathbf{A}_1(t) - \mathbf{B}_1\mathbf{K}_1(t) & \mathbf{B}_1\mathbf{K}_1(t) \\ 0 & \mathbf{A}_1(t) - \mathbf{L}_1(t)\mathbf{C}_1 \end{bmatrix} \mathbf{z}_1 \qquad (24)$$

$$f_2(t, \mathbf{z}_2) = \begin{bmatrix} \mathbf{A}_2 - \mathbf{B}_2\mathbf{K}_2 & \mathbf{B}_2\mathbf{K}_2 \\ 0 & \mathbf{A}_2 - \mathbf{L}_2\mathbf{C}_2 \end{bmatrix} \mathbf{z}_2 \qquad (25)$$

$$g(t, \mathbf{z}_1, \mathbf{z}_2) = (x_{21} + \xi_{2d}) \begin{bmatrix} 0 & 0 & 0 & 0 \\ -k_1 & -k_2 & k_1 & k_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad (26)$$

where $\tilde{x}_{ij} = x_{ij} - \hat{x}_{ij}$.

Verifying the three assumptions stated in (Lefeber *et al.*, 2000, Theorem 2.7):

(1) Assumption on Σ_1: Due to the assumption that u_{1d} is uniformly bounded in t, continuously differentiable and persistently exciting, it is proved that $\mathbf{A}_1(t) - \mathbf{L}_1(t)\mathbf{C}_1$ is GUES and it was already proved that $\mathbf{A}_1(t) - \mathbf{B}_1\mathbf{K}_1(t)$ is GUES by (Aneke, 2003a). When $\mathbf{A}_1(t) - \mathbf{B}_1\mathbf{K}_1(t)$ and $\mathbf{A}_1(t) - \mathbf{L}_1(t)\mathbf{C}_1$ are GUES then the subsystem (24) is GUES if the term $\mathbf{B}_1\mathbf{K}_1(t)$ is bounded. Under the assumption that the signals $u_{1d}(t)$, $\dot{u}_{1d}(t)$, $\ddot{u}_{1d}(t)$, $u_{1d}^{(3)}(t)$ are bounded the term $\mathbf{B}_1\mathbf{K}_1(t)$ is bounded. Hence subsystem Σ_1 is GUES.

(2) Assumption on the connection term: By assumption the signal ξ_{2d} is bounded, i.e., $|\xi_{2d}(t)| \leq M \ \forall\ t \geq 0$. Therefore it holds that

$$\begin{aligned} \|g(t, \mathbf{z}_1, \mathbf{z}_2)\| &\leq \|k\|(|x_{21}| + M) \\ \|g(t, \mathbf{z}_1, \mathbf{z}_2)\| &\leq \|k\|M + \|k\|\|\mathbf{z}_1\| \end{aligned} \tag{27}$$

where $\|k\| = [k_1, k_2]$.

(3) Assumption on Σ_2: The characteristic polynomial of the Σ_2 subsystem is given by

$$\begin{aligned} & det[\lambda\mathbf{I} - \mathbf{A}_2 + \mathbf{B}_2\mathbf{K}_2] \cdot \\ & \quad det[\lambda\mathbf{I} - \mathbf{A}_2 + \mathbf{L}_2\mathbf{C}_2] \end{aligned} \tag{28}$$

So the $2 \times n$ eigenvalues of the closed loop system are given by the n eigenvalues of the observer and the n eigenvalues that would be obtained by linear state feedback. Because the system is controllable and observable the two characteristic polynomials can both be chosen to be Hurwitz in which case the Σ_2 subsystem becomes GES.

Therefore GUAS is concluded for the complete controller/observer design.

5. EXPERIMENTAL RESULTS

To validate the controller/observer design an underactuated H-drive manipulator is used. The H-drive, as seen in figures 1 and 2, consists of two parallel Y-axes, that are connected to the X-axis by two joints. An additional link, with encoder for measuring the link orientation θ, is mounted on top of the X-sledge along the X-axis to make the system underactuated. The origin is located near the center of the H-drive, the generalized coordinates, i.e., $\mathbf{q} = [r_x, r_y, \theta]$ are given by the joint coordinates and orientation of the link. The system has three inputs, i.e., the currents i_X, i_{Y1} and i_{Y2}, and four position coordinates, i.e., the positions $X, Y1, Y2$ and the rotation of the link θ. The mass and inertia of the link are denoted by m_3 and I_3 respectively. The masses of the

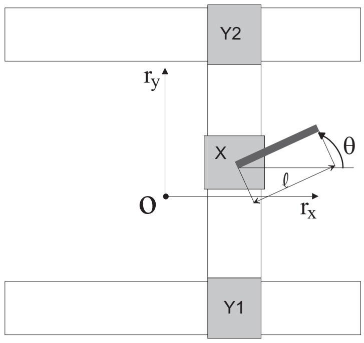


Fig. 1. H-drive with gen. coordinates $[r_x \ r_y \ \theta]$.

two Y-sledges, X-sledge and X-beam are defined as m_{Y1}, m_{Y2}, m_X and m_B respectively. The positions $Y1(t)$ and $Y2(t)$ will be controlled to follow the same reference position. It is assumed that the positions $Y1$ and $Y2$ are equal. A simplified dynamical model, without friction, is given by

$$\begin{aligned} & m_x\ddot{r}_x - \frac{m_3 l}{2}\sin(\theta)\ddot{\theta} - \frac{m_3 l}{2}\cos(\theta)\dot{\theta}^2 = k_m i_Y \\ & m_y\ddot{r}_y + m_3 l\cos(\theta)\ddot{\theta} - m_3 l\sin(\theta)\dot{\theta}^2 = -k_m i_X \\ & I\ddot{\theta} - m_3 l\sin(\theta)\ddot{r}_x + m_3 l\cos(\theta)\ddot{r}_y = 0 \end{aligned} \tag{29}$$

with motor constant k_m and where $m_x = \frac{m_{Y1}+m_{Y2}}{2} + \frac{m_b}{2} + \frac{(m_x+m_3)}{2}$, $m_y = m_X + m_3$ and $I = I_3 + m_3 l^2$. The dynamical system (29) can be transformed into the second-order chained form (1) by the coordinate- and feedback transformation given in (Imura *et al.*, 1996). The relation between ξ and the generalized coordinates $\mathbf{q}$ is denoted by

$$\begin{aligned} \xi_1 &= r_x + \frac{I}{m_3 l}(\cos(\theta) - 1) \\ \xi_2 &= \tan(\theta) \\ \xi_3 &= r_y + \frac{I}{m_3 l}\sin(\theta) \end{aligned} \tag{30}$$

and the feedback transformation is given by

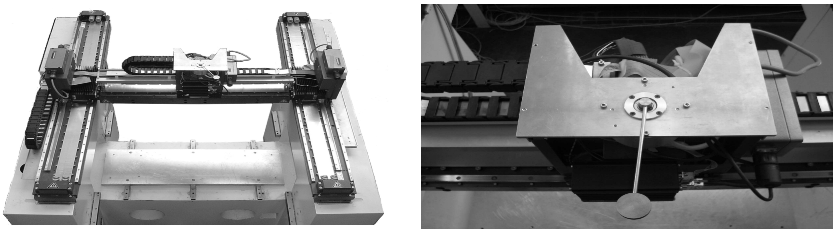
$$\begin{bmatrix} i_X \\ i_Y \end{bmatrix} = \frac{1}{k_m}\begin{bmatrix} m_3 l\sin(\theta)\dot{\theta}^2 - a\nu_x - b\nu_y \\ -\frac{m_3 l}{2}\cos(\theta)\dot{\theta}^2 + c\nu_x + d\nu_y \end{bmatrix} \tag{31}$$

where $a = \frac{m_3 l}{\lambda}\sin(\theta)\cos(\theta)$, $b = m_y - \frac{m_3 l}{\lambda}\cos^2(\theta)$, $c = m_x - \frac{m_3 l}{2\lambda}\sin^2(\theta)$, $d = \frac{m_3 l}{2\lambda}\sin(\theta)\cos(\theta)$ and $\lambda = \frac{I}{m_3 l}$. By taking the new inputs ν_x and ν_y as

$$\begin{bmatrix} \nu_x \\ \nu_y \end{bmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ \sin(\theta) & -\cos(\theta) \end{bmatrix}\begin{bmatrix} \frac{u_1}{\cos(\theta)} + \lambda\dot{\theta}^2 \\ \lambda e \end{bmatrix} \tag{32}$$

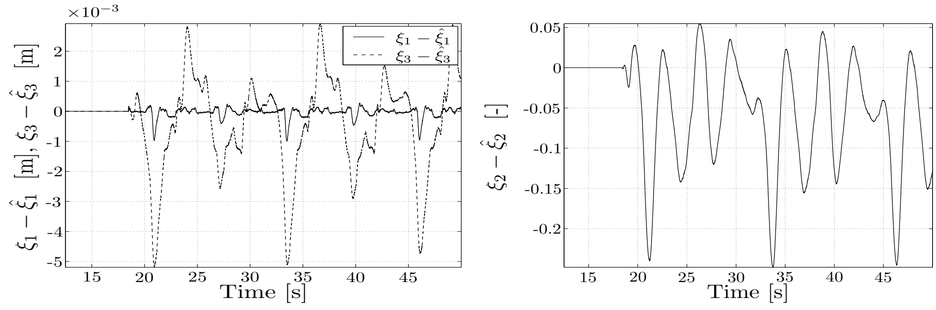
where $e = u_2\cos^2(\theta) - 2\dot{\theta}^2\tan(\theta)$, the system is transformed into the second-order chained form. It can be seen from (30) that the transformation is only valid for $\theta \in (-\pi/2, \pi/2)$.

In practice the H-drive manipulator is influenced by friction forces in the joints. It is assumed that friction, cogging, reluctance forces and the coupling of mass between the X-axis and Y-axes are



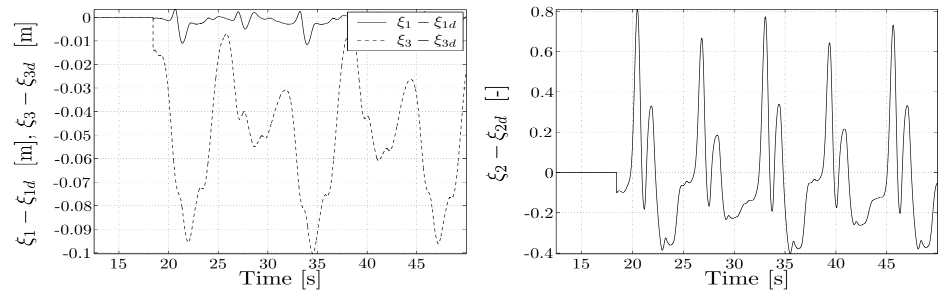
(a) The H-drive servo system. (b) The unactuated rotational link.

Fig. 2. The underactuated H-drive servo system.



(a) (b)

Fig. 3. a) Observer errors $\xi_1 - \hat{\xi}_1$ and $\xi_3 - \hat{\xi}_3$. b) Observer error $\xi_2 - \hat{\xi}_2$.



(a) (b)

Fig. 4. a) Tracking errors $\xi_1 - \xi_{1d}$ and $\xi_3 - \xi_{3d}$. b) Tracking error $\xi_2 - \xi_{2d}$.

compensated for by the servo-controllers. A so-called 'virtual internal model following control' approach is used to accomplish this. This means that the X and Y axes are controlled by a combination of a high-level controller and a low-level servo-loop. For more details about the experimental set-up the reader is referred to (Aneke *et al.*, 2004).

With an additional friction term for the rotational link the dynamical model (29) changes to

$$\begin{aligned} m_x \ddot{r}_x - \frac{m_3 l}{2}\sin(\theta)\ddot{\theta} - \frac{m_3 l}{2}\cos(\theta)\dot{\theta}^2 &= k_m \tilde{i}_Y \\ m_y \ddot{r}_y + m_3 l\cos(\theta)\ddot{\theta} - m_3 l \sin(\theta)\dot{\theta}^2 &= -k_m \tilde{i}_X \\ I\ddot{\theta} - m_3 l \sin(\theta)\ddot{r}_x + m_3 l\cos(\theta)\ddot{r}_y &= \tau_{f,\theta}(\dot{\theta}). \end{aligned} \quad (33)$$

Note that friction in the r_x and r_y direction is directly compensated in $\tilde{i}_X = i_X + \tau_{f,y}(\dot{r}_y)$ and $\tilde{i}_Y = i_Y + \tau_{f,x}(\dot{r}_x)$. Using the coordinate- and feedback transformation presented in (Imura *et al.*, 1996) in this dynamical system a perturbed second-order chained form is obtained

$$\begin{aligned} \ddot{\xi}_1 &= u_1 + \Gamma_1(\xi_2, \dot{\xi}_2) \\ \ddot{\xi}_2 &= u_2 + \Gamma_2(\xi_2, \dot{\xi}_2) \\ \ddot{\xi}_3 &= \xi_2 u_1 + \Gamma_3(\xi_2, \dot{\xi}_2), \end{aligned} \quad (34)$$

where the perturbation terms due to the friction in the unactuated link are given by

$$\begin{aligned} \Gamma_1 &= -\frac{\xi_2}{\sqrt{1+\xi_2^2}} \frac{\tau_{f,\theta}(\xi_2, \dot{\xi}_2)}{m_3 l} \\ \Gamma_2 &= (1+\xi_2^2)\frac{\tau_{f,\theta}(\xi_2, \dot{\xi}_2)}{I} \\ \Gamma_3 &= \frac{1}{\sqrt{1+\xi_2^2}} \frac{\tau_{f,\theta}(\xi_2, \dot{\xi}_2)}{m_3 l}. \end{aligned} \quad (35)$$

Therefore the observer is modified, by adding the coupling term $(\hat{x}_{21} + \xi_{2d})(u_1 - u_{1d})$, see (2), in equation (7) and by estimations of the Γ functions (35), to cope with the friction. The friction term in (33) is approximated by the following model

$$\tau_{f,\theta} = -c_s \frac{2}{\pi}\arctan(100 \cdot \dot{\theta}) - c_v \dot{\theta} \quad (36)$$

Table 1. Parameters.

Controller	Observer	Reference
$k_1 = 4$	$l_1 = 20$	$u_{1d} = -0.4\cos(t)$
$k_2 = 2\sqrt{2}$	$l_2 = 100$	$u_{2d} = 0$
$k_3 k_4 = 40$	$l_3 l_4 = 4.5$	$\xi_{1d} = 0.4\cos(t)$
$k_3 + k_4 = 9$	$l_3 + l_4 = 3$	$\xi_{2d} = 0$
$k_5 = 5$	$l_5 = 10$	$\xi_{3d} = 0$
$k_6 = 100$	$l_6 = 50$	

where c_s and c_v denote the static and viscous friction coefficients respectively. By doing this the global stability is not guaranteed anymore. A form of practical tracking is obtained, as presented in (Aneke, 2003a), the tracking and observer errors are globally uniformly ultimately bounded.

The control and observer parameters and the reference trajectories used in experiments are given in table 5.1. An initial tracking error is given by setting the angle θ of the link at approximately $-6°$, while the initial observer error is set to zero to avoid peaking. This is done because huge inputs could cause the link to pass through $\pm\frac{\pi}{2}$ and then the transformation (30) into the second-order chained form is not possible anymore.

Results of an experiment are shown in figures 3 and 4. The desired trajectories are started after about 18 seconds. It can be seen that both the tracking and observer errors are bounded. The size of the bounds in the observer errors can be influenced by the static and viscous friction parameters (36). The infinity norm of the tracking errors, [0.01 0.81 0.10], obtained with this controller/observer combination are comparable with the results of the state feedback controller in (Aneke, 2003a).

6. CONCLUSIONS

In this paper theoretic and experimental results for output based tracking control of an underactuated manipulator are presented. The underactuated H-drive manipulator can be transformed into a so-called second-order chained form by a coordinate- and feedback transformation. An observer is used to solve the output feedback tracking problem for systems in second-order chained form. For the designed observer global stability of the closed loop system is proved. The controller can be used for tracking problems for systems with a second-order nonholonomic constraint that can be transformed into the second-order chained form, under the condition that the desired trajectory does not converge to a point.

A heuristic modification of the observer is made to cope with friction in the rotational link. Although global stability is no longer guaranteed the tracking and observer errors are lower and upper bounded. Experimental results on the underactuated H-drive manipulator show the performance of the output feedback controller.

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