SIMULTANEOUS IDENTIFICATION OF TIME-VARYING PARAMETERS AND ESTIMATION OF SYSTEM STATES USING ITERATIVE LEARNING OBSERVER

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Abstract: This paper presents the design of an Iterative Learning Observer (ILO) for the purpose of estimating system states while simultaneously identifying time-varying parameters. The proposed ILO uses a novel updating mechanism to identify time-varying parameters instead of using integrators which are commonly used in classical adaptive observers to identify constant parameters while estimating system states. The main idea behind the design of the ILO is the use of *learning*, i.e. previous information is combined into the ILO for identifying online time-varying parameters. Stability of estimation error dynamics and convergence of parameter estimation error are established and proven. An illustrative example exhibits the effectiveness of the ILO.

Keywords: Iterative learning observer, identification of time-varying parameters, adaptive observers, learning mechanism, simulation.

1. INTRODUCTION

An observer, which is driven by measurable system outputs and inputs, can achieve state reconstruction for the purpose of control applications when the entire state vector is not available for the use of feedback. To perform the twin tasks of state estimation and parameter identification, adaptive observers have been extensively studied since 1970s (Bastin & Gevers, 1988; Friedland, 1997; Ioannou & Kokotovic, 1982; Kreisselmeier, 1977; Luders & Narendra, 1973; Marino at al., 2001; Rimon & Narendra, 1992; Zhang, 2002). Successful stories in identifying constant or slowly time-varying parameters have been reported by Bastin & Gevers (1988). The main principle is to combine integrators into a Luenberger observer (Luenberger, 1966) in order to identify constant parameters while estimating system states. Many applications, however, involve time-varying parameters acting as disturbances or/and uncertainties. Estimation of constant parameters may not be sufficient for applications. It is highly desired to identify time-varying parameters for the purpose of feedback control and other applications.

In this paper, an Iterative Learning Observer (ILO) is proposed to alleviate the constraint of identifying constant and slowly time-varying parameters by adaptive observers. The ILO was first suggested by Chen & Saif (2002) to achieve detection and estimation of constant faults. The purpose of this work is to explore its ability of identifying time-varying parameters that may be periodic or aperiodic signals while estimating system states. The concept of *learning* is used as updating mechanism for parameter estimation in

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the proposed ILO, i.e. the previous information is used to adjust the parametric update law online.

The remainder of this paper is organized as follows: in Section 2, the problems are stated and the considered system is formulated. The main results are presented in Section 3 where an ILO is proposed, and stability, as well as convergence is also proven. An example is used to illustrate effectiveness of the ILO in estimating system states and simultaneously identifying time-varying parameter in Section 4. Finally, conclusions are drawn in Section 5.

2. PROBLEM STATEMENT

Consider a linear system described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + F\theta(t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where $x(t) \in \Re^n$ is the system state, $y(t) \in \Re^p$ is the output, $u(t) \in \Re^m$ is the control input, $\theta(t) \in \Re^q$ is the unknown time-varying parameter that may be constant, periodic or aperiodic signals, $A \in \Re^{n\times n}$, $B \in \Re^{n\times m}$, $F \in \Re^{n\times q}$, and $C \in \Re^{p\times n}$ are constant matrices. It is assumed that the pair $\{A, C\}$ is detectable, that $\theta(t)$ may vary quickly, and that the derivative of $\theta(t)$ may not exist at some time instants.

Throughout this paper, the following assumption is made

Assumption 1. The time-varying signal $\theta(t)$ is bounded.

Remark 1. Assumption 1 is the only requirement for identifying the time-varying parameter $\theta(t)$.

The objective of this paper is to design an ILO for estimating system states and simultaneously identifying time-varying parameters that act as disturbances or uncertainties.

An adaptive observer is an excellent tool for reconstructing system states and unknown constants or slowly time-varying parameters. Integrators are employed to update parameter estimation laws, which has turned out to be effective for constant parameters. However, derivatives of time-varying parameters are usually not available for the use of integrators and they may not exist at some time instants. To break through these constraints caused by the integrators, this work proposes an ILO to identify time-varying parameters while system states are being estimated.

3. MAIN RESULTS

In this section, the ILO will be presented and its stability as well as convergence of parameter estimation, will be proven.

3.1 Design of ILO

Using the concept of learning, an ILO for dealing with time-varying parameters is proposed. This is in contrast with the use of integrators in classical adaptive observers. Time-varying parameters will be learnt online by an additional term in the ILO, and their effects on state estimation errors will be accordingly attenuated by their estimates. In line with this, an ILO is designed as follows:

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + Bu(t) + L(y(t) - \hat{y}(t)) + Fv(t) \\ \hat{y}(t) &= C\hat{x}(t) \\ v(t) &= K_1 v(t-\tau) + K_2(y(t) - \hat{y}(t)) \end{aligned} \tag{2}$$

where $\hat{x}(t)$ and $\hat{y}(t)$ are the estimated state and output. The matrix L can be selected such that $(A - LC)$ is stable. The signal $v(t)$ is used to estimate time-varying parameter $\theta(t)$. It is updated online by both its previous information and output estimation errors. The parameter $\tau > 0$ is the updating interval. It may be chosen as the sampling-time interval in a sampled-data control system, or as an integer multiple of the sampling-time interval. A better choice of parameter τ is the sampling interval because a larger τ may lead to inaccurate estimate when the estimated parameter $\theta(t)$ varies quickly. The gain matrices are $K_1 \in \Re^{q\times q}$ and $K_2 \in \Re^{q\times p}$.

System state estimation error can be obtained by subtracting (2) from (1) as follows:

$$\begin{aligned} \dot{\tilde{x}}(t) &= (A - LC)\tilde{x}(t) + F(\theta(t) - v(t)) \\ \tilde{y}(t) &= C\tilde{x}(t) \end{aligned} \tag{3}$$

where $\tilde{x}(t) = x(t) - \hat{x}(t)$ and $\tilde{y}(t) = y(t) - \hat{y}(t)$.

3.2 Analysis of Convergence and Stability

The following lemma will be used in the proof of Theorem 1.

Lemma 1. If the parameter update law $v(t)$ is designed in (2), the following equation then holds

$$\begin{aligned} e^\top(t)e(t) = &\; e^\top(t-\tau)K_1^\top K_1 e(t-\tau) \\ &+\tilde{x}^\top(t)(K_2C)^\top K_2C\tilde{x}(t) \\ &-2\tilde{x}^\top(t)(K_2C)^\top K_1 e(t-\tau) \\ &+2e^\top(t-\tau)K_1^\top d(t) \\ &-2\tilde{x}^\top(t)(K_2C)^\top d(t) + d^\top(t)d(t) \end{aligned} \tag{4}$$

where $e(\cdot) = \theta(\cdot) - v(\cdot)$ and $d(t) = \theta(t) - K_1\theta(t - \tau)$.

Proof: Starting with

$$\begin{aligned} e(t) &= \theta(t) - K_1 v(t-\tau) - K_2 C\tilde{x}(t) \\ &= K_1 e(t-\tau) - K_2 C\tilde{x}(t) + d(t), \end{aligned} \tag{5}$$

it is straightforward to have

$$\begin{aligned} e^\top(t)e(t) &= [K_1 e(t-\tau) - K_2 C\tilde{x}(t) + d(t)]^\top \\ &\quad [K_1 e(t-\tau) - K_2 C\tilde{x}(t) + d(t)] \\ &= e^\top(t-\tau)K_1^\top K_1 e(t-\tau) \\ &\quad +\tilde{x}^\top(t)(K_2C)^\top K_2 C\tilde{x}(t) \\ &\quad -2\tilde{x}^\top(t)(K_2C)^\top K_1 e(t-\tau) \\ &\quad +2e^\top(t-\tau)K_1^\top d(t) \\ &\quad -2\tilde{x}^\top(t)(K_2C)^\top d(t) + d^\top(t)d(t). \end{aligned} \tag{6}$$

∎

Remark 2. If $\theta(t)$ is a constant, then $d(t)$ can be zero by selecting $K_1 = I$, an identity matrix. As a result, (4) can be simplified to

$$\begin{aligned} e^\top(t)e(t) &= e^\top(t-\tau)e(t-\tau) \\ &\quad +\tilde{x}^\top(t)(K_2C)^\top(K_2C)\tilde{x}(t) \\ &\quad -2\tilde{x}^\top(t)(K_2C)^\top e(t-\tau). \end{aligned} \tag{7}$$

Remark 3. Rewrite state estimation error equation by combining (5) into (3) as follows

$$\begin{aligned} \dot{\tilde{x}}(t) &= (A-LC)\tilde{x}(t) + FK_1 e(t-\tau) \\ &\quad -FK_2C\tilde{x}(t) + Fd(t). \end{aligned} \tag{8}$$

This will be used in the proof of Theorem 1.

Theorem 1. (Boundedness). Consider system (1) satisfying Assumption 1 and the ILO is designed in (2). If there exists a positive definite matrix P satisfying $(A-LC)^\top P + P(A-LC) = -Q$, where Q is a positive definite matrix, and K_1 and K_2 can be selected such that $0 < \alpha K_1^\top K_1 \leq I$ and $PF = \epsilon(K_2C)^\top$, where $\alpha > \epsilon > 1$, then both state estimation error and parameter estimation error are bounded.

Proof: Consider the following Lyapunov function candidate

$$V(\tilde{x}, e) = \tilde{x}^\top(t)P\tilde{x}(t) + \int_{t-\tau}^{t} e^\top(s)e(s)ds. \tag{9}$$

Substituting (8) into the derivative of the Laypunov candidate leads to

$$\begin{aligned} \dot{V} &= \tilde{x}^\top[(A-LC)^\top P + P(A-LC)]\tilde{x} \\ &\quad +2\tilde{x}^\top(t)PFK_1e(t-\tau) \\ &\quad -2\tilde{x}^\top(t)PFK_2C\tilde{x}(t) \\ &\quad +2\tilde{x}^\top(t)PFd(t) - \varepsilon e^\top(t)e(t) \\ &\quad +\epsilon e^\top(t)e(t) - e^\top(t-\tau)e(t-\tau) \end{aligned} \tag{10}$$

where ε is a positive constant and $\epsilon = 1 + \varepsilon$.

Using Lemma 1, (11) can be extended as

$$\begin{aligned} \dot{V} &= \tilde{x}^\top[(A-LC)^\top P + P(A-LC)]\tilde{x} \\ &\quad +2\tilde{x}^\top(t)PFK_1e(t-\tau) \\ &\quad -2\tilde{x}^\top(t)PFK_2C\tilde{x}(t) \\ &\quad +2\tilde{x}^\top(t)PFd(t) - \varepsilon e^\top(t)e(t) \\ &\quad +\epsilon e^\top(t-\tau)K_1^\top K_1 e(t-\tau) + \epsilon d^\top(t)d(t) \\ &\quad +\epsilon\tilde{x}^\top(t)(K_2C)^\top K_2C\tilde{x}(t) \\ &\quad -2\epsilon\tilde{x}^\top(t)(K_2C)^\top K_1 e(t-\tau) \\ &\quad +2\epsilon e^\top(t-\tau)K_1^\top d(t) \\ &\quad -2\epsilon\tilde{x}^\top(t)(K_2C)^\top d(t) \\ &\quad -e^\top(t-\tau)e(t-\tau). \end{aligned} \tag{11}$$

For $Q = Q^\top > 0$, there exists a $P = P^\top > 0$ satisfying $(A-LC)^\top P + P(A-LC) = -Q$, and K_2 can be selected such that $PF = \epsilon(K_2C)^\top$, (11) is simplified to

$$\begin{aligned} \dot{V} &\leq -\lambda_{min}(Q)\|\tilde{x}\|^2 - \varepsilon\|e(t)\|^2 \\ &\quad -e^\top(t-\tau)e(t-\tau) \\ &\quad +\epsilon e^\top(t-\tau)K_1^\top K_1 e(t-\tau) \\ &\quad +2\epsilon e^\top(t-\tau)K_1^\top d(t) + \epsilon k_d^2 \end{aligned} \tag{12}$$

where k_d is the norm bound of $d(t)$.

By considering the following inequality

$$\begin{aligned} 2\epsilon e^\top(t-\tau)K_1^\top d(t) &\leq \gamma e^\top(t-\tau)K_1^\top K_1 e(t-\tau) \\ &\quad +\frac{\epsilon^2}{\gamma}d^\top(t)d(t), \quad \gamma > 0, \end{aligned} \tag{13}$$

inequality (12) can be rearranged as follows

$$\begin{aligned} \dot{V} &\leq -\lambda_{min}(Q)\|\tilde{x}\|^2 - \varepsilon\|e(t)\|^2 \\ &\quad +e^\top(t-\tau)(\alpha K_1^\top K_1 - I)e(t-\tau) + \beta k_d^2 \end{aligned} \tag{14}$$

where $\alpha = \epsilon + \gamma$, $\beta = \epsilon + \frac{\epsilon^2}{\gamma}$, and I is an identity matrix.

If $0 < \alpha K_1^\top K_1 \leq I$, then both system state estimation error $\tilde{x}(t)$ and estimation error $e(t)$ are bounded. ∎

Remark 4. Theorem 1 has suggested a method for selecting K_1 and K_2.

Remark 5. If K_1 is designed as a zero matrix, the ILO will then lose the capability of identifying time-varying parameters.

Remark 6. When the parameter $\theta(t)$ is a constant, state estimation error $\tilde{x}(t)$ and parameter estimation error $e(t)$ can be easily proven to converge to zero by substituting (7) into (10). As such, the ILO can also achieve the same task as a classical adaptive observer does. However, the ILO consumes less computing power than the adaptive observer because an algebraic equation is used for parameter identification in the ILO while integrators are the updating mechanism in the adaptive observer.

4. AN ILLUSTRATIVE EXAMPLE

The following example illustrates the effectiveness of the ILO for estimating system states while simultaneously identifying time-varying parameters.

Consider an airplane model (Zhang & Jiang, 2003) with

$$A = \begin{bmatrix} -3.598 & 0.1968 & -35.18 & 0 \\ -0.0377 & -0.3576 & 5.884 & 0 \\ 0.0688 & -0.9957 & -0.2163 & 0.0733 \\ 0.9947 & 0.1027 & 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 14.65 & 6.538 \\ 0.2179 & -3.087 \\ -0.0054 & 0.0516 \\ 0 & 0 \end{bmatrix}, \text{ and}$$

$$F = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

A constant parameter $\theta(t) = 1$ is first employed to show the effectiveness of the ILO for both state and parameter estimation. Figure 1 exhibits that four state estimation errors all converge to zero under the condition that gain matrix $K_1 = 1$, and Figure 3 demonstrates accurate parameter estimation by the ILO. These verify that the proposed ILO can achieve the same task as a classical adaptive observer does.

A sinusoidal parameter $sin(4t)$ is further used to test the ILO. Figure 2 shows convergence of state estimation errors. This verifies that the accurate parameter estimate $v(t)$, which is generated from the ILO, has counteracted the effect of the sinusoidal signal on state estimation errors. Figure 3 clearly reveals that the sinusoidal signal is accurately reconstructed.

A more general time-varying parameter is employed to show parameter estimation ability of the ILO. Figure 5 displays system state estimation errors where the effect of the time-varying parameter on estimation errors has been attenuated by the parameter estimation $v(t)$ that is demonstrated in Figure 4. This verifies the effectiveness of the ILO for estimating states while simultaneously identifying the time-varying parameter.

5. CONCLUSIONS

This paper has proposed an ILO for estimating system states while simultaneously identifying time-varying parameters. In order to circumvent the deficiency of identifying constant parameters while estimating system states by classical adaptive observers, the concept of integrators is abandoned and a *learning* mechanism is adopted to design the ILO, i.e. the previous information is used to adjust the updating law online . The only *a priori* required for designing the ILO is the bound of time-varying parameters. By virtue of the proposed ILO, system state estimation errors and parameter estimation errors are convergent and the effect of time-varying parameter on state estimation errors is successfully attenuated by the accurate parameter estimates. The illustrative example has clearly demonstrated the effectiveness of the ILO for identifying time-varying parameters which include constant, periodic and aperiodic signals.

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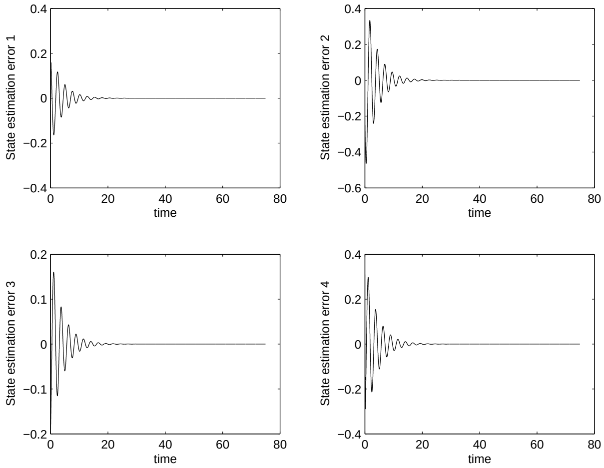

Fig. 1. Case I: a constant parameter.

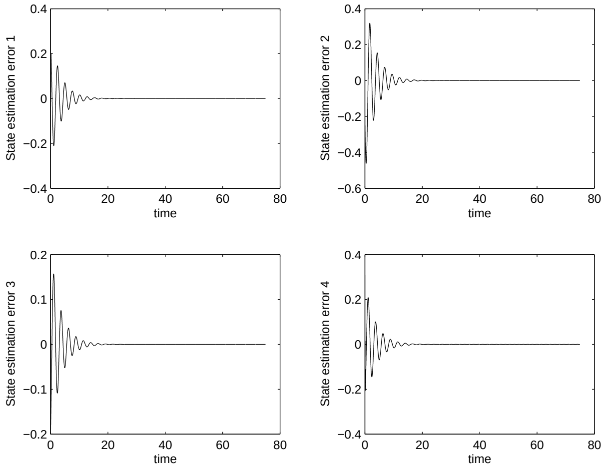

Fig. 2. Case II: a periodic signal

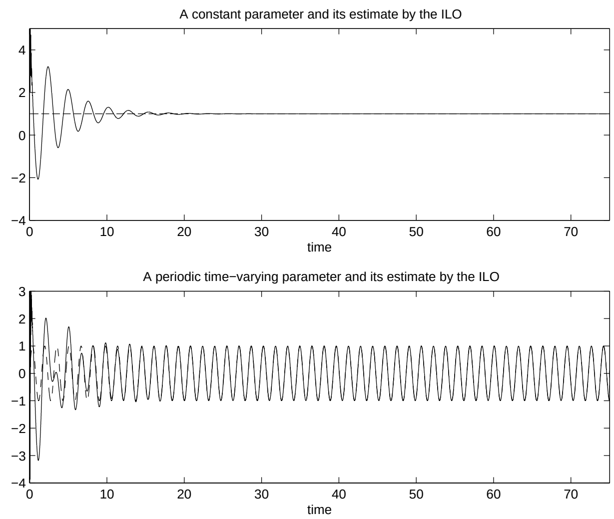

Fig. 3. Parameter estimation by the ILO (The solid line is the estimate).

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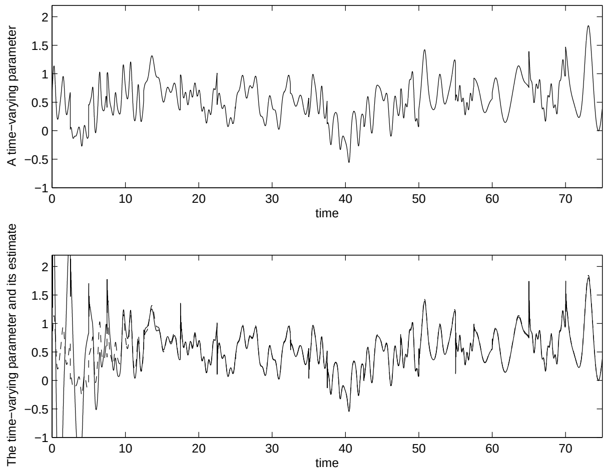

Fig. 4. A time-varying parameter and its estimate (The solid line is the estimate).

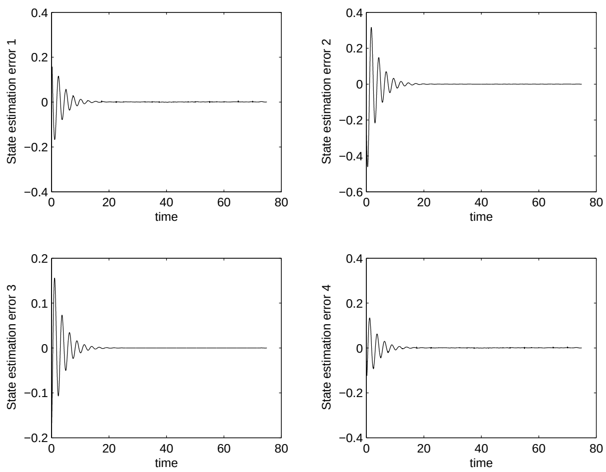

Fig. 5. System state estimation errors.