# ROBUST ADAPTIVE FUZZY CONTROL BASED ON GENERALIZED FUZZY HYPERBOLIC MODEL

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Abstract: In this paper, a robust adaptive fuzzy control algorithm based on the generalized fuzzy hyperbolic model (GFHM) for nonlinear system with uncertainties is proposed. The proposed control is a smooth control with no chattering phenomena, which consists of two control terms. One is the certainty equivalent control and the other is the compensated control, which is obtained by bounded estimation in the on-line approximation of the uncertainty. The main advantage of the proposed control law is that the human knowledge about the plant under control is be used to design the controller and only one parameter vector in the adaptive mechanism is on-line adjusted. 

Keywords: fuzzy control, nonlinear control systems, robust control, Lyapunov equation, uncertainty, stability analysis

## 1. INTRODUCTION

For many real systems there are highly nonlinear and with uncertainties, it is generally difficult to develop accurate mathematical models for them i.e., there are inevitable uncertainties in the constructed models. Therefore, the design of a robust controller that can deal with model uncertainties is very important.

Fuzzy system has been successfully applied to many control problems because it needs no accurate mathematical models of the system under control and it can cooperate with human experts' knowledge. There exists voluminous literature on the subject of making use of various fuzzy control techniques for nonlinear systems, from adaptive control based fuzzy basis functions (L.-X.Wang, 1993; L.-X.Wang, 1996), to model reference fuzzy control (C.-S. Chen, et al., 1996; J.R. Layen, 1993), neural-fuzzy control (J.T. Spooner, et al., 1996), sliding model control (B. Yoo, W. Ham, 1998), and fuzzy adaptive control ( H.-J. Kang, et al., 1998; S.C.,Tong, 2000; Yansheng Yang, et al., 2002; Parka J.H., et al., 2003). However, when the dimensions of system states or the number of fuzzy rules for the description of the unknown function increase, the number of on-line adjusting parameters in the controller increases rapidly and the computing-load becomes very heavy. Meanwhile, since the fuzzy descriptions are imprecise and may be insufficient to achieve the desired accuracy, the approximation error introduced into the feedback loop makes it difficult to guarantee the stability of the closed-loop control system (L.-X.Wang, 1996). This problem was solved in (C.-S. Chen, et al., 1996; J.T. Spooner,et al., 1996) by sliding mode-like estimation of the reconstruction error bound, but nonsmooth control input is generated. In general, such discontinuous adaptive control schemes are avoided since it is well known that they

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not only create problems of existence and uniqueness of solutions (M.M. Polycarpou, et al., 1993) but are also known to display chattering phenomena and to excite high-frequency unmodeled dynamics (J.-J.E. Slotine, et al., 1991). Another problem in (L.-X.Wang, 1996; C.-S. Chen, et al., 1996; J.T. Spooner, et al, 1996) is that bounds on the unknown plant dynamics must be known. Generally, this calculation may require an exact model of the plant, which defeats the purpose of using a model-free technique.

Recently, Zhang et al. proposed a class of fuzzy model, i.e., the fuzzy hyperbolic model(FHM) and the generalized fuzzy hyperbolic model(GFHM) (H.G. Zhang, et al., 2001; H.G. Zhang, et al., 2003; H.G. Zhang, et al., 2004). It is proved that the generalized fuzzy hyperbolic model is an universal approximator and can be used to establish the model for the nonlinear dynamic systems. There are several good examples using this class of fuzzy model (H.G.Zhang, et al., 2001; H.G. Zhang, et al., 2003). The purpose of this paper is to develop a robust adaptive control algorithm using the GFHM for single-input single-output (SISO) nonlinear dynamical systems with uncertainties. We proposed a smooth control with no chattering phenomena. The overall control system guarantees that the tracking error converges in the small neighborhood of zero and that all signals involved are uniformly bounded. The main advantage of the proposed control law is that the human knowledge about the plant under control is to be used to design the controller and only one parameter vector in the adaptive mechanism is on-line adjusted.

The paper is organized as follows. In Section 2, preliminaries about the GFHM are reviewed. In Section 3, the GFHM is used to descript nonlinear systems with uncertainty. In Section 4, a robust adaptive control algorithm is proposed. In Section 5, simulation examples are provided to demonstrate the design procedure for robust adaptive controller. Finally, in Section 6, the conclusion is given.

## 2. PRELIMINARIES

In this section we review some necessary preliminaries for the GFHM.

In (H.G. Zhang, et al., 2001), the membership function of the GFHM, $P_x$ and $N_x$, is defined as:

$$\begin{aligned} \mu_{P_x}(x_z) &= e^{-\frac{1}{2}(x_z-k_z)^2}, \\ \mu_{N_x}(x_z) &= e^{-\frac{1}{2}(x_z+k_z)^2}, \end{aligned} \tag{1}$$

where $k_z > 0$. We can see that only two fuzzy sets are used to represent the input variable, and that the fuzzy sets cannot cover the whole input space. Thus, we transform the input variable $x_z$ as follows:

$$\bar{x}_j = x_z - d_{zj} \tag{2}$$

where $j = 1, \cdots, w$ ($w$ is a positive integer) and $d_{zj}$ is a constant. We can see that after the linear transformation of $x_z$, the fuzzy sets may cover the whole input space if $w$ is large enough.

*Definition 1.* (H.G. Zhang, et al., 2004) Given a plant with $n$ input variables $x = (x_1(t), ..., x_n(t))^T$ (where $x$ is any state variable or input variable), and $n$ output variable $\dot{x} = (\dot{x}_1, \cdots \dot{x}_n)^T$, define the generalized input variables as follows:

$$\begin{aligned} \bar{x}_1 &= x_1 - d_{11}, \\ &\cdots \\ \bar{x}_{w_1} &= x_1 - d_{1w_1}, \\ \bar{x}_{w_1+1} &= x_2 - d_{21}, \\ &\cdots \\ \bar{x}_{w_1+w_2} &= x_2 - d_{2w_2}, \\ &\cdots \\ \bar{x}_m &= x_n - d_{nw_n}, \end{aligned}$$

where $m = \sum_{i=1}^{n} w_i$ are the numbers of generalized input variables, $w_z (z = 1, \ldots, n)$ are the numbers to be transformed about $x_z$, $d_{zj} (z = 1, \ldots, n,\ j = 1, \ldots, w_z)$ are the constants where $x_z$ is transformed. If the fuzzy rule base satisfies the following conditions, we call it a type of generalized fuzzy hyperbolic rule base:

(1) For each output variable $\dot{x}_l, l = 1, \ldots, n$, the corresponding group of fuzzy rules has the following form:

$R^l$: IF $(x_1 - d_{11})$ is $F_{x_{11}}$ and ... and $(x_1 - d_{1w_1})$ is $F_{x_{1w_1}}$ and $(x_2 - d_{21})$ is $F_{x_{21}}$ and ... and $(x_2 - d_{2w_2})$ is $F_{x_{2w_2}}$ and ... and $(x_n - d_{nw_n})$ is $F_{x_{nwn}}$

$$\text{THEN } \dot{x}_l = c_{F_{11}} + \cdots + c_{F_{1w_1}} + c_{F_{21}} + \cdots + c_{F_{nwn}}, \tag{3}$$

where $F_{x_{zj}}$ are fuzzy sets of $x_z - d_{zj}$, which include $P_x$ (Positive) and $N_x$ (Negative) subsets. $c_{F_{zj}}$ are constants corresponding to $F_{x_{zj}}$.

(2) The constants $c_{F_{zj}}$ $(z = 1, \ldots, n,\ j = 1, \ldots, w_z)$ in the "THEN" part correspond to $F_{x_{zj}}$ in the "IF" part; that is, if there is $F_{x_{zj}}$ in the "IF" part, $c_{F_{zj}}$ must appear in the "THEN" part. Otherwise, $c_{F_{zj}}$ does not appear in the "THEN" part.

(3) There are $2^m$ fuzzy rules in the rule base, where $m = \sum_{i=1}^{n} w_i$; that is, all the possible $P_x$ and $N_x$ combinations of input variables in the "IF" part and all the linear combinations of constants in the "THEN" part.

*Lemma 1.* (H.G. Zhang, et al.,2004) For a multi-input dynamic system, $x = (x_1(t), \cdots, x_n(t))^T$ is the state variable vector, $u = (u_1, \cdots, u_p)$ is the input variable vector. If we define a generalized fuzzy hyperbolic rule base and generalized input variables as definition 1, define the membership function of the generalized input variables $P_x$ and $N_x$ as (1), then we can derive the following model:

$$
\begin{aligned}
\dot{x}_l &= \sum_{i=1}^{m} \frac{c_{P_{x_i}} e^{k_{x_i}\bar{x}_i} + c_{N_{x_i}} e^{-k_{x_i}\bar{x}_i}}{e^{k_{x_i}\bar{x}_i} + e^{-k_{x_i}\bar{x}_i}} \\
&\quad + \sum_{j=1}^{q} \frac{c_{P_{u_j}} e^{k_{u_j}\bar{u}_j} + c_{N_{u_j}} e^{-k_{u_j}\bar{u}_j}}{e^{k_{u_j}\bar{u}_j} + e^{-k_{u_j}\bar{u}_j}} \\
&= \sum_{i=1}^{m} p_i + \sum_{i=1}^{m} a_i \frac{e^{k_{x_i}\bar{x}_i} - e^{-k_{x_i}\bar{x}_i}}{e^{k_{x_i}\bar{x}_i} + e^{-k_{x_i}\bar{x}_i}} + \sum_{j=1}^{q} q_j \\
&\quad + \sum_{j=1}^{q} b_j \frac{e^{k_{u_j}\bar{u}_j} - e^{-k_{u_j}\bar{u}_j}}{e^{k_{u_j}\bar{u}_j} + e^{-k_{u_j}\bar{u}_j}} \\
&= A_0 + A_1 \tanh(K_x \bar{x}) + B \tanh(K_u \bar{u}) \\
&= F(x), \qquad (4)
\end{aligned}
$$

where $p_i = (c_{P_{x_i}} + c_{N_{x_i}})/2$, $a_i = (c_{P_{x_i}} - c_{N_{x_i}})/2$, $q_j = (c_{P_{u_j}} + c_{N_{u_j}})/2$, $b_j = (c_{P_{u_j}} - c_{N_{u_j}})/2$, $A_0 = (\sum_{i=1}^{m} p_i + \sum_{j=1}^{q} q_j) \in R^{1\times 1}$, $A_1 = [a_1, \cdots, a_m] \in R^{m\times 1}$, $B = [b_1, \cdots, b_q] \in R^{q\times 1}$, $\bar{x}_i (i = 1, \cdots, m; m = \sum_{i=1}^{n} w_i)$ is the generalized state variable after the linear transformation of $x_z (z = 1, \cdots, n)$, $\bar{u}_j (j = 1, \cdots, q; q = \sum_{l=1}^{p} r_l)$ is the generalized input variable after the linear transformation of $u_l (l = 1, \cdots, p)$, $\tanh(K_x\bar{x})$ and $\tanh(K_u\bar{u})$ are defined by $\tanh(K_x\bar{x}) = [\tanh(k_1\bar{x}_1), \cdots, \tanh(k_m\bar{x}_m)]^T$ and $\tanh(K_u\bar{u}) = \left[\tanh(k_{u_1}\bar{u}_1), \cdots, \tanh(k_{u_q}\bar{u}_q)\right]^T$, respectively; $K_x = diag[k_{x_1}, \ldots, k_{x_m}]$, $K_u = diag[k_{u_1}, \ldots, k_{u_q}]$. We call (4) the generalized fuzzy hyperbolic model (GFHM).

Let $Y$ be the space composed of all the functions in the form of the right-hand side of (4). We then have the following conclusion.

*Lemma 2.* (H.G. Zhang, et al., 2004) For any given real continuous $g(x)$ on the compact set $U \subset R^n$ and any arbitrary $\varepsilon > 0$, there exists an $F(x) \in Y$ such that

$$\sup_{x\in U} |g(x) - F(x)| < \varepsilon.$$

*Remark 1.* There are some distinguishing characteristics in the GFHM:

1) The GFHM is nonlinear model in nature. Unlike the T-S fuzzy model, which is the combination of local linear models, the GFHM is a global nonlinear model.

2) The GFHM can be proved to be an universal approximator.

3) The GFHM is a fuzzy model that can easily be derived from known linguistic information.

4) The GFHM is equivalent to a series expansion of fuzzy hyperbolic basis functions, $[1, \tanh(k_{x_1}\bar{x}_1), \cdots, \tanh(k_{x_m}\bar{x}_m), \tanh(k_{u_1}\bar{u}_1), \cdots, \tanh(k_{u_q}\bar{u}_q)]^T$. This basis function expansion is linear in its adjustable parameters; therefore, we can use the least squares algorithm to determine the parameters.

In this paper, we will design a robust adaptive fuzzy controller based on the GFHM in the form of (4).

## 3. DESCRIPTION OF NONLINEAR SYSTEMS WITH UNCERTAINTIES

Consider a class of SISO nonlinear systems with uncertainties in the following form:

$$
\begin{aligned}
&x^{(n)} = f(x, \dot{x}, \cdots, x^{(n-1)}) + g(x, \dot{x}, \cdots, x^{(n-1)})u + d, \\
&y = x,
\end{aligned} \qquad (5)
$$

where $f, g$ are unknown nonlinear functions, and $u(k) \in R$, $y(k) \in R$ are the input and output variable of the system, respectively, $x = (x, \dot{x}, \cdots x^{(n-1)})^T = (x_1, \cdots, x_n)^T \in R^n$ is the state variable vector, where $x$ is assumed to be available for measurement. $d \in R$ is a bounded uncertainty including external disturbance, unmodeled dynamic and measurement noise.

Next, we derive the fuzzy model in the form of (4) from partial knowledge about the system. If we define the generalized fuzzy hyperbolic rule base and the generalized input variables as definition 1, then we can derive the following model:

$$x^{(n)} = A_0 + A_1 \tanh(K_x\bar{x}) + B \tanh(K_u\bar{u}) + \varepsilon + d, \quad (6)$$

where $A_0 \in R^{1\times 1}$, $A_1 \in R^{m\times 1}$, $B \in R^{q\times 1}$, $\tanh(K_x\bar{x}) = [\tanh(k_1\bar{x}_1), \cdots, \tanh(k_m\bar{x}_m)]^T$, where $\bar{x} = (\bar{x}_1, \cdots, \bar{x}_m)^T$ is the generalized state variables , $\bar{x}_i (i = 1, \ldots, m, m = \sum_{i=1}^{n} w_i)$ is the generalized state variable after the linear transformation of $x_z (z = 1, \ldots, n)$, $\tanh(K_u\bar{u}) = \left[\tanh(k_{u_1}\bar{u}_1), \cdots, \tanh(k_{u_q}\bar{u}_q)\right]^T$, where $\bar{u}_j = u - d_{u_j}$ $(j = 1, \ldots, q)$ is the generalized input variable after the linear transformation of $u$, $\varepsilon$ is the model error.

We assume that the control $u$ is bounded. Since the variables of real physical systems are always bounded, it seems reasonable. After linearization (6) in $\bar{u}$ we get the following form:

$$
\begin{aligned}
x^{(n)} &= A_0 + A_1 \tanh(K_x\bar{x}) + BK_u(u - d_u) + \Delta + \varepsilon + d \\
&= \bar{A}_0 + A_1 \tanh(K_x\bar{x}) + bu + d_s,
\end{aligned} \qquad (7)
$$

where $\bar{A}_0 = A_0 - BK_u d_u$, $b = BK_u$, $\Delta$ is the linearizated error, $d_s = \Delta + \varepsilon + d$ is a combining uncertainty. Since the control $u$ is bounded, the error $\Delta$ is also bounded, $d_s$ is a bounded uncertainty.

Let $\bar{A} = [\bar{A}_0^T, A_1^T]^T$, $\bar{f}(x) = [1, \tanh(k_{x_1}\bar{x}, \cdots, \tanh(k_{x_m}\bar{x}_m)]^T$, then (7) is rewritten in the following form

$$x^{(n)} = \bar{A}\ \bar{f}(x) + bu + d_s. \qquad (8)$$

From the above derive; we can easily obtain the fuzzy model from priori knowledge about the plant such that we can incorporate priori information into later controller design.

## 4. DESIGN OF ROBUST ADAPTIVE FUZZY CONTROLLER

For a class of SISO nonlinear systems with uncertainties in the form of (8), the control objective is to force the output $y(t)=x$ to track a given bounded reference signal $y_m(t)$, under the constraint that all signals involved must be bounded.

Let $e=y_m-y=y_m-x$, $\mathbf{e}=[e,\dot{e},\cdots,e^{(n-1)}]^T$ and $\mathbf{k}=[k_n,\cdots,k_1]^T\in R^n$ be such that all roots of the polynomial $h(s)=s^n+k_1s^{n-1}+\cdots+k_n$ are in the open left-half complex plane, and choose the control law as

$$u=u_i-u_s/b, \tag{9}$$

where $u_i$ is the so-called certainty equivalent controller given by

$$u_i=\frac{1}{b}(-\bar{A}\,\bar{f}(x)+y_m^{(n)}+\mathbf{k}^T\mathbf{e}), \tag{10}$$

and $u_s$ is an additional robust control term which is mainly used to compensate the effect of uncertainty in the system. The design approach for $u_s$ is given as follows.

Substituting (9) (10) into (8), we obtain the closed-loop dynamics of the fuzzy control system as

$$\dot{\mathbf{e}}=\Lambda_c\mathbf{e}+B_cu_s-B_cd_s, \tag{11}$$

where $\Lambda_c=\begin{bmatrix}0 & 1 & 0 & \cdots & 0\\ 0 & 0 & 1 & \cdots & 0\\ & & \cdots & & \\ -k_n & -k_{n-1} & \cdots & & -k_1\end{bmatrix}$, $B_c=\begin{bmatrix}0\\0\\ \vdots\\ 1\end{bmatrix}$.

For the system (11), the uncertainty $d_s$ is a bounded function in the control engineering. In order to design robust control law, a GFHM is employed to approximate the uncertain function $d_s$. From lemma 2, we may conclude that there exists a fuzzy system (4) that can be used to approximate the uncertain function $d_s$ in (11). Hence, we can obtain the following bounded function:

$$|d_s|\le\sigma^T|\xi(x)|+\varepsilon_d, \tag{12}$$

where $\xi(x)=[1,\xi_1(x),\cdots,\xi_k(x)]^T$=[1, $\tanh(k_{d1}\bar{x}_1)$, $\cdots$, $\tanh(k_{dk}\bar{x}_k)]^T$ is the known fuzzy base function, $\sigma=[\sigma_0,\sigma_1,\cdots,\sigma_k]^T$ is the weight parameters of fuzzy system. $\varepsilon_d$ is a parameter for respecting approximating accuracy. Suppose $\sigma$ and $\varepsilon_d$ are unknown, the above bound function $d_s$ can be rewritten in the following form:

$$|d_s|\le\theta^T\psi(x), \tag{13}$$

where $\psi(x)=[1,1,|\xi_1(x)|,\cdots,|\xi_k(x)|]^T$ is a known vector and $\theta=[\varepsilon_d,|\sigma_0|,\cdots,|\sigma_k|]^T$.

For the uncertain system (11) with bound as (13), a following robust adaptive fuzzy control law is designed:

$$u_s=-\hat{\theta}\psi(x)\tanh(\hat{\theta}\psi(x)B_c^TP\mathbf{e}/\varepsilon_d), \tag{14}$$

$$\dot{\hat{\theta}}=-\lambda\hat{\theta}+r\psi(x)\left\|B_c^TP\mathbf{e}\right\|, \tag{15}$$

where $\lambda\in(0,\infty)$, $r=diag[r_1,r_2,\cdots,r_l]$, $r_i\in(0,\infty)$, $l$ is the dimension of $\theta$, $\hat{\theta}=\theta+\tilde{\theta}$, $\hat{\theta}$ is an estimate of $\theta$. $\lambda,r_i,\varepsilon_d$ are parameters determined by the designer. $P$ is a positive definite solution $P=P^T\ge0$ of the Lyapunov equation:

$$\Lambda_c^TP+P\Lambda_c+Q=0, \tag{16}$$

where $Q=Q^T\ge0$ is chosen by the designer.

The following theorem shows that the above control scheme (14)(15) is a robust adaptive fuzzy control law.

*Theorem 1.* Suppose the uncertainty in the system (11) satisfies (12) and parameters $\theta$ are unknown, then the robust adaptive tracking design described by the control scheme (9)(14) and the bounding parameter adaptive law (15) guarantees that

(a) all the signals and parameter estimates in the on-line approximation based control scheme are uniformly bounded;

(b) given any $\rho>\sqrt{\frac{\bar{\varepsilon}}{\mu}}$, there exists $T(\rho)$ such that for all $t>T$, $|e(t)|\le\rho$, where $\mu=\frac{1}{2}\min\{\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)},\lambda\}$, $\bar{\varepsilon}=\kappa\varepsilon_d+\frac{\lambda}{2r_{\min}}\|\theta\|$, $r_{\min}=\min\{r_1,\cdots,r_p\}$.

*proof*: Choosing the following Lyapunov function candidate:

$$V=\mathbf{e}^TP\mathbf{e}+\frac{1}{r}\tilde{\theta}^T\tilde{\theta}=z^T\bar{P}z=\bar{V}(z,t), \tag{17}$$

where $z=[\mathbf{e}^T,\tilde{\theta}^T]^T$ and $\bar{P}=\begin{bmatrix}P & 0\\ 0 & \frac{1}{r}\end{bmatrix}$.

The adaptive laws in (15) can be modified as

$$\dot{\tilde{\theta}}=-\lambda(\tilde{\theta}+\theta)+r\psi(x)\left\|B_c^TP\mathbf{e}\right\|. \tag{18}$$

The derivative of $V$ along the trajectory of the system is given by

$$\dot{\bar{V}}=\dot{V}=-\mathbf{e}^TQ\mathbf{e}+2\mathbf{e}^TPB_cu_s-2\mathbf{e}^TPB_cd_s+\frac{2}{r}\tilde{\theta}^T\dot{\tilde{\theta}}. \tag{19}$$

For the second and third term at the right-hand side of (19), the following equation and inequality hold:

$$2\mathbf{e}^TPB_cu_s=-2\hat{\theta}\psi(x)B_c^TP\mathbf{e}\tanh(\hat{\theta}\psi(x)B_c^TP\mathbf{e}/\varepsilon_d), \tag{20}$$

$$-2\mathbf{e}^TPB_cd_s\le2\|d_s\|\left\|B_c^TP\mathbf{e}\right\|\le2\theta\psi(x)\left\|B_c^TP\mathbf{e}\right\|. \tag{21}$$

Substituting (20) and (21) into (19), we get

$$\begin{aligned}\dot{V}\le&-\mathbf{e}^TQ\mathbf{e}+2[\theta^T\psi(x)\left\|B_c^TP\mathbf{e}\right\|-\hat{\theta}\psi(x)B_c^TP\mathbf{e}\cdot\\ &\tanh(\hat{\theta}\psi(x)B_c^TP\mathbf{e}/\varepsilon_d)]+\frac{2}{r}\tilde{\theta}^T\dot{\tilde{\theta}}\\ =&-\mathbf{e}^TQ\mathbf{e}+2[\hat{\theta}^T\psi(x)\left\|B_c^TP\mathbf{e}\right\|-\hat{\theta}\psi(x)B_c^TP\mathbf{e}\cdot\\ &\tanh(\hat{\theta}\psi(x)B_c^TP\mathbf{e}/\varepsilon_d)]+\tilde{\theta}^T[\frac{2}{r}\dot{\tilde{\theta}}-2\psi(x)\left\|B_c^TP\mathbf{e}\right\|].\end{aligned} \tag{22}$$

It can be shown that the following inequality holds for any $\varepsilon > 0$ and for any $\eta \in R$

$$0 \le |\eta| - \eta \tanh(\frac{\eta}{\varepsilon}) \le \kappa\varepsilon, \qquad (23)$$

where $\kappa$ is a constant that satisfies $\kappa = \exp(-(\kappa+1))$, i.e., $\kappa = 0.2785$ (Yansheng Yang, et al., 2002).

Setting $\eta = \hat{\theta}^T \psi(x) B_c^T P\mathbf{e}$, applying (23) and substituting (15) into (22), we have

$$\dot{V} \le -\mathbf{e}^T Q\mathbf{e} + 2\kappa\varepsilon_d - 2\frac{\lambda}{r}\tilde{\theta}^T(\tilde{\theta}+\theta). \qquad (24)$$

Owing to

$$\frac{1}{2}(\tilde{\theta}+\theta)(\tilde{\theta}+\theta) \ge 0,$$

it can be obtained as

$$\tilde{\theta}^T\tilde{\theta} + \tilde{\theta}^T\theta \ge \frac{1}{2}(\tilde{\theta}^T\tilde{\theta} - \theta^T\theta).$$

Therefore, we get

$$\dot{V} \le -\mathbf{e}^T Q\mathbf{e} - \frac{\lambda}{r}\tilde{\theta}^T\tilde{\theta} + \frac{\lambda}{r}\theta^T\theta + 2\kappa\varepsilon_d. \qquad (25)$$

Furthermore, let

$$\bar{Q} = \begin{bmatrix} Q & 0 \\ 0 & \frac{\lambda}{r} \end{bmatrix},$$

then (25) is given as

$$\dot{\bar{V}} \le -z^T\bar{Q}z + 2\bar{\varepsilon}.$$

Substituting the parameters given in theorem 1 into above expression, we get

$$\dot{\bar{V}} \le -2\mu\bar{V} + 2\bar{\varepsilon} = -2\mu(\bar{V} - \frac{\bar{\varepsilon}}{\mu}). \qquad (26)$$

Now, if we let $\frac{\bar{\varepsilon}}{\mu} > 0$, then (26) satisfies

$$0 \le V(t) \le \frac{\bar{\varepsilon}}{\mu} + (V(0) - \frac{\bar{\varepsilon}}{\mu})\exp(-2\mu t). \qquad (27)$$

Therefore, $x, \theta$ are uniformly bounded. Furthermore, using (17) and (27) we obtain that given any $\rho > \sqrt{\frac{\bar{\varepsilon}}{\mu}}$, there exists $T(\rho)$ such that for all $t \ge T$ satisfies $|e(t)| \le \rho$.

The proof completes. □

In summarizing the above discussions, the design procedure is described as follows:

Step1: Construct the fuzzy plant rules in the form of(3), and obtain the GFHM in the form of (8).

Step2: Select the feedback gain vectors $K$, such that the matrices $\Lambda_c = A - BK^T$ is Hurwitz matrix.

Step3: Select $Q$ and solve (16), obtain the matrix $P$.

Step4: Choose the appropriate values $r, \lambda, \varepsilon_d$ in (14)(15).

Step5: Construct fuzzy basis vector $\psi(x)$ and select an initial $\hat{\theta}$.

Step6: Obtain the control law (9),(14) and the adaptive law (15).

## 5. SIMULATION EXAMPLE

To illustrate the design procedure of the controller and its performance we apply our robust adaptive fuzzy controller to control the inverted pendulum to track a sine wave trajectory. The dynamic equations of the system are given by(J.-J.E. Slotine, W. Li,1991):

$$\dot{x}_1 = x_2,$$
$$\dot{x}_2 = f(x_1,x_2) + g(x_1,x_2)u + d,$$

where

$$f(x_1,x_2) = \frac{g\sin(x_1) - mlx_2^2\cos(x_1)\sin(x_1)/(m_c+m)}{l(\frac{4}{3} - m\cos^2(x_1)/(m_c+m))}$$

and

$$g(x_1,x_2) = \frac{\cos(x_1)/(m_c+m)}{l(\frac{4}{3} - m\cos^2(x_1)/(m_c+m))},$$

$x_1$ represents the angle of the pendulum, $x_2$ represents angular velocity, $m_c$ is the mass of cart, $m$ is the mass of pole, $l$ is the half-length of pole, and $u$ is the applied force (control). We choose $m_c$ =1 kg, $m = 0.1$kg, $l = 0.5$m, $g = 9.8m/s^2$ and $d = 0.1e^{-t}\sin(t)$. We also choose the reference signal $y_m(t) = (\pi/30)\sin(t)$ in the following simulations.

First, we derive the GFHM in the form of (7) from the priori knowledge about the system. By the linguistic knowledge, we can obtain the following the fuzzy rules:

$R^l$: IF $x_1 - d_1$ is $P_{x_1}$ and $x_2 - d_2$ is $P_{x_2}$ and $u$ is $P_u$

THEN $\dot{x}_2 = c_{P_1} + c_{P_2} + c_{P_u}$,

……

$R^8$: IF$x_1 - d_1$ is $N_{x_1}$and $x_2 - d_2$ is $N_{x_2}$and $u$ is $N_u$

THEN $\dot{x}_2 = -c_{N_1} - c_{N_2} - c_{N_u}$,

where the membership function of the fuzzy sets is in the form of (1), where $k_{x_1} = 0.6, k_{x_2} = 0.2, k_u = 1, d_1 = d_2 = 0$, and $a_0 = 0.0048, a_1 = 26.72, a_2 = -0.165, b = 1.4125$. By the above fuzzy rules we derive a GFHM as follows:

$$\dot{x}_2 = a_0 + a_1\tanh(k_{x_1}x_1) + a_2\tanh(k_{x_2}x_2) + b\tanh(k_u u).$$

After liberalization in $\bar{u}$, we get the form of (7), where $b = 1.4125$. Our fuzzy controller applies to this system. Let $k_1 = 2$; $k_2 = 1$(so that $s^2 + k_1 s +$

$k_2$ is stable), and $Q = 5I_{2\times2}$, then we have Lyapunov equation (16) and obtain $P = \begin{bmatrix} 7.5 & 2.5 \\ 2.5 & 2.5 \end{bmatrix}$. Let $\lambda = 0.02, \varepsilon_d = 0.5$, $r = diag[0.1, 0.1, 0.9, 0.8]$, $\psi(x) = [1, 1, |\tanh(0.1x_1)|, |\tanh(0.3x_2)|]^T$, and the initial state is $x(0) = [-0.15, 0]^T$. The result of Simulation is illustrated in Fig. 1.

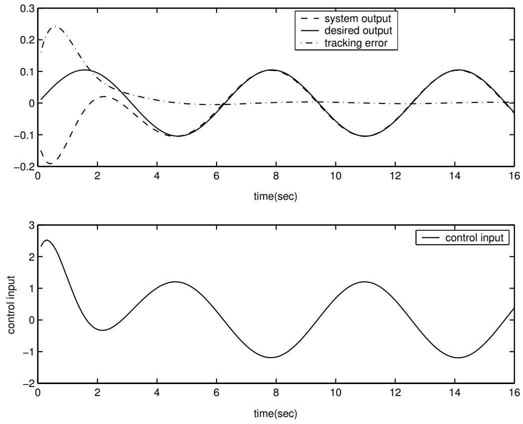


Fig. 1. (a) system output, desired output, and tracking error ( in the upper figure); (b) control input(in the lower figure).

## 6. CONCLUSIONS

This paper developed a robust adaptive control algorithm based on the GFHM for single-input single-output (SISO) nonlinear dynamical systems with uncertainties. The proposed control is a smooth control with no chattering phenomena. The overall control system guarantees that the tracking error converges in the small neighborhood of zero and that all signals involved are uniformly bounded. The main advantage of the proposed control law is that the human knowledge about the plant under control is to be used to design the controller and only one parameter vector in the adaptive mechanism is on-line adjusted. Simulation example that the inverted pendulum to track a sine wave trajectory is provided to illustrate the design procedure of the proposed controller and performance.

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