ROBUST STABILITY OF UNCERTAIN DISCRETE SYSTEMS WITH TIME-VARYING DELAY

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Abstract: Robust stability is considered for uncertain discrete-time systems with time-varying delays from given intervals. A new construction of Lyapunov-Krasovskii functions (LKFs), which has been recently introduced in the continuous case, is applied: to a nominal LKF, which is appropriate to the system with nominal delays, terms are added that correspond to the system with the perturbed delays and that vanish when the delay perturbations approach 0. The nominal LKF is chosen in two forms: the descriptor type and the 'exact' one. The delay-independent result is derived via Razumikhin approach. The advantage of the new tests is demonstrated via illustrative examples.

Keywords: time-varying delay, discrete systems, Lyapunov-Krasovskii method, norm-bounded uncertainties.

1. INTRODUCTION

Stability and control of continuous-time linear systems with delays have been studied by many authors (see e.g. Li & de Souza (1997), Kolmanovskii & Richard (1999), Niculescu (2001), Fridman (2001), Fridman & Shaked (2002) and the references therein). Delay-independent and, less conservative, delay-dependent sufficient stability conditions in terms of Riccati or linear matrix inequalities (LMIs) have been derived by using Lyapunov-Krasovskii functionals or Lyapunov-Razumikhin functions. Delay-dependent conditions are based on different model transformations. The most recent one, a descriptor representation of the system Fridman (2001), minimizes the overdesign that stems from the model transformation used. The conservatism that stems from the bounding of the cross-terms in the derivation of the derivative of the Lyapunov-Krasovskii functional has also been significantly reduced in the past few years. An important result that improves the standard bounding technique has been proposed in (Moon *et al.*, 2001).

Less attention has been drawn to the corresponding results for discrete-time delay systems (Verriest & Ivanov, 1995), (Kapila & Haddad, 1998), (Song *et al.*, 1999), (Mahmoud, 2000), (Lee & Kwon, 2002), (Chen, Guan & Lu, 2003). This is mainly due to the fact that such systems can be transformed into augmented systems without delay. This augmentation of the system is, however, inappropriate for systems with unknown delays or systems with time-varying delays (such systems appear e.g. in the field of communication networks).

For the case of 'small' delay from $[0, \mu]$ the delay-dependent conditions were derived in (Lee & Kwon, 2002) and (Chen *et al.*, 2003) by applying the discrete counterparts of the methods developed in (Moon *et al.*, 2001) and Fridman & Shaked, 2002) correspondingly. The case of uncertain *'non-small'* time-varying delay, where the *nominal delay value is non-zero* and constant, has been recently considered in (Xu & Chen, 2004). A Lyapunov function has been used there with a 'nominal' part that corresponds to delay-independent stability of the nominal system (with a nominal value of the delay). Thus, the necessary condition for the feasibility of the LMIs derived in (Xu & Chen, 2004) for stability is the delay-independent stability of the nominal system, which is very restrictive.

For continuous systems with uncertain non-small delay a new construction of the LKF has been introduced recently (Fridman, 2004): to a nominal LKF, which is appropriate to the nominal system (with nominal delays), the terms are added which correspond to the perturbed system and which vanish when the delay perturbations approach 0. In the present paper we apply such construction of LKF to the discrete case. We consider both, the descriptor type and the exact nominal LKF and derive LMI conditions for robust stability.

The delay-independent conditions are derived via Razumikhin approach. Examples are given that show that our conditions are less conservative than those that have appeared in the literature.

2. ROBUST STABILITY

We consider the following unforced discrete-time state-delayed system

$$\begin{aligned} x(k+1) &= (A+H\Delta(k)E)x(k) \\ &+(A_1+H\Delta(k)E_1)x(k-\tau(k)), \\ x(k) &= \phi(k),\ -h-\mu_2 \le k \le 0 \end{aligned} \tag{1}$$

where $x(k) \in \mathcal{R}^n$ is the state vector, $\tau(k)$ is a positive number representing the delay $\tau(k) = h + \eta(k)$ with the nominal constant value $h > 0$ and a time-varying perturbation $\eta(k) \in [-\mu_1, \mu_2]$, $h \ge \mu_1 \ge 0, \mu_2 \ge 0$. Matrices A, A_1, H, E and E_1 are constant and $\Delta(k) \in \mathcal{R}^{r_1 \times r_2}$ is a time-varying uncertain matrix satisfying the following inequality:

$$\Delta(k)^T \Delta(k) \le I. \tag{2}$$

2.1 *Lyapunov-Krasovskii method for discrete systems with delays*

Denoting

$$y(k) = x(k+1) - x(k) \tag{3}$$

and taking into account that

$$x(k-\tau(k)) = x(k-h) - \sum_{j=k-h-\eta(k)}^{k-h-1} y(j)$$

we represent (1) in the following descriptor form:

$$\begin{aligned} &x(k+1) = y(k) + x(k), \\ &0 = -y(k) + (A + H\Delta E - I)x(k) \\ &+(A_1 + H\Delta E_1)x(k-h) \\ &-\sum_{j=k-h-\eta(k)}^{k-h-1} (A_1 + H\Delta E_1)y(k), \end{aligned} \tag{4}$$

$$\begin{aligned} &x(0) = \phi(0), \\ &y(0) = (A + H\Delta E - I)\phi(0) \\ &+(A_1 + H\Delta E_1)\phi(-\tau(0)), \\ &y(k) = \phi(k+1) - \phi(k), \quad k = -h-\mu_2, ..., -1. \end{aligned} \tag{5}$$

Thus, if $x(k)$ is a solution of (1), then $\{x(k), y(k)\}$, where $y(k)$ is defined by (3), is a solution of (4), (5) and vise versa.

Lemma 1. If there exist positive numbers α, β and a continuous functional

$$V(k) = V(x(k-h), ..., x(k), y(k-h-\mu_2), ..., y(k-1))$$

such that

$$\begin{aligned} &0 \le V(k) \le \beta \max\{\max_{j\in[k-h-\mu_2,k]} |x(j)|^2, \\ &\max_{j\in[k-h-\mu_2,k-1]} |y(j)|^2\}, \\ &\Delta V(k) \stackrel{\Delta}{=} V(k+1) - V(k) \le -\alpha |x(k)|^2, \end{aligned} \tag{6}$$

for $x(k)$ and $y(k)$ satisfying (4), then (1) is asymptotically stable.

We suggest to construct the LKF for (4) in the form of

$$V(k) = V_n(k) + V_a(k), \tag{7}$$

where

$$V_a(k) = \sum_{m=-\mu_2}^{\mu_1-1} \sum_{j=k+m-h}^{k-1} y(j)^T R_a y(j), \quad 0 < R_a. \tag{8}$$

Similarly to the continuous-time case, we intend to construct the nominal Lyapunov function V_n which corresponds to (4), where $\eta(k) = 0$ in two forms: 1) the form of 'descriptor type' as considered in (Chen *et al.*, 2003), 2) the form of the exact (the discrete counterpart of the 'complete' LKF) Lyapunov function. Unlike the continuous-time case (see Kharitonov & Zhabko, 2003), (Fridman, 2004b) and references therein), the exact Lyapunov function may be easily found by representing the nominal system (4) (with $\eta(k) \equiv 0$) in the form of an augmented non-delay descriptor system. However, this may lead to high-dimensional LMIs. To derive a reduced-order LMIs we will consider the descriptor type V_n.

2.2 *The case of descriptor type nominal LKF*

The nominal LKF (which corresponds to (4) with $\eta(k) = 0,\ H = 0$) is given by (see e.g. Chen *et al.* (2003)):

$$\begin{aligned} V_n(k) &= x^T(k) P_1 x(k) \\ &+ \sum_{m=-h}^{-1} \sum_{j=k+m}^{k-1} y(j)^T R y(j) \\ &+ \sum_{j=k-h}^{k-1} x(j)^T S x(j),\ P_1 > 0,\ R > 0,\ S > 0. \end{aligned} \tag{9}$$

The nominal system is asymptotically stable if there exist $n \times n$ matrices $0 < P_1,\ P_2,\ P_3,\ S,\ Y,\ Z_1,\ Z_2, Z_3, R$ such that the following LMIs are feasible

$$\Gamma_n = \begin{bmatrix} \Psi_n + hZ & P^T \begin{bmatrix} 0 \\ A_1 \end{bmatrix} - Y^T \\ * & -S \end{bmatrix} < 0, \quad \begin{bmatrix} R & Y \\ * & Z \end{bmatrix} \geq 0, \tag{10}$$

where

$$P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix},$$

$$Y = [Y_1\ Y_2], \quad Z = \begin{bmatrix} Z_1 & Z_2 \\ * & Z_3 \end{bmatrix}, \quad i = 1, 2,$$

$$\begin{aligned} \Psi_n = P^T &\begin{bmatrix} 0 & I \\ A - I & -I \end{bmatrix} + \begin{bmatrix} 0 & I \\ A - I & -I \end{bmatrix}^T P \\ &+ \begin{bmatrix} S & 0 \\ 0 & hR + P_1 \end{bmatrix} + \begin{bmatrix} Y \\ 0 \end{bmatrix} + \begin{bmatrix} Y \\ 0 \end{bmatrix}^T, \end{aligned} \tag{11}$$

We obtain:

Lemma 2. Eq. (1) with $\Delta \equiv 0$ is asymptotically stable for $0 \leq h - \mu_1 \leq \tau(k) \leq h + \mu_2$ if there exist $n \times n$ matrices $0 < P_1,\ P_2,\ P_3,\ S,\ Y_1,\ Y_2, R$ and $R_a > 0$ that satisfy the following LMI:

$$\Gamma_1 = \begin{bmatrix} \Psi & P^T \begin{bmatrix} 0 \\ A_1 \end{bmatrix} - Y^T & \mu P^T \begin{bmatrix} 0 \\ A_1 \end{bmatrix} & hY^T \\ * & -S & 0 & 0 \\ * & * & -\mu R_a & 0 \\ * & * & * & -hR \end{bmatrix} < 0, \tag{12}$$

where $\mu = max\{\mu_1, \mu_2\}$, Y and Ψ_n are given by (11) and

$$\Psi = \Psi_n + \begin{bmatrix} 0 & 0 \\ 0 & (\mu_1 + \mu_2) R_a \end{bmatrix}. \tag{13}$$

Proof. We find when $\Delta V(k)$ is strictly negative. The difference $\Delta V_n(k)$ along the trajectories of the nominal system satisfies the following inequality (Chen *et al.*, 2003):

$$\Delta V_n(k) \leq \xi^T(k) \Gamma_n \xi(k), \tag{14}$$

where Γ_n is given by (10a) and

$$\xi(k) = col\{x(k),\ y(k),\ x(k-h)\}, \tag{15}$$

provided (10b) is satisfied. Note that along the trajectories of (4)

$$\begin{aligned} &x^T(k+1) P_1 x(k+1) - x^T(k) P_1 x(k) \\ &= 2x^T(k) P_1 y(k) + y^T(k) P_1 y(k) \\ &= 2\bar{x}^T(k) P^T \begin{bmatrix} y(k) \\ 0 \end{bmatrix} + y^T(k) P_1 y(k) \\ &= 2\bar{x}^T(k) P^T \times \\ &\times \begin{bmatrix} y(k) \\ -y(k) + (A - I)x(k) + A_1 x(k-h) \end{bmatrix} \\ &+ y^T(k) P_1 y(k) + \delta(k), \end{aligned} \tag{16}$$

where $\bar{x}(k) = col\{x(k), y(k)\}$,

$$\delta(k) = -2\bar{x}^T(k) P^T \sum_{j=k-h-\eta(k)}^{k-h-1} \begin{bmatrix} 0 \\ A_1 \end{bmatrix} y(j),$$

while along the trajectories of the nominal system with $\tau(k) \equiv h$ gives (16) with $\delta(k) \equiv 0$.

Applying the standard bounding of δ and Schur complements, we find

$$\Delta V(k) \leq \xi_1(k)^T(\Gamma_1 + diag\{hZ,\ 0,\ 0,\ 0\})\xi_1(k), \tag{17}$$

where $\xi_1(k) = col\{\xi(k), y(k), 0\}$. From (10b) it follows that $Z \geq Y^T R^{-1} Y$. Replacing therefore Z in (17) by $Y^T R^{-1} Y$ it is obtained that (12) implies $\Delta V(k) < 0$ and the asymptotic stability of (1). □

We have thus proved the following:

Theorem 3. Consider (1), where $0 \leq h - \mu_1 \leq \tau(k) \leq h + \mu_2$. This system is asymptotically stable if there exist $n \times n$ matrices $0 < P_1,\ P_2,\ P_3,\ S,\ Y_1,\ Y_2,\ R, R_a$ and a scalar ρ_0 that satisfy

$$\begin{bmatrix} \Gamma_1 & \begin{bmatrix} P_2^T H \\ P_3^T H \\ 0 \\ 0 \\ 0 \end{bmatrix} & \rho_0 \begin{bmatrix} E^T \\ 0 \\ E_1^T \\ \mu E_1^T \\ 0 \end{bmatrix} \\ * & -\rho_0 I & 0 \\ * & * & -\rho_0 I \end{bmatrix} < 0, \tag{18}$$

where $\mu = max\{\mu_1, \mu_2\}$.

2.3 *Augmentation and descriptor nominal LKF*

In the case when the non-delayed system is not asymptotically stable or $h - \mu_1$ is not large, we represent (1) in the form of the augmented system

$$\zeta(k+1) = (\mathcal{A} + \mathcal{H}\Delta(k)\mathcal{E})\zeta(k) + (\mathcal{A}_1 + \mathcal{H}\Delta(k)\mathcal{E}_1)\zeta(k - \mu_1 - \eta(k)), \tag{19}$$

where

$$\zeta(k) = \begin{bmatrix} x(k - h + \mu_1) \\ x(k - h + \mu_1 - 1) \\ \dots \\ x(k) \end{bmatrix},\ \mathcal{H} = \begin{bmatrix} 0 \\ \dots \\ 0 \\ H \end{bmatrix},$$
$$\mathcal{A} = \begin{bmatrix} 0 & I_n & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & I_n \\ 0 & 0 & \dots & A \end{bmatrix},\ \mathcal{A}_1 = \begin{bmatrix} 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 \\ A_1 & 0 & \dots & 0 \end{bmatrix},$$
$$\mathcal{E} = [0\ 0\ \dots\ 0\ E],\ \mathcal{E}_1 = [E_1\ 0\ \dots\ 0]. \tag{20}$$

Note that for $\mu_1 = 0$, the nominal system (19), where $\eta(k) \equiv 0$ and $\Delta \equiv 0$, has no delay and the nominal *exact Lyapunov function* $V_n(k) = \zeta^T(k) P_1 \zeta(k)$ should be used. This is different from the continuous case, where the exact (complete) LKF has a complicated form and leads to complicated robust stability conditions (Kharitonov and Zhabko, 2003).

In the general case of $\mu_1 \geq 0$ we apply Theorem 1 to (19), where $h = \mu_1$, and obtain the following:

Theorem 4. Consider (1), where $0 \leq h - \mu_1 \leq \tau(k) \leq h + \mu_2$. This system is asymptotically stable if there exist $(h - \mu_1 + 1)n \times (h - \mu_1 + 1)n$ matrices $0 < P_1,\ P_2,\ P_3,\ S,\ Y_1,\ Y_2,\ R,\ R_a$ and scalars $\rho_i > 0,\ i = 0, 1$ that satisfy (18) with $\mu = max\{\mu_1, \mu_2\}$ and $h = \mu_1$, where A, A_1, E, E_1 and H should be changed correspondingly to $\mathcal{A}, \mathcal{A}_1, \mathcal{E}, \mathcal{E}_1$ and $\mathcal{H}$.

2.4 *Augmentation and discrete descriptor Lyapunov function*

We consider $\mu_1 = 0$ and $\Delta = 0$. To reduce the size and the number of the decision variables by the previous augmented method, we consider $h \geq 1$ and the state vector $\zeta = [\zeta_1 \dots \zeta_{h+1}]^T$ given by (20). Defining $y(k) = x(k+1-h) - x(k-h) = \zeta_2(k) - \zeta_1(k)$ and representing (1) in the form

$$x(k+1) = Ax(k) + A_1 x(k-h) - A_1 \sum_{j=k-\eta(k)}^{k-1} y(j), \tag{21}$$

we obtain the following descriptor form:

$$E\bar{\zeta}(k+1) = \mathcal{A}_d \bar{\zeta}(k) + \mathcal{A}_1 \sum_{j=k-\eta(k)}^{k-1} y(j),$$
$$E = diag\{I_{(h+1)n},\ 0_{n\times n}\},\ \bar{\zeta}(k) = \begin{bmatrix} \zeta(k) \\ y(k) \end{bmatrix},$$
$$\mathcal{A}_d = \begin{bmatrix} I_n & 0 & 0 & \dots & 0 & I_n \\ 0 & 0 & I_n & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & I_n & 0 \\ A_1 & 0 & 0 & \dots & A & 0 \\ -I_n & I_n & 0 & \dots & 0 & -I_n \end{bmatrix},$$
$$\mathcal{A}_1 = -\begin{bmatrix} 0 & 0 & \dots & 0 & A_1^T & 0 \end{bmatrix}^T. \tag{22}$$

We construct the LKF for (4) in the form of $V(k) = V_n(k) + V_a(k)$, where

$$V_a(k) = \mu_2 \sum_{m=-\mu_2}^{-1} \sum_{j=k+m}^{k-1} y(j)^T R_a y(j), \quad 0 < R_a \tag{23}$$

and V_n is a nominal Lyapunov function which corresponds to (22a), with $\eta(k) = 0$:

$$V_n = \bar{\zeta}^T(k) E P E \bar{\zeta}(k),\ P = P^T,\ EPE \geq 0. \tag{24}$$

Lemma 5. Consider (1), where $\Delta \equiv 0$, $1 \leq h \leq \tau(k) \leq h + \mu_2$. This system is asymptotically stable if there exist a $(h+2)n\times(h+2)n$ matrix $P=P^T$, such that $[I_{(h+1)n}\ 0]P[I_{(h+1)n}\ 0]^T>0$, and a $n\times n$ matrix R_a that lead to

$$\begin{aligned} &\begin{bmatrix} \Psi & \mathcal{A}_d^T P\mathcal{A}_1 \\ * & -R_a+\mathcal{A}_1^T P\mathcal{A}_1 \end{bmatrix}<0, \\ &\Psi=\mathcal{A}_d^T P\mathcal{A}_d-EPE+\begin{bmatrix} 0 & 0 \\ 0 & \mu_2^2 R_a \end{bmatrix}, \end{aligned} \tag{25}$$

where $\mathcal{A}_d$ and $\mathcal{A}_1$ are given by (22d) and (22e), correspondingly.

The condition of Lemma 3 can also be adopted to the systems with norm-bounded uncertainties.

2.5 *Delay-independent conditions*

As in the continuous-time situation, this case is treated adopting the Lyapunov-Razumikhin approach (Zhang & Chen, 1998).

Theorem 6. Consider the system (1) with $\Delta(k)$ that satisfies (2). This system is asymptotically stable for all delays $\tau(k)$ if there exist $P=P^T\in\mathcal{R}^{n\times n}$, $\alpha\in(0,1), q>1$ and $\epsilon>0$ that satisfy the following LMI:

$$\begin{bmatrix} -\alpha P+\epsilon E^T E & \epsilon E^T E_1 & A^T P & 0 \\ * & -\frac{1-\alpha}{q}P+\epsilon E_1^T E_1 & A_1^T P & 0 \\ * & * & -P & PH \\ * & * & * & -\epsilon I \end{bmatrix}$$
$<0.$

2.6 *Examples*

Example 1: We consider the system (1) where:

$$A=\begin{bmatrix} 0.8 & 0 \\ 0 & 0.97 \end{bmatrix},\ A_1=\begin{bmatrix} -0.1 & 0 \\ -0.1 & -0.1 \end{bmatrix}, \tag{26}$$

where $H=0$. Assuming that h is constant, we seek the maximum value of $\bar{h}$ for which the asymptotic stability of the system is guaranteed. The maximum value of $\bar{h}$, achievable by the method of Lee & Kwon (2002), is 12, whereas a value of $\bar{h}=16$ was obtained by applying Chen *et al.* (2003).Using augmentation it is found that the system considered is asymptotically stable for all $h\le 18$. The criterion of Theorem 3 did not provide a solution, so that no delay-independent solution has been found.

Allowing τ to be time-varying we apply Lemma 2, where $h=\mu_1=1$ and $\mu_2=7$. We obtain thus that asymptotic stability is guaranteed for all $0\le\tau(k)\le 8$. The same result is obtained by Corollary 1 via discrete descriptor Lyapunov function. Choosing $h=8,\mu_1=\mu_2=3$; $h=10$, $\mu_1=\mu_2=2$ and $h=11$, $\mu_1=1,\mu_2=2$ we verified that conditions of Lemma 2 are feasible. Hence the system is asymptotically stable for all $\tau(k)$ from the following intervals: $[3,10]$, $[5,11]$, $[8,12]$ and $[10,13]$. Note that conditions of Xu and Chen (2004) are not feasible even for $0\le\tau(k)\le 1$.

By augmentation via the discrete descriptor Lyapunov function we verify that the conditions of Lemma 3 are feasible $\tau(k)$ from larger intervals: $[3,10]$, $[5,11]$, $[7,12]$ and $[9,13]$. The augmented approach via descriptor LKF of Lemma 2 leads to the same stability intervals as Lemma 3, but needs essentially more time for computation.

Treating next the case where the system parameters are uncertain, with A and A_1 given in (26) and with

$$H=\begin{bmatrix} 0.1 & 0 \\ 0 & 0.02 \end{bmatrix},\quad E=I_2\quad\text{and}\quad E_1=0.5I_2,$$

we apply Theorem 1 for $h=0,3$ and 5 and verify that the system (1)is asymptotically stable for all $\Delta(k)$ that satisfy (**??**) and for $\tau(k)$ from the following segments: $[0,4]$, $[3,5]$ and $[5,6]$. By the augmented system approach via descriptor LKF, we find that the conditions of Theorem 2 are feasible for $h=3$, $\mu_1=1,\mu_2=2$ and for $h=5,\mu_1=1$, and $\mu_2=1$. Thus the stability intervals, starting from non-zero values, are larger $[2,5]$ and $[4,6]$.

Example 2 Wu & Hong (1994): We consider the system (1) where

$$A=\begin{bmatrix} 0 & 0.5 \\ 0.5 & 0.2 \end{bmatrix},\quad A_1=\begin{bmatrix} -0.4 & 0 \\ 0 & 0 \end{bmatrix}\quad\text{and}\quad H=0.$$

In the case of constant delay, this system is delay-independently stable by the conditions of Wu & Hong (1994) and by Corollary 1 of the present paper. In the case of time-varying delay, by conditions of Song *et al.* (1999) the system is asymptotically stable for $0<\tau(k)\le 2$. By Theorem 3, it is verified that also in the case of time-varying delay the system is delay-independently stable. This is achieved by taking $\alpha=0.5$ and $q=1.01$.

3. CONCLUSIONS

New sufficient stability conditions have been derived for discrete-time systems with uncertain delay and norm-bounded uncertainties. The delay is assumed to be time-varying either bounded or not. In the first case the Lyapunov-Krasovskii method is applied, while the second (delay-independent) case is treated by Laypunov-Razumikhin technique. Illustrative examples demonstrate the efficiency of the method. The method can be effectively applied to guaranteed cost control and to H_∞ control.

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