ADAPTIVE OUTPUT FEEDBACK CONTROL OF UNCERTAIN MIMO NONLINEAR SYSTEMS WITH UNKNOWN ORDERS

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Abstract: In this paper, we propose a design method for a robust adaptive output feedback control system for uncertain MIMO nonlinear systems with a higher order relative degree and an uncertain controlled system order. By using a virtual input filter method, this method allows an adaptive output feedback control of MIMO nonlinear systems without constructing a state observer in the controller.

Keywords: adaptive control, output feedback, MIMO nonlinear systems, model following control, uncertain nonlinear systems

1. INTRODUCTION

It is well known that one can stabilize uncertain nonlinear systems, which satisfy OFEP (output feedback exponentially passive) conditions, by high-gain output feedback based adaptive controls with a simple structure and strong robustness with respect to bounded disturbances (Fradkov and Hill 1998, Allgower *et al.* 1997, Fradkov 1996). The system is said to be OFEP if there exists an output feedback such that the resulting closed-loop system is exponentially passive(Fradkov and Hill 1998). The sufficient conditions for the MIMO nonlinear system to be OFEP are that (1) the system has relative degree $\{1, 1, \cdots, 1\}$, (2) the system is exponential minimum-phase, (3) the nonlinearities of the system satisfy the Lipschitz condition and (4) the high frequency gain matrix of the system is positive definite. Unfortunately, however the OFEP conditions impose very severe restrictions on practical applications of the abovementioned adaptive schemes because most practical systems do not satisfy the OFEP conditions, in particular the restriction of the relative degree. The backstepping strategy has been recognized as a powerful design tool for relieving the restriction on the relative degree of the controlled system. Concerning output feedback based adaptive controls for SISO systems, adaptive output feedback controller designs based on backstepping strategy have been widely developed (Marino and Tomei 1993, Mizumoto *et al.* 2003). As for MIMO systems, a backstepping strategy has been applied to linear systems (Mizumoto *et al.* 1996, Ling and Tao 1997, Takahashi *et al.* 1999) in order to solve the problem of the relative degree. The method proposed by Ling and Tao (1997) is a design scheme basically for a system with relative degree $\{r, r, \cdots, r\}$, having the same relative degrees in each subsystem. This method also requires information about state variables for designing the controller and therefore a state observer has to be designed into the controller. The methods in Mizumoto *et al.* (1996) and Takahashi *et al.* (1999) can design an adaptive output feedback controller without an observer and these are directly applied to m-input m-output systems with any relative degrees $\{r_1, r_2, \cdots, r_m\}$. Recently, the method in Ling and Tao (1997) is extended to a system with uncertain parametric nonlinearities (Wu and Zhou 2004). In this method, although the restriction on the high frequency gain matrix has been relaxed to be Hurwitz while it had been assumed in general MIMO cases that the high frequency gain matrix should be positive definite, a state observer is still required for designing an adaptive output feedback controller. This implies that the order of the controlled system must be known. Further, since this method is basically a design strategy for systems with relative degree $\{r, r, \cdots, r\}$ having the same relative degrees in each subsystem, information about an upper triangular interactor polynomial matrix is required in the case where the system has any relative degree $\{r_1, r_2, \cdots, r_m\}$.

In this paper, we propose a design method for an adaptive control system based on high gain output feedback for uncertain MIMO nonlinear systems with higher order relative degree and an uncertain system order. A condition for designing an adaptive controller for MIMO nonlinear systems without the use of an interactor matrix will appear. The use of a virtual input filter (Marino and Tomei 1993, Mizumoto *et al.* 2003) will make it possible to design a controller without a state observer. This means that the proposed method can design a controller independent of the order of the controlled system. The proposed method can also deal with nonparametric uncertainties because the proposed method is based on the high gain output feedback control of OFEP nonlinear systems.

2. PROBLEM STATEMENT

Consider a nonlinear system which can be represented by the following canonical form:

$$\begin{aligned} \dot{\boldsymbol{z}} &= A\boldsymbol{z} + \tilde{B}B\boldsymbol{u} + \boldsymbol{f}_1(\boldsymbol{z}, \boldsymbol{\eta}) \\ \dot{\boldsymbol{\eta}} &= \boldsymbol{q}(\boldsymbol{y}, \boldsymbol{\eta}) + \boldsymbol{f}_2(\boldsymbol{z}, \boldsymbol{\eta}) \\ \boldsymbol{y} &= C\boldsymbol{z} \end{aligned} \tag{1}$$

with a relative degree:

$$\{\overbrace{\underbrace{r_1, \cdot\cdot, r_1}_{k_1}, \underbrace{r_2, \cdot\cdot, r_2}_{k_2}, \cdots, \underbrace{r_i, \cdot\cdot, r_i}_{k_i}, \cdots, \underbrace{r_f, \cdot\cdot, r_f}_{k_f}}^{m}\} \tag{2}$$

where $\boldsymbol{z}^T \in R^r$ and $\boldsymbol{\eta}^T \in R^{n-r}$ are the state variables, $r = r_1k_1 + r_2k_2 + \cdots + r_fk_f$. $\boldsymbol{u}$ and $\boldsymbol{y} \in R^m$ are the control inputs and outputs, respectively, and $\boldsymbol{f}_1, \boldsymbol{f}_2$ and $\boldsymbol{q}$ are uncertain vector nonlinear functions. $\tilde{B} \in R^{m\times m}$ is an unknown constant matrix. Without loss of generality, we assume that $r_1 < r_2 < \cdots < r_f$ and that $A = \text{diag}\,[A_j]_{j=1,\cdot\cdot,m}$, $\tilde{B} = \text{diag}\left[\tilde{\boldsymbol{b}}_j\right]_{j=1,\cdot\cdot,m}$, $C = \text{diag}\left[\boldsymbol{c}_j^T\right]_{j=1,\cdot\cdot,m}$ has the following form:

$$A_j = \begin{bmatrix} \boldsymbol{0} & I_{r_i-1\times r_i-1} \\ a_{j,1} & \cdots \; a_{j,r_i} \end{bmatrix} \in R^{r_i\times r_i},$$

$$\tilde{\boldsymbol{b}}_j = \begin{bmatrix} \boldsymbol{0} \\ 1 \end{bmatrix} \in R^{r_i}, \; \boldsymbol{c}_j = \begin{bmatrix} 1 \\ \boldsymbol{0} \end{bmatrix} \in R^{r_i}$$

for $l_i + 1 \le j \le l_i + k_i = l_{i+1}$. The system outputs y_j, $j = 1, \cdots, m$, then can be obtained as an output of the following subsystem:

$$\begin{aligned} \dot{\boldsymbol{z}}_j &= A_j\boldsymbol{z}_j + \tilde{\boldsymbol{b}}_j\boldsymbol{b}_j^T\boldsymbol{u} + \boldsymbol{f}_{1j}(\boldsymbol{z}, \boldsymbol{\eta}) \\ y_j &= \boldsymbol{c}_j^T\boldsymbol{z}_j \end{aligned} \tag{3}$$

where $\boldsymbol{z}_j$ is a vector element of $\boldsymbol{z}$ such as $\boldsymbol{z} = [\boldsymbol{z}_1^T, \cdots, \boldsymbol{z}_m^T]^T$ and $[\boldsymbol{b}_1, \cdots, \boldsymbol{b}_m]^T = B$.

Here, we impose the following assumption on input coefficient vectors $\boldsymbol{b}_j$.

Assumption 1. Defining the numbers: $l_1 = 0$ and $l_i = k_1 + k_2 + \cdots + k_{i-1}$ for $i = 2, 3, \cdot\cdot, f$, $\tilde{\boldsymbol{b}}_j$ for $l_i + 1 \le j \le l_i + k_i = l_{i+1}$ has the following form:

$$\boldsymbol{b}_j^T = [b_{j1}, b_{j2}, \cdots, b_{j,l_i+k_i}, 0, \cdots, 0]$$

Under this assumption, defining a new variable $\boldsymbol{z}_j^{r_i}$ by $[\boldsymbol{z}_1^{r_i}, \cdots, \boldsymbol{z}_{r_i}^{r_i}] = [\boldsymbol{z}_{l_i+1}, \cdots, \boldsymbol{z}_{l_i+k_i}]^T$, the augmented subsystem containing all the subsystems with relative degree r_i associated with the outputs $\boldsymbol{y}^{r_i} = [y_{l_i+1}, \cdots, y_{l_i+k_i}]^T = \boldsymbol{z}_1^{r_i}$ can be represented by

$$\begin{aligned} \dot{\boldsymbol{z}}_1^{r_i} &= \boldsymbol{z}_2^{r_i} + \boldsymbol{f}_1^{r_i} \\ &\vdots \\ \dot{\boldsymbol{z}}_{r_i-1}^{r_i} &= \boldsymbol{z}_{r_i}^{r_i} + \boldsymbol{f}_{r_i-1}^{r_i} \\ \dot{\boldsymbol{z}}_{r_i}^{r_i} &= \sum_{d=1}^{r_i} A_{r_i-d+1}^{r_i}\boldsymbol{z}_{r_i-d+1}^{r_i} + \sum_{l=r_1,r_2,\cdot\cdot,r_i} B_l^{r_i}\boldsymbol{u}_l + \boldsymbol{f}_{r_i}^{r_i} \end{aligned} \tag{4}$$

where $\boldsymbol{u}_{r_j} = [u_{l_j+1}, u_{l_j+2}, \cdot\cdot, u_{l_j+k_j}]^T$, $[\boldsymbol{f}_1^{r_i}, \cdots, \boldsymbol{f}_{r_i}^{r_i}] = [\boldsymbol{f}_{1l_i+1}, \cdots, \boldsymbol{f}_{1l_i+k_i}]^T$ and

$$A_d^{r_i} = \begin{bmatrix} a_{l_i+1,d} & 0 & \cdots & 0 \\ 0 & a_{l_i+2,d} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & a_{l_i+k_i,d} \end{bmatrix},$$

$$B_{r_j}^{r_i} = \begin{bmatrix} b_{l_i+1,l_j+1} & b_{l_i+1,l_j+2} & \cdots & b_{l_i+1,l_j+k_j} \\ b_{l_i+2,l_j+1} & b_{l_i+2,l_j+2} & & \vdots \\ \vdots & & \ddots & \vdots \\ b_{l_i+k_i,l_j+1} & \cdots & \cdots & b_{l_i+k_i,l_j+k_j} \end{bmatrix}$$

Furthermore, we suppose that the system (1) satisfies the following assumptions.

Assumption 2. The nominal part of the system (1) is exponentially minimum phase, that is, the zero dynamics of the nominal system

$$\dot{\boldsymbol{\eta}} = \boldsymbol{q}(\boldsymbol{0}, \boldsymbol{\eta}) \tag{5}$$

is exponentially stable.

Assumption 3. The uncertain funcion $\boldsymbol{q}(\boldsymbol{y}, \boldsymbol{\eta})$ is globally Lipschitz with respect to $(\boldsymbol{y}, \boldsymbol{\eta})$, i.e., there exist a positive constant L_1 such that

$$\|\boldsymbol{q}(\boldsymbol{y}_1, \boldsymbol{\eta}_1) - \boldsymbol{q}(\boldsymbol{y}_2, \boldsymbol{\eta}_2)\| \le L_1(\|\boldsymbol{y}_1 - \boldsymbol{y}_2\| + \|\boldsymbol{\eta}_1 - \boldsymbol{\eta}_2\|) \tag{6}$$

for any variables $\boldsymbol{y}_1, \boldsymbol{y}_2$ and $\boldsymbol{\eta}_1, \boldsymbol{\eta}_2$.

Assumption 4. There exists symmetric positive definite matrices P_l such that

$$B_l^{l^T} P_l + P_l B_l^l = M_l > 0 \quad (7)$$

for $l = r_1, \cdots, r_f$.

Assumption 5. The uncertain functions $\boldsymbol{f}_k^l, (l = r_1, \cdots, r_f)$ and $\boldsymbol{f}_2$ can be evaluated by

$$\begin{aligned} &\|\boldsymbol{f}_k^l\| \le d_{1k}^l |\phi_k^l(\boldsymbol{y}^l)| + d_{0k}^l + d_{\xi k}^l \|\boldsymbol{\eta}\|, \ (1 \le k \le l) \\ &\|\boldsymbol{f}_2\| \le \sum_{l=r_1,\cdots,r_f} \left(d_{1\eta}^l |\phi_\eta^l(\boldsymbol{y}^l)| \right) + d_{0\eta} \end{aligned} \quad (8)$$

with unknown positive constants $d_{1k}^l, d_{1\eta}^l, d_{0k}^l, d_{0\eta}$ and known smooth functions ϕ_k^l, ϕ_η^l that have the following properties for any variables $\boldsymbol{y}_1^l$ and $\boldsymbol{y}_2^l$

$$\begin{aligned} |\phi_k^l(\boldsymbol{y}_1^l + \boldsymbol{y}_2^l)| &\le \|\boldsymbol{y}_1^l\| |\psi_{1k}^l(\boldsymbol{y}_1^l, \boldsymbol{y}_2^l)| + |\psi_{2k}^l(\boldsymbol{y}_2^l)| \\ |\phi_\eta^l(\boldsymbol{y}_1^l + \boldsymbol{y}_2^l)| &\le \|\boldsymbol{y}_1^l\| |\psi_{1\eta}^l(\boldsymbol{y}_1^l, \boldsymbol{y}_2^l)| + |\psi_{2\eta}^l(\boldsymbol{y}_2^l)| \end{aligned} \quad (9)$$

with known smooth functions $\psi_{1k}^l(\boldsymbol{y}_1^l, \boldsymbol{y}_2^l)$, $\psi_{1\eta}^l(\boldsymbol{y}_1^l, \boldsymbol{y}_2^l)$ and functions $\psi_{2k}^l(\boldsymbol{y}_2^l)$, $\psi_{2\eta}^l(\boldsymbol{y}_2^l)$ that are bounded for all bounded variables $\boldsymbol{y}_2^l$.

The control objective is to have the output $\boldsymbol{y}$ track a bounded reference signal $\boldsymbol{y}_m$ such as $\|\boldsymbol{y}_m\| \le d_m$ and $\|\dot{\boldsymbol{y}}_m\| \le \bar{d}_m$.

3. CONTROLLER DESIGN

3.1 Virtual system

For each subsystem with a relative degree $\{r_i, \cdots, r_i\}$ given in (4), we introduce the following virtual input filter:

$$\begin{aligned} \dot{\boldsymbol{u}}_{f,j}^{r_i} &= -\lambda \boldsymbol{u}_{f,j}^{r_i} + \boldsymbol{u}_{f,j+1}^{r_i}, (1 \le j \le r_i - 2) \\ \dot{\boldsymbol{u}}_{f,r_i-1}^{r_i} &= -\lambda \boldsymbol{u}_{f,r_i-1}^{r_i} + \boldsymbol{u}_{r_i}, \end{aligned} \quad (10)$$

with any positive constant λ, where $\boldsymbol{u}_{f,j}^{r_i} \in R^{k_i}$.

The virtual system, which is obtained by considering $\boldsymbol{u}_{f_1} = [\boldsymbol{u}_{f,1}^{r_1\,T}, \boldsymbol{u}_{f,1}^{r_2\,T}, \cdots, \boldsymbol{u}_{f,1}^{r_f\,T}]^T$ as a control input, can be expressed by the following form with an apporopriate variable transformation.

$$\begin{aligned} \dot{\boldsymbol{y}} &= M_{0y}\boldsymbol{y} + \boldsymbol{\xi}_2 + \bar{B}\boldsymbol{u}_{f_1} + \bar{\boldsymbol{f}}_1 \\ \dot{\boldsymbol{\xi}} &= \bar{A}_{u_f}\boldsymbol{\xi} + M_{1\xi}\boldsymbol{y} + \bar{\boldsymbol{f}} - M_1\bar{\boldsymbol{f}}_1 \\ \dot{\boldsymbol{\eta}} &= \boldsymbol{q}(\boldsymbol{y}, \boldsymbol{\eta}) + \boldsymbol{f}_2 \end{aligned} \quad (11)$$

Where $\boldsymbol{\xi}_2 = [\boldsymbol{\xi}_2^{r_1^T}, \boldsymbol{\xi}_2^{r_2^T}, \cdots, \boldsymbol{\xi}_2^{r_f^T}]^T$, $\boldsymbol{\xi} = [\boldsymbol{\xi}^{r_1^T}, \boldsymbol{\xi}^{r_2^T}, \cdots, \boldsymbol{\xi}^{r_f^T}]^T$ and $\boldsymbol{\xi}^{r_i} = [\boldsymbol{\xi}_2^{r_i^T}, \boldsymbol{\xi}_3^{r_i^T}, \cdots, \boldsymbol{\xi}_{r_i}^{r_i^T}]^T$. Denoting $m_{i0} = r_i$ and $m_{ij} = r_i - (r_j - 1)$, $j = 1, \cdots, i-1$ for $i = 2, \cdots, f$, the variables $\boldsymbol{\xi}_k^{r_i}$ are defined as

$$\begin{aligned} \boldsymbol{\xi}_{r_i}^{r_i} &= \boldsymbol{z}_{r_i}^{r_i} - \sum_{l=r_1,\cdots,r_i} B_l^{r_i} \boldsymbol{u}_{f,l-1}^l - \sum_{d=1}^{r_i-1} \chi_{r_i,d}^{r_i} \boldsymbol{z}_{r_i-d}^{r_i} \\ \boldsymbol{\xi}_{m_{ij}}^{r_i} &= \boldsymbol{z}_{m_{ij}}^{r_i} - \sum_{l=r_j,\cdots,r_i} B_l^{r_i} \boldsymbol{u}_{f,l-r_i+m_{ij}-1}^l \\ &\quad - \sum_{d=1}^{m_{ij}-1} \chi_{m_{ij},d}^{r_i} \boldsymbol{z}_{m_{ij}-d}^{r_i} - B_{r_j}^{r_i} B_{r_j}^{r_j\,-1} \boldsymbol{z}_1^{r_j} \end{aligned}$$

and

$$\boldsymbol{\xi}_k^{r_i} = \boldsymbol{z}_k^{r_i} - \sum_{l=r_j,\cdots,r_i} B_l^{r_i} \boldsymbol{u}_{f,l-r_i+k-1}^l - \sum_{d=1}^{k-1} \chi_{k,d}^{r_i} \boldsymbol{z}_{k-d}^{r_i}$$

for $m_{ij} + 1 \le k \le m_{ij-1} - 1$. Where

$$\begin{aligned} &\chi_{r_i,1}^{r_i} = \lambda I + A_{r_i}^{r_i} \\ &\chi_{r_i,d+1}^{r_i} = -\lambda \chi_{r_i,d}^{r_i} + A_{r_i-d}^{r_i} \ (1 \le d \le r_i - 1) \\ &\chi_{k,1}^{r_i} = \lambda I + \chi_{k+1,1}^{r_i} \\ &\chi_{k,d+1}^{r_i} = -\lambda \chi_{k,d}^{r_i} + \chi_{k+1,d+1}^{r_i} \\ &(2 \le k \le r_i - 1, \ 1 \le d \le k - 1) \end{aligned} \quad (12)$$

Further,

$$\bar{\boldsymbol{f}}_1 = [\boldsymbol{f}_1^{r_1^T}, \boldsymbol{f}_1^{r_2^T}, \cdots, \boldsymbol{f}_1^{r_f^T}]^T, \bar{\boldsymbol{f}} = [\boldsymbol{f}^{r_1^T}, \boldsymbol{f}^{r_2^T}, \cdots, \boldsymbol{f}^{r_f^T}]^T,$$

$$\boldsymbol{f}^{r_i} = [\bar{\boldsymbol{f}}_2^{r_i T}, \bar{\boldsymbol{f}}_3^{r_i T}, \cdots, \bar{\boldsymbol{f}}_{r_i}^{r_i T}]^T, \bar{\boldsymbol{f}}_k^{r_i} = \boldsymbol{f}_k^{r_i} - \sum_{d=1}^{k-1} \chi_{k,d}^{r_i} \boldsymbol{f}_{k-d}^{r_i}$$

and

$$\begin{aligned} M_{0y} &= \chi_{2,1} + M_0 \\ M_{1\xi} &= \chi - M_1(\lambda I + \chi_{2,1} + M_2) + \bar{M}_1 \end{aligned}$$

where

$$\begin{aligned} \chi_{2,1} &= \mathrm{diag}[\chi_{2,1}^{r_1}, \chi_{2,1}^{r_2}, \cdots, \chi_{2,1}^{r_f}] \\ \chi &= \mathrm{diag}[\chi^{r_1}, \chi^{r_2}, \cdots, \chi^{r_f}] \\ \bar{B} &= \mathrm{diag}[B_{r_1}^{r_1}, B_{r_2}^{r_2}, \cdots, B_{r_f}^{r_f}] \end{aligned}$$

$$M_0 = \begin{bmatrix} \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ B_{r_1}^{r_2} B_{r_1}^{r_1\,-1} & \ddots & & \vdots \\ \mathbf{0} & \ddots & \mathbf{0} & \vdots \\ \mathbf{0} & \mathbf{0} & B_{r_{f-1}}^{r_f} B_{r_{f-1}}^{r_{f-1}\,-1} & \mathbf{0} \end{bmatrix}$$

$$M_1 = \begin{bmatrix} \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ N_{r_1}^{r_2} B_{r_1}^{r_2} B_{r_1}^{r_1\,-1} & \ddots & & \vdots \\ \vdots & \ddots & \mathbf{0} & \vdots \\ N_{r_1}^{r_f} B_{r_1}^{r_f} B_{r_1}^{r_1\,-1} & \cdots & N_{r_{f-1}}^{r_f} B_{r_{f-1}}^{r_f} B_{r_{f-1}}^{r_{f-1}\,-1} & \mathbf{0} \end{bmatrix}$$

$$\bar{M}_1 = \begin{bmatrix} \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \bar{N}_{r_1}^{r_2} B_{r_1}^{r_2} B_{r_1}^{r_1\,-1} & \ddots & & \vdots \\ \vdots & \ddots & \mathbf{0} & \vdots \\ \bar{N}_{r_1}^{r_f} B_{r_1}^{r_f} B_{r_1}^{r_1\,-1} & \cdots & \bar{N}_{r_{f-1}}^{r_f} B_{r_{f-1}}^{r_f} B_{r_{f-1}}^{r_{f-1}\,-1} & \mathbf{0} \end{bmatrix}$$

$$M_2 = \begin{bmatrix} \mathbf{0} & \cdots & \cdots & \mathbf{0} & \mathbf{0} \\ B_{r_1}^{r_2} B_{r_1}^{r_1\,-1} & & & \vdots & \vdots \\ \mathbf{0} & \ddots & & \vdots & \vdots \\ \vdots & \ddots & B_{r_{f-2}}^{r_{f-1}} B_{r_{f-2}}^{r_{f-2}\,-1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

$$\chi^{r_i} = \begin{bmatrix} \chi_{2,2}^{r_i} \\ \chi_{3,3}^{r_i} \\ \vdots \\ \chi_{r_i,r_i}^{r_i} \end{bmatrix}, \quad \begin{aligned} N_l^{r_i^T} &= [\overbrace{\underbrace{\mathbf{0}, \cdots, \mathbf{0}, I}_{r_i-(l-1)}, \mathbf{0}, \cdots, \mathbf{0}}^{r_i-1}] \\ \bar{N}_l^{r_i^T} &= [\overbrace{\underbrace{\mathbf{0}, \cdots, \mathbf{0}, I}_{r_i-l}, \mathbf{0}, \cdots, \mathbf{0}}^{r_i-1}] \end{aligned}$$

Furthermore

$$\bar{A}_{u_f}=\begin{bmatrix} A_{u_f}^{r_1} & \mathbf{0} & \cdots & \cdots & \cdots & \mathbf{0} \\ H_{r_1}^{r_2} & A_{u_f}^{r_2} & \mathbf{0} & & & \vdots \\ H_{r_1}^{r_3} & H_{r_2}^{r_3} & A_{u_f}^{r_3} & \mathbf{0} & & \vdots \\ \vdots & & \ddots & \ddots & \ddots & \vdots \\ H_{r_1}^{r_f-1} & & & H_{r_f-2}^{r_f-1} & A_{u_f}^{r_f-1} & \mathbf{0} \\ H_{r_1}^{r_f} & \cdots & \cdots & \cdots & H_{r_f-1}^{r_f} & A_{u_f}^{r_f} \end{bmatrix}$$

where $A_{u_f}^{r_i}$ is the system matrix of the filter (10) and

$$H_{l_2}^{l_1}=[N_{l_2}^{l_1}B_{l_2}^{l_1}B_{l_2}^{l_2}{}^{-1}\ \mathbf{0}\ \cdots\ \mathbf{0}]$$
$$r_2\le l_1\le r_i,\ r_1\le l_2\le l_1-1$$

The uncertain nonlinearity $\bar{\boldsymbol{f}}$ can be evaluated from assumption 5 by

$$\|\bar{\boldsymbol{f}}\|\le\sum_{l=r_1,\cdots,r_f}\left(d_1^l|\phi^l(\boldsymbol{y}^l)|\right)+d_0+d_\xi\|\boldsymbol{\eta}\| \quad (13)$$

with unknown constants d_0, d_1^l, d_ξ and known functions $\phi^l(\boldsymbol{y}^l)$, which have the following properties for any variables $\boldsymbol{y}_1^l$ and $\boldsymbol{y}_2^l$:

$$\|\phi^l(\boldsymbol{y}_1^l+\boldsymbol{y}_2^l)\|\le\|\boldsymbol{y}_1^l\||\psi_1^l(\boldsymbol{y}_1^l,\boldsymbol{y}_2^l)|+|\psi_2^l(\boldsymbol{y}_2^l)| \quad (14)$$

with a known smooth function $\psi_1^l(\boldsymbol{y}_1^l,\boldsymbol{y}_2^l)$ and functions $\psi_2^l(\boldsymbol{y}_2^l)$ that are bounded for all bounded $\boldsymbol{y}_2^l$. Furthermore, since $\bar{A}_{u_f}$ is a stable matrix, there exists a positive symmetric matrix P_ξ for any positive matrix Q_ξ such that

$$P_{u_f}\bar{A}_{u_f}+\bar{A}_{u_f}^T P_{u_f}=-Q_{u_f}. \quad (15)$$

Moreover, since the system (1) is exponentially minimum-phase from assumption 2, there exists a positive definite function $W(\boldsymbol{\eta})$ and positive constants κ_1 to κ_4 such that

$$\frac{\partial W(\boldsymbol{\eta})}{\partial\boldsymbol{\eta}}\boldsymbol{q}(0,\boldsymbol{\eta})\le-\kappa_1\|\boldsymbol{\eta}\|^2,\ \left\|\frac{\partial W(\boldsymbol{\eta})}{\partial\boldsymbol{\eta}}\right\|\le\kappa_2\|\boldsymbol{\eta}\|$$
$$\kappa_4\|\boldsymbol{\eta}(t)\|^2\le\|W(\boldsymbol{\eta})\|\le\kappa_3\|\boldsymbol{\eta}(t)\|^2 \quad (16)$$

from the converse theorem of Lyapunov(Khalil 1996).

3.2 Adaptive controller design

Consider a subsystem with a relative degree $\{r_i, r_i, \cdots, r_i\}$ and an output $\boldsymbol{y}^{r_i}$.

Step1: Defining an error signal between $\boldsymbol{y}^{r_i}$ and the corresponding reference signal $\boldsymbol{y}_m^{r_i}$ by $\boldsymbol{\nu}^{r_i}=\boldsymbol{y}^{r_i}-\boldsymbol{y}_m^{r_i}$, the error system, $\boldsymbol{\nu}^{r_i}$-system, is given from (11) that

$$\dot{\boldsymbol{\nu}}^{r_i}=\chi_{2,1}^{r_i}\boldsymbol{y}^{r_i}+\boldsymbol{\xi}_2^{r_i}+B_{r_i}^{r_i}\boldsymbol{u}_{f,1}^{r_i}+\boldsymbol{f}_1^{r_i}+\bar{B}^{r_i}\boldsymbol{y}^{r_{i-1}}-\dot{\boldsymbol{y}}_m^{r_i} \quad (17)$$

where $\bar{B}^{r_i}=B_{r_{i-1}}^{r_i}B_{r_{i-1}}^{r_{i-1}}{}^{-1}$. For this error system, we first design a virtual control input $\boldsymbol{\alpha}_1^{r_i}$ for the filter signal $\boldsymbol{u}_{f,1}^{r_i}$ as follows:

$$\boldsymbol{\alpha}_1^{r_i}=-K^{r_i}\boldsymbol{\nu}^{r_i} \quad (18)$$
$$K^{r_i}=(k_I^{r_i}+k_p^{r_i}+k_r^{r_i})I_{k_i} \quad (19)$$
$$\dot{k}_I^{r_i}=\gamma_I^{r_i}\|\boldsymbol{\nu}^{r_i}\|^2-\sigma_I^{r_i}k_I^{r_i},\ k_I^{r_i}(0)\ge 0 \quad (20)$$
$$k_p^{r_i}=\gamma_p^{r_i}\phi_1^{r_i}{}^2 \quad (21)$$
$$k_r^{r_i}=\gamma_r^{r_i}(\psi_1^{r_i}{}^4+\psi_{1\eta}^{r_i}{}^4)\|\boldsymbol{\nu}^{r_i}\|^2 \quad (22)$$

Now consider the following positive definite function:

$$V_1^{r_i}=\boldsymbol{\nu}^{r_i\,T}P_{r_i}\boldsymbol{\nu}^{r_i}+\frac{m_0^{r_i}}{2\gamma_I^{r_i}}(k_I^{r_i}-k_I^{*r_i})^2 \quad (23)$$

where $k_I^{*r_i}$, which will be determined later, is an ideal feedback gain of $k_I^{r_i}$. The time derivative of $V_1^{r_i}$ is given by

$$\begin{aligned}\dot{V}_1^{r_i}=&[\chi_{2,1}^{r_i}(\boldsymbol{\nu}^{r_i}+\boldsymbol{y}_m^{r_i})+\boldsymbol{\xi}_2^{r_i}+B_{r_i}^{r_i}(\boldsymbol{\omega}_1^{r_i}+\boldsymbol{\alpha}_1^{r_i})+\boldsymbol{f}_1^{r_i}\\&+\bar{B}^{r_i}(\boldsymbol{\nu}^{r_{i-1}}+\boldsymbol{y}_m^{r_{i-1}})-\dot{\boldsymbol{y}}_m^{r_i}]^T P_{r_i}\boldsymbol{\nu}^{r_i}\\&+\boldsymbol{\nu}^{r_i^T}P_{r_i}[\chi_{2,1}^{r_i}(\boldsymbol{\nu}^{r_i}+\boldsymbol{y}_m^{r_i})+\boldsymbol{\xi}_2^{r_i}+B_{r_i}^{r_i}(\boldsymbol{\omega}_1^{r_i}\\&+\boldsymbol{\alpha}_1^{r_i})+\boldsymbol{f}_1^{r_i}+\bar{B}^{r_i}(\boldsymbol{\nu}^{r_{i-1}}+\boldsymbol{y}_m^{r_{i-1}})-\dot{\boldsymbol{y}}_m^{r_i}]\\&+\frac{m_0^{r_i}}{\gamma_I^{r_i}}(k_I^{r_i}-k_I^{*r_i})(\gamma_I^{r_i}\|\boldsymbol{\nu}\|^2-\sigma_I^{r_i}k_I^{r_i})\end{aligned} \quad (24)$$

where $\boldsymbol{\omega}_1^{r_i}=\boldsymbol{u}_{f,1}^{r_i}-\boldsymbol{\alpha}_1^{r_i}$. Since $\|\boldsymbol{\xi}\|\ge\|\boldsymbol{\xi}_2^{r_i}\|$, we have from (18) that

$$\begin{aligned}\dot{V}_1^{r_i}\le&-\boldsymbol{\nu}^{r_i^T}(K^{r_i^T}B_{r_i}^{r_i^T}P_{r_i}+P_{r_i}B_{r_i}^{r_i}K_i^r)\boldsymbol{\nu}^{r_i}\\&+2\|P_{r_i}\chi_{2,1}^{r_i}\|\|\boldsymbol{\nu}^{r_i}\|+2d_m^{r_i}\|P_{r_i}\chi_{21}^{r_i}\|\|\boldsymbol{\nu}^{r_i}\|\\&+2\|P_{r_i}\|\|\boldsymbol{\nu}^{r_i}\|\|\boldsymbol{\xi}\|+2\boldsymbol{\nu}^{r_i^T}P_{r_i}B_{r_i}^{r_i}\boldsymbol{\omega}_1^{r_i}\\&+2\|P_{r_i}\|(d_{11}^{r_i}|\phi_1^{r_i}|+d_{01}^{r_i})\|\boldsymbol{\nu}^{r_i}\|\\&+2\|P_{r_i}\bar{B}^{r_i}\|\|\boldsymbol{\nu}^{r_{i-1}}\|+2d_m^{r_{i-1}}\|P_{r_i}\bar{B}^{r_i}\|\|\boldsymbol{\nu}^{r_i}\|\\&+2\bar{d}_m^{r_i}\|P_{r_i}\|\|\boldsymbol{\nu}^{r_i}\|+2d_{\xi 1}^{r_i}\|P_{r_i}\|\|\boldsymbol{\nu}^{r_i}\|\|\boldsymbol{\eta}\|\\&+m_0^{r_i}k_I^{r_i}\|\boldsymbol{\nu}^{r_i}\|^2-m_0^{r_i}k_I^{r_i*}\|\boldsymbol{\nu}^{r_i}\|^2\\&-\frac{m_0^{r_i}\sigma_I^{r_i}}{\gamma_I^{r_i}}(k_I^{r_i}-k_I^{r_i*})k_I^{r_i*}\end{aligned} \quad (25)$$

Here, from the fact that $k_I^{r_1}, k_p^{r_1}, k_r^{r_1}\ge 0$ and from the structure of K^{r_i} and assumption 4, it follows that

$$\begin{aligned}&-\boldsymbol{\nu}^{r_i^T}(K^{r_i^T}B_{r_i}^{r_i^T}P_{r_i}+P_{r_i}B_{r_i}^{r_i}K_i^r)\boldsymbol{\nu}^{r_i}\\&=-(k_I^{r_i}+k_p^{r_i}+k_r^{r_i})\boldsymbol{\nu}^{r_i^T}M_{r_i}\boldsymbol{\nu}^{r_i}\\&\le-(k_I^{r_i}+k_p^{r_i}+k_r^{r_i})m_0^{r_i}\|\boldsymbol{\nu}^{r_i}\|^2\end{aligned} \quad (26)$$

Thus, the time derivative of $V_1^{r_i}$ can be evaluated by

$$\begin{aligned}\dot{V}_1^{r_i}\le&-(m_0^{r_i}k_I^{*r_i}-v_0^{r_i})\|\boldsymbol{\nu}^{r_i}\|^2+\rho_2^{r_i}\|\boldsymbol{\xi}\|^2+\rho_3^{r_i}\|\boldsymbol{\eta}\|^2\\&-\frac{m_0^{r_i}\sigma_I^{r_i}}{\gamma_I^{r_i}}(1-\rho_4^{r_i})(k_I^{r_i}-k_I^{r_i*})^2\\&-m_0^{r_i}k_r^{r_i}\|\boldsymbol{\nu}^{r_i}\|^2+2\|P_{r_i}B_{r_i}^{r_i}\|\|\boldsymbol{\nu}^{r_i}\|\|\boldsymbol{\omega}_1^{r_i}\|\\&+\frac{\|P_{r_i}B_{r_{i-1}}^{r_i}B_{r_{i-1}}^{r_{i-1}}{}^{-1}\|}{\rho_0^{r_i}}\|\boldsymbol{\nu}^{r_{i-1}}\|^2+R_1^{r_i}\end{aligned} \quad (27)$$

where $\rho_0^{r_i}\sim\rho_4^{r_i}$ are any positive constants and

$$v_0^{r_i}=2\|P_{r_i}\chi_{2,1}^{r_i}\|+\rho_1^{r_i}+\frac{\|P_{r_i}\|}{\rho_2^{r_i}}+\frac{(d_{\xi 1}^{r_i}\|P_{r_i}\|)^2}{\rho_3^{r_i}}+\rho_0^{r_i}$$

$$\begin{aligned}R_1^{r_i}=&\frac{\sigma_R^{r_i^2}}{\rho_1^{r_i}}+\frac{m_0^{r_i}\sigma_I^{r_i}k_I^{r_i*2}}{4\rho_4^{r_i}\gamma_I^{r_i}}+\frac{(d_{11}^{r_i}\|P_{r_i}\|)^2}{m_0^{r_i}\gamma_p^{r_i}}\\\sigma_R^{r_i}=&\|P_{r_i}\|(d_m^{r_i}\|\chi_{2,1}^{r_i}\|+\bar{d}_m^{r_i}+d_{01}^{r_i}\\&+d_m^{r_{i-1}}\|B_{r_{i-1}}^{r_i}B_{r_{i-1}}^{r_{i-1}}{}^{-1}\|)\end{aligned}$$

Step k ($2\le k\le r_i-1$): Setting $\boldsymbol{\omega}_{k-1}^{r_i}=\boldsymbol{u}_{f,k-1}^{r_i}-\boldsymbol{\alpha}_{k-1}^{r_i}$, the $\boldsymbol{\omega}_{k-1}^{r_i}$-system is given by

$$\dot{\boldsymbol{\omega}}_{k-1}^{r_i} = -\lambda \boldsymbol{u}_{f,k-1}^{r_i} + \boldsymbol{u}_{f,k}^{r_i} - \frac{\partial \boldsymbol{\alpha}_{k-1}^{r_i}}{\partial \boldsymbol{y}_m^{r_i}} \dot{\boldsymbol{y}}_m^{r_i} - \frac{\partial \boldsymbol{\alpha}_{k-1}^{r_i}}{\partial k_I^{r_i}} (\gamma_I^{r_i} \|\boldsymbol{\nu}^{r_i}\| - \sigma_I^{r_i} k_I^{r_i}) - \sum_{j=1}^{k-2} \frac{\partial \boldsymbol{\alpha}_{k-1}^{r_i}}{\partial \boldsymbol{u}_{f,j}^{r_i}} (-\lambda \boldsymbol{u}_{f,j}^{r_i} + \boldsymbol{u}_{f,j+1}^{r_i}) - \frac{\partial \boldsymbol{\alpha}_{k-1}^{r_i}}{\partial \boldsymbol{y}^{r_i}} [\chi_{2,1}(\boldsymbol{\nu}^{r_i} + \boldsymbol{y}_m^{r_i}) + \boldsymbol{\xi}_2^{r_i} + B_{r_i}^{r_i} \boldsymbol{u}_{f,1}^{r_i} + \boldsymbol{f}_1^{r_i} + B_{r_{i-1}}^{r_i} B_{r_{i-1}}^{r_{i-1}\,-1} (\boldsymbol{\nu}^{r_{i-1}} + \boldsymbol{y}_m^{r_{i-1}})] \tag{28}$$

Taking this into consideration, we design a virtual control input $\boldsymbol{\alpha}_k^{r_i}$ for the filter signal $\boldsymbol{u}_{f,k}^{r_i}$ as follows:

$$\boldsymbol{\alpha}_k^{r_i} = -c_{k-1}^{r_i} \boldsymbol{\omega}_{k-1}^{r_i} - \boldsymbol{\omega}_{k-2}^{r_i} + \lambda \boldsymbol{u}_{f,1}^{r_i} - \epsilon_{k-1}^{r_i} \Psi_{k-1}^{r_i} \boldsymbol{\omega}_{k-1}^{r_i} + \frac{\partial \boldsymbol{\alpha}_{k-1}^{r_i}}{\partial k_I^{r_i}} (\gamma_I^{r_i} \|\boldsymbol{\nu}^{r_i}\|^2 - \sigma_I^{r_i} k_I^{r_i}) + \sum_{j=1}^{k-2} \frac{\partial \boldsymbol{\alpha}_{k-1}^{r_i}}{\partial \boldsymbol{u}_{f,j}^{r_i}} (-\lambda \boldsymbol{u}_{f,j}^{r_i} + \boldsymbol{u}_{f,j+1}^{r_i}) \tag{29}$$

$$\Psi_{k-1}^{r_i} = (\phi_1^{r_i\,2} + \|\boldsymbol{u}_{f_1}^{r_i}\|^2 + l_{k-1}^{r_i}) \left\| \frac{\partial \boldsymbol{\alpha}_{k-1}^{r_i}}{\partial \boldsymbol{y}^{r_i}} \right\|^2 + \left\| \frac{\partial \boldsymbol{\alpha}_{k-1}^{r_i}}{\partial \boldsymbol{y}_m^{r_i}} \right\|^2$$

where $c_{k-1}^{r_i}, \epsilon_{k-1}^{r_i}, l_{k-1}^{r_i}$ are any positive constants.

Consider a positive definite function $V_k^{r_i}$:

$$V_k^{r_i} = V_{k-1}^{r_i} + \frac{1}{2} \boldsymbol{\omega}_{k-1}^{r_i\,T} \boldsymbol{\omega}_{k-1}^{r_i} \tag{30}$$

The time derivative of $V_k^{r_i}$ can be evaluated by

$$\begin{aligned}\dot{V}_k^{r_i} \leq & -(m_0^{r_i} k_I^{r_i *} - \bar{v}_0^{r_i} - \sum_{j=1}^{k-1} \frac{\|\chi_{2,1}\|^2}{\epsilon_j^{r_i} l_j^{r_i}}) \|\boldsymbol{\nu}^{r_i}\|^2 \\ & + (\rho_2^{r_i} + \sum_{j=1}^{k-1} \frac{1}{\epsilon_j^{r_i} l_j^{r_i}}) \|\boldsymbol{\xi}\|^2 - m_0^{r_i} k_r^{r_i} \|\boldsymbol{\nu}^{r_i}\|^2 \\ & + (\rho_3^{r_i} + \sum_{j=1}^{k-1} \frac{d_{\xi 1}^{r_i\,2}}{\epsilon_j^{r_i} l_j^{r_i}}) \|\boldsymbol{\eta}\|^2 + \boldsymbol{\omega}_k^{r_i\,T} \boldsymbol{\omega}_{k-1}^{r_i} \\ & - \frac{m_0^{r_i} \sigma_I^{r_i}}{\gamma_I^{r_i}} (1 - \rho_4^{r_i})(k_I^{r_i} - k_I^{r_i *})^2 \\ & - (c_1^{r_i} - \rho_5^{r_i}) \|\boldsymbol{\omega}_1^{r_i}\|^2 - \sum_{j=2}^{k-1} c_j^{r_i} \|\boldsymbol{\omega}_j^{r_i}\|^2 + R_k^{r_i} \\ & + \left(\sigma_\rho^{r_i} + \sum_{j=1}^{k-1} \frac{\|B_{r_{i-1}}^{r_i} B_{r_{i-1}}^{r_{i-1}\,-1}\|^2}{\epsilon_j^{r_i} l_j^{r_i}} \right) \|\boldsymbol{\nu}^{r_{i-1}}\|^2 \end{aligned} \tag{31}$$

where $\rho_5^{r_i}$ is any positive constant and

$$\sigma_\rho^{r_i} = \frac{\|P_{r_i} B_{r_{i-1}}^{r_i} B_{r_{i-1}}^{r_{i-1}\,-1}\|}{\rho_0^{r_i}}$$

$$R_k^{r_i} = R_{k-1}^{r_i} + \frac{1}{\epsilon_{k-1}^{r_i}} \left[\frac{1}{l_{k-1}^{r_i}} (d_m^{r_i} \|\chi_{2,1}^{r_i}\| + d_{01}^{r_i} + d_m^{r_{i-1}} \|B_{r_{i-1}}^{r_i} B_{r_{i-1}}^{r_{i-1}\,-1}\|)^2 + \|B_{r_i}^{r_i}\|^2 + d_{11}^{r_i\,2} + \bar{d}_m^{r_i\,2} \right]$$

Step r_i: In this step we can design an actual control input $\boldsymbol{u}_{r_i}$ as

$$\boldsymbol{u}_{r_i} = \boldsymbol{\alpha}_{r_i}^{r_i} \tag{32}$$

$\boldsymbol{\alpha}_{r_i}^{r_i}$ is designed in (29) with $k = r_i$.

Consider the following positive function:

$$V_{r_i}^{r_i} = V_{r_i-1}^{r_i} + \frac{1}{2} \boldsymbol{\omega}_{r_i-1}^{r_i\,T} \boldsymbol{\omega}_{r_i-1}^{r_i} \tag{33}$$

The time derivative of $V_{r_i}^{r_i}$ can be evaluated by

$$\begin{aligned}\dot{V}_{r_i}^{r_i} \leq & -(m_0^{r_i} k_I^{r_i *} - \tilde{v}_0^{r_i}) \|\boldsymbol{\nu}^{r_i}\|^2 + v_1^{r_i} \|\boldsymbol{\xi}\|^2 \\ & + v_2^{r_i} \|\boldsymbol{\eta}\|^2 - \frac{m_0^{r_i} \sigma_I^{r_i}}{\gamma_I^{r_i}} (1 - \rho_4^{r_i})(k_I^{r_i} - k_I^{r_i *})^2 \\ & - (c_1^{r_i} - \rho_5^{r_i}) \|\boldsymbol{\omega}_1^{r_i}\|^2 - \sum_{j=2}^{r_i-1} c_j^{r_i} \|\boldsymbol{\omega}_j^{r_i}\|^2 \\ & - m_0^{r_i} k_r^{r_i} \|\boldsymbol{\nu}^{r_i}\|^2 + v_3^{r_i} \|\boldsymbol{\nu}^{r_{i-1}}\|^2 + R_{r_i}^{r_i} \end{aligned} \tag{34}$$

where

$$\tilde{v}_0^{r_i} = \bar{v}_0^{r_i} + \sum_{j=1}^{r_i-1} \frac{\|\chi_{2,1}^{r_i}\|^2}{\epsilon_j^{r_i} l_j^{r_i}},\ v_1^{r_i} = \rho_2^{r_i} + \sum_{j=1}^{r_i-1} \frac{1}{\epsilon_j^{r_i} l_j^{r_i}}$$

$$v_2^{r_i} = \rho_3^{r_i} + \sum_{j=1}^{r_i-1} \frac{d_{\xi 1}^{r_i\,2}}{\epsilon_j^{r_i} l_j^{r_i}},$$

$$v_3^{r_i} = \sigma_\rho^{r_i} + \sum_{j=1}^{r_i-1} \frac{\|B_{r_{i-1}}^{r_i} B_{r_{i-1}}^{r_{i-1}\,-1}\|^2}{\epsilon_j^{r_i} l_j^{r_i}}$$

$$R_{r_i}^{r_i} = R_{r_i-1}^{r_i} + \frac{1}{4\epsilon_{r_i-1}^{r_i}} \left[\frac{4}{l_{r_i-1}^{r_i}} (d_m^{r_i} \|\chi_{2,1}^{r_i}\| + d_{01}^{r_i} + d_m^{r_{i-1}} \|B_{r_{i-1}}^{r_i} B_{r_{i-1}}^{r_{i-1}\,-1}\|)^2 + \|B_{r_i}^{r_i}\|^2 + d_{11}^{r_i\,2} + \bar{d}_m^{r_i\,2} \right]$$

4. MAIN RESULTS

Theorem 1. Under assumptions 1 to 5 on the controlled system (1), all the signals in the resulting closed-loop system with a control input $\boldsymbol{u} = [\boldsymbol{u}_{r_1}^T, \boldsymbol{u}_{r_2}^T, \cdot\cdot, \boldsymbol{u}_{r_f}^T]^T$ designed in (32) are bounded. Further, an appropriate choice of controller parameters makes it possible for the tracking error $\boldsymbol{\nu}^l$, $l = r_1, \cdots, r_f$, to converge to any given bound such as

$$\lim_{t \to \infty} \|\boldsymbol{\nu}^l\| \leq \delta^l \tag{35}$$

Proof : Consider the following positive definite function:

$$V = V_y + V_\xi \tag{36}$$

where $V_y = \sum_{l=r_1,\cdots,r_f} V_l^l$ and

$$V_\xi = \mu_0 \boldsymbol{\xi}^T P_{u_f} \boldsymbol{\xi} + \mu_1 W(\boldsymbol{\eta}) \tag{37}$$

where μ_0 and μ_1 are any positive constants to be determined later. From (15) and (16), the time derivative of V_ξ can be evaluated by

$$\begin{aligned}\dot{V}_\xi \leq & -(\mu_0 \lambda_Q - \sum_{l=r_1,\cdots,r_f} (\rho_6^l + \rho_8^l + \rho_{10}^l + \frac{\beta_6^2}{4\rho_{12}^l})) \|\boldsymbol{\xi}\|^2 \\ & - (\mu_1 \kappa_1 - \sum_{l=r_1,\cdots,r_f} (\rho_7^l + \rho_9^l + \rho_{11}^l + \rho_{12}^l)) \|\boldsymbol{\eta}\|^2 \\ & + \sum_{l=r_1,\cdot\cdot,r_f} \left[\frac{\beta_2^2}{\rho_6^l} + \frac{\beta_3^2}{4\rho_7^l} \right] \|\boldsymbol{\nu}^l\|^2 + R_\xi \\ & + \sum_{l=r_1,\cdots,r_f} \left[\frac{\beta_4^2}{\rho_8^l} |\psi_1^l|^2 + \frac{\beta_5^2}{4\rho_9^l} |\psi_{1\eta}^l|^2 \right] \|\boldsymbol{\nu}^l\|^2 \end{aligned} \tag{38}$$

with any positive constants ρ_6^l to ρ_{12}^l. Where,

$$\beta_0 = \|P_{u_f}\|\|M_{1\xi}\|,\ \beta_1 = \|P_{u_f}\|(1+\|M_1\|)$$
$$\beta_2 = \mu_0\beta_0,\ \beta_3 = \mu_1 L_1\kappa_2,\ \beta_4 = \mu_0\beta_1 d_1^l$$
$$\beta_5 = \mu_0\kappa_2 d_{1\eta}^l,\ \beta_6 = 2\mu_0\beta_1 d_\xi,$$
$$\lambda_Q = \lambda_{min}[Q_{u_f}]$$

$$R_\xi = \sum_{l=r_1,\cdots,r_f}\left[\frac{\mu_0^2[\beta_0 d_m^l + \beta_1(d_1^l\psi_{2M}^l + d_0)]^2}{\rho_{10}^l} + \frac{\mu_0^2\kappa_2^2[L_1 d_m^l + d_{1\eta}^l\psi_{2\eta M}^l + d_{0\eta}]^2}{\rho_{11}^l}\right]$$

and ψ_{2M}^l and $\psi_{2\eta M}^l$ are positive constants such that $|\psi_2(\boldsymbol{y}_m^l)^l| \le \psi_{2M}^l, |\psi_{2\eta}(\boldsymbol{y}_m^l)^l| \le \psi_{2\eta M}^l$. Since $\boldsymbol{y}_m^l(l = r_1,\cdot\cdot,r_f)$ are bounded, such constants exist from assumption 5.

Consequently, from (34),(38) and (22), we have

$$\dot V \le -\sum_{l=r_1,\cdots,r_f}\left[(m_0^l k_I^{l*} - \hat v_0^l)\|\boldsymbol{\nu}^l\|^2 + \frac{m_0^l\sigma_I^l}{\gamma_I^l}(1-\rho_4^l)(k_I^l - k_I^{l*})^2\right] - (\mu_0\lambda_Q - v_\xi)\|\boldsymbol{\xi}\|^2 - (\mu_1\kappa_1 - v_\eta)\|\boldsymbol{\eta}\|^2 + R - \sum_{l=r_1,\cdots,r_f}\left[(c_1^l - \rho_5^l)\|\boldsymbol{\omega}_1\|^2 + \sum_{j=2}^{l-1} c_j^l\|\boldsymbol{\omega}_j^l\|^2\right] \tag{39}$$

where

$$\hat v_0^l = \tilde v_0^l + v_3^{l+1} + \frac{\beta_2^2}{{\rho_6^l}^2} + \frac{\beta_3^2}{4\rho_7^l}\ (l = r_1,\cdot\cdot,r_{f-1})$$
$$\hat v_0^{r_f} = \tilde v_0^{r_f} + \frac{\beta_2^2}{{\rho_6^{r_f}}^2} + \frac{\beta_3^2}{4\rho_7^{r_f}}$$
$$v_\xi = \sum_{l=r_1,\cdot\cdot,r_f}(\rho_6^l + \rho_8^l + \rho_{10}^l + \frac{\beta_6^2}{4\rho_{12}^l} + v_1^l)$$
$$v_\eta = \sum_{l=r_1,\cdot\cdot,r_f}(\rho_7^l + \rho_9^l + \rho_{11}^l + \rho_{12}^l + v_2^l)$$
$$R = \sum_{l=r_1,\cdot\cdot,r_f} R_l^l + R_\xi + \sum_{l=r_1,\cdot\cdot,r_f}\left[\frac{\beta_4^4}{4m_0^l\gamma_r^l{\rho_8^l}^2} + \frac{\beta_5^4}{m_0^l\gamma_r^l{\rho_9^l}^2}\right]$$

Finally, setting $\rho_2^l = \rho_6^l = \rho_8^l = \rho_{10}^l = \frac{\mu_0\lambda_Q}{10f}, \rho_3^l = \rho_7^l = \rho_9^l = \rho_{11}^l = \rho_{12}^l = \frac{\mu_1\kappa_1}{15f}, \rho_4^l = \frac{1}{2}, \rho_5^l = \frac{c_1^l}{2}, \mu_0 = \frac{5}{\lambda_Q}[\frac{2\beta_6^2}{\mu_1\kappa_1} + \sum_{l=r_1,\cdot\cdot,r_f}\sum_{j=1}^{l}\frac{1}{\epsilon_j^l l_j^l}], \mu_1 = \frac{6}{\kappa_1}\sum_{l=r_1,\cdot\cdot,r_f}\sum_{j=1}^{l}\frac{{d_{\xi 1}^l}^2}{\epsilon_j^l l_j^l}]$ and considering ideal feedback gains ${k_I^l}^*$ which satisfy the following inequality for $\gamma_v^l > 0$

$$m_0^l {k_I^l}^* \ge \hat v_0^l + \gamma_v^l\lambda_{max}[P_l],\ (l = r_1,\cdots,r_f) \tag{40}$$

the time derivative of V can be evaluated by

$$\dot V \le -\alpha_v V + R \tag{41}$$

$$\alpha_v = min[\gamma_v^l, \sigma_I^l, \frac{\lambda_Q}{2\lambda_{max}[P_{u_f},]}, \frac{\kappa_1}{2\kappa_3}, c_1^l, 2c_j^l]$$
$$(l = r_1,\cdots,r_f,\ 1 \le j \le l-1)$$

It is apparent from (41) that all the signals in the closed-loop system are bounded. We also obtain from (41) that

$$\lim_{t\to\infty} V \le R/\alpha_v \tag{42}$$

From this result, we have

$$\lim_{t\to\infty}\|\nu^l\| \le 2R/\alpha_v\lambda_{max}[P_l] \tag{43}$$

This means that the goal (35) can be achieved for δ^l such as ${\delta^l}^2 \ge 2R/\alpha_v\lambda_{max}[P_l]$. It is also easily confirmed that an appropriate choice of controller parameters ensures the control objective (35) for any δ^l. ∎

5. CONCLUSIONS

In this paper, we proposed a design scheme for a robust adaptive tracking control system for uncertain MIMO nonlinear systems. The proposed method can be applied to a system with a higher order relative degree and unknown system order.

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