ANALYSIS AND SYNTHESIS OF NETWORKED CONTROL SYSTEMS: TOPOLOGICAL ENTROPY, OBSERVABILITY, ROBUSTNESS AND OPTIMAL CONTROL

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Abstract: This paper extends the concept of topological entropy to the case of uncertain dynamical systems. We address problems of observability and optimal control via limited capacity digital communication channels. The main results are given in terms of inequalities between date rate of the communication channel and topological entropy of the open-loop system. Topological entropy is calculated for some classes of uncertain dynamical systems.

Keywords: Networked Control Systems, Topological Entropy, Uncertain Systems, Observability, Optimal Control

1. INTRODUCTION

A standard assumption in the classical control theory is that the data transmission required by the control or state estimation algorithm can be performed with infinite precision. However, due to the growth in communication technology, it is becoming more common to employ digital limited capacity communication networks for exchange of information between system components. The resources available in such systems for communication between sensors, controllers and actuators can be severely limited due to size or cost.

In recent years there has been a significant interest in the problem of control and state estimation via a digital communication channel with bit-rate constraint; e.g., see (Wong and Brockett, 1997; Brockett and Liberzon, 2000; Tatikonda, 2000; Nair and Evans, 2002; Nair and Evans, 2003; Savkin and Petersen, 2003; Jain *et al.*, 2002; Matveev and Savkin, 2004*c*; Nair *et al.*, 2004; Matveev and Savkin, 2004*b*; Matveev and Savkin, 2004*a*). A stochastic setting for the problem of stabilization via a perfect noiseless digital channel was investigated by Nair and Evans in (Nair and Evans, 2002), where an important fundamental result on minimum data rates was obtained. Minimum data rates for stabilization and state estimation via such channels were also studied in (Wong and Brockett, 1997; Tatikonda, 2000; Nair and Evans, 2003). The problems of state estimation and stabilization via noisy digital channels were studied in (Matveev and Savkin, 2004*b*; Matveev and Savkin, 2004*a*), and necessary and sufficient conditions were given in terms of the classic Shannon's communication channel capacity.

In this paper, we study connections between observability and optimal control via digital channels and topological entropy of the open-loop system. The concept of entropy of dynamical systems originated in the work of Kolmogorov (Kolmogorov, 1958; Kolmogorov, 1959) and was inspired by the Shannon's pioneering paper (Shannon,

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1948). Kolmogorov's work started a whole new research direction in which entropy appears as a numerical invariant of a class of deterministic dynamical systems. Later, Adler and his co-authors introduced topological entropy of dynamical systems (Adler *et al.*, 1965) which is a modification of Kolmogorov's metric entropy. The pioneering paper (Nair *et al.*, 2004) imported the concept of topological entropy into the theory of networked control systems. The concept of feedback topological entropy was introduced and condition of local stabilizability of nonlinear systems via a limited capacity channel was given. In this paper, we extend the concept of topological entropy to the case of uncertain dynamical systems with non-compact state space. Unlike (Nair *et al.*, 2004), we use a less common "metric" definition of topological entropy, which is, in our opinion, more suitable to the theory of networked control systems. The main results of the paper are necessary and sufficient conditions of robust observability and solvability of the optimal control problem that are given in terms of inequalities between the communication channel data rate and the topological entropy of the open-loop system.

Due to page limitation, all the results are given without proofs. The proofs will be given in the full version of the paper.

2. OBSERVABILITY VIA COMMUNICATION CHANNELS

In this section, we consider a nonlinear uncertain discrete-time dynamical system of the form:

$$x(t+1) = F(x(t), \omega(t)), \qquad x(1) \in \mathcal{X}_1, \qquad x(t) \in \mathcal{X}, \tag{1}$$

where $t = 1, 2, 3, \ldots$, $x(t) \in \mathbf{R}^n$ is the state, $\omega(t) \in \Omega$ is the uncertainty input, $\mathcal{X} \subset \mathbf{R}^n$ is a given set, $\mathcal{X}_1 \subset \mathcal{X}$ is a given non-empty compact set, and $\Omega \subset \mathbf{R}^m$ is a given set. Notice that we do not assume that the function $F(\cdot, \cdot)$ is continuous.

In our observability problem, a sensor measures the state $x(t)$ and is connected to the controller that is at the remote location. Moreover, the only way of communicating information from the sensor to that remote location is via a digital communication channel which carries one discrete-valued symbol $h(jT)$ at time jT, selected from a coding alphabet $\mathcal{H}$ of size l. Here $T \geq 1$ is a given integer period, and $j = 1, 2, 3, \ldots$.

This restricted number l of codewords $h(jT)$ is determined by the transmission data rate of the channel. For example, if μ is the number of bits that our channel can transmit at any time instant, then $l = 2^\mu$ is the number of admissible codewords. We assume that the channel is a perfect noiseless channel and there is no time delay. Let

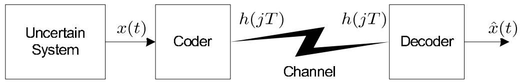

Fig. 1. State estimation via digital communication channel

$R \geq 0$ be a given constant. We consider the class $\mathcal{C}_R$ of such channels with any period T satisfying the following transmission data rate constraint:

$$\frac{\log_2 l}{T} \leq R. \tag{2}$$

The rate $R = 0$ corresponds to the case when the channel does not transmit data at all.

We consider the problem of estimation of the state $x(t)$ via a digital communication channel with a bit-rate constraint. Our state estimator consists of two components. The first component is developed at the measurement location by taking the measured state $x(\cdot)$ and coding to the codeword $h(jT)$. This component will be called "coder". Then the codeword $h(jT)$ is transmitted via a limited capacity communication channel to the second component which is called "decoder". The second component developed at the remote location takes the codeword $h(jT)$ and produces the estimated state $\hat{x}((j-1)T+1), \ldots, \hat{x}(jT-1), \hat{x}(jT)$. This situation is illustrated in Figure 1.

The coder and the decoder are of the following form:

Coder:

$$h(jT) = \mathcal{F}_j \left(x(\cdot)|_1^{jT} \right); \tag{3}$$

Decoder:

$$\begin{pmatrix} \hat{x}((j-1)T+1) \\ \vdots \\ \hat{x}(jT-1) \\ \hat{x}(jT) \end{pmatrix} = \mathcal{G}_j \left(h_T, h_{2T}, ..., h_{(j-1)T}, h_{jT} \right). \tag{4}$$

Here $j = 1, 2, 3, \ldots$.

Notation 1. Let $x = \begin{bmatrix} x_1 & \ldots & x_n \end{bmatrix}$ be a vector from $\mathbf{R}^n$. Then

$$\|x\|_\infty := \max_{j=1,\ldots,n} |x_j|. \tag{5}$$

Furthermore, $\|\cdot\|$ denotes the standard Euclidean vector norm:

$$\|x\| := \sqrt{\sum_{j=1}^{n} x_j^2}.$$

Definition 1. The system (1) is said to be observable in the communication channel class $\mathcal{C}_R$ if for any $\epsilon > 0$ there exists a period $T \geq 1$ and a coder-decoder pair (3), (4) with a coding alphabet of size l satisfying the constraint (2) such that

$$\|x(t) - \hat{x}(t)\|_\infty < \epsilon \quad \forall t = 1, 2, 3, \dots \qquad (6)$$

for any solution of (1).

3. TOPOLOGICAL ENTROPY AND OBSERVABILITY OF UNCERTAIN SYSTEMS

In this section, we introduce the concept of topological entropy for the system (1). In general, we follow the scheme of (Pollicott and Yuri, 1998), however, unlike (Pollicott and Yuri, 1998) we consider uncertain dynamical systems.

Notation 2. For any $k \geq 1$, let $\mathcal{X}_k := \{x(1), \dots, x(k)\}$ is the set of solutions of (1) with uncertainty inputs from Ω.

Definition 2. Consider the system (1). For $k \geq 1$ and $\epsilon > 0$ we call a finite set $Q \subset \mathcal{X}_k$ an $(k, \epsilon)-$spanning set if for any $x_a(\cdot) \in \mathcal{X}_k$ there exists an element $x_b(\cdot) \in Q$ such that $\|x_a(t) - x_b(t)\|_\infty < \epsilon$ for all $t = 1, 2, \dots, k$. Let $q(k, \epsilon)$ denotes the least cardinality of any $(k, \epsilon)-$spanning set.

Now we are in a position to give a definition of topological entropy for the uncertain dynamical system (1).

Definition 3. The quantity

$$H(F(\cdot,\cdot), \mathcal{X}_1, \mathcal{X}, \Omega) := \lim_{\epsilon \to 0} \limsup_{k \to \infty} \frac{1}{k} \log_2(q(k, \epsilon)) \qquad (7)$$

is called the topological entropy of the uncertain system (1).

Remark 1. We use "metric" definition of topological entropy that is different from the more common "topological" definition (see e.g. p. 20 of (Pollicott and Yuri, 1998)). In the case of continuous system without uncertainty both these definitions are equivalent (Pollicott and Yuri, 1998). Notice that the topological entropy may be equal to infinity. In the case of a system without uncertainty with continuous $F(\cdot,\cdot)$ and compact $\mathcal{X}$, the topological entropy is always finite (Pollicott and Yuri, 1998).

Now we are in a position to present the main result of this section.

Theorem 1. Consider the system (1) and let $R \geq 0$ be a given constant. Then the following two statements hold.

(1) If $R < H(F(\cdot,\cdot), \mathcal{X}_1, \mathcal{X}, \Omega)$ then the system (1) is not observable in the communication channel class $\mathcal{C}_R$.
(2) Assume that $\mathcal{X} = \mathcal{X}_1$ (hence, $\mathcal{X}$ is compact). If $R > H(F(\cdot,\cdot), \mathcal{X}_1, \mathcal{X}, \Omega)$ then the system (1) is observable in the communication channel class $\mathcal{C}_R$.

The proof will be given in the full version of the paper.

Definition 4. The system (1) is said to be robustly stable if for any $\epsilon > 0$ there exists an integer $k \geq 1$ such that

$$\|x(t)\|_\infty < \epsilon \quad \forall t \geq k \qquad (8)$$

for any solution $x(\cdot)$ of the system (1).

Proposition 1. Consider the system (1) and assume that Ω is compact, $F(\cdot,\cdot)$ is continuous and the system (1) is robustly stable. Then, $H(F(\cdot,\cdot), \mathcal{X}_1, \mathcal{X}, \Omega) = 0$.

The proof will be given in the full version of the paper.

Definition 5. Let $x(\cdot)$ be a solution of (1). The system (1) is said to be locally reachable along the trajectory $x(\cdot)$ if there exists a constant $\delta > 0$ and an integer $N \geq 1$ such that for any $k \geq 1$ and any $a, b \in \mathcal{X}$ satisfying

$$\|x(k) - a\| \leq \delta \|x(k)\|;$$
$$\|x(k+N) - b\| \leq \delta \|x(k+N)\|$$

there exists a solution $\tilde{x}(\cdot)$ of (1) with

$$\tilde{x}(k) = a, \quad \tilde{x}(k+N) = b.$$

Definition 6. A solution $x(\cdot)$ of (1) is said to be separated from the origin, if there exist a constant $\delta_0 > 0$ such that

$$\|x(t)\| \geq \delta_0 \quad \forall t \geq 1.$$

We will use the following assumptions.

Assumption 1. The system (1) is locally reachable along a trajectory separated from the origin.

Assumption 2. The system (1) is locally reachable along a trajectory $x(\cdot)$ such that $\|x(t)\|_\infty \to \infty$ as $t \to \infty$.

Theorem 2. Consider the system (1). The following two statements hold.

(1) If Assumption 1 is satisfied, then $H(F(\cdot,\cdot), \mathcal{X}_1, \mathcal{X}, \Omega)$ equals infinity, hence, according to Theorem 1, the system (1) is not observable in the communication channel class $\mathcal{C}_R$ with any R.

(2) If Assumption 2 is satisfied, then for any coder-decoder pair of the form (3), (4) with any T, R

$$\sup_{t, x(\cdot)} \|x(t) - \hat{x}(t)\|_\infty = \infty,$$

where the supremum is taken over all times t and all solutions $x(\cdot)$ of the system (1).

The proof will be given in the full version of the paper.

4. THE CASE OF LINEAR SYSTEMS

In this section, we first consider a linear system without uncertainty:

$$x(t+1) = Ax(t), \quad x(1) \in \mathcal{X}_1 \tag{9}$$

where $t = 1, 2, 3, \ldots,$ $x(t) \in \mathbf{R}^n$ is the state, $\mathcal{X}_1 \subset \mathbf{R}^n$ is a given compact set, and A is a given square matrix.

We will suppose that the following assumption holds.

Assumption 3. The origin is an interior point of the set $\mathcal{X}_1$: there exists a constant $\delta > 0$ such that

$$\|a\|_\infty < \delta \quad \Rightarrow a \in \mathcal{X}_1.$$

Furthermore, let $S(A) = \{\lambda_1, \lambda_2, \ldots, \lambda_n\}$ be the set of eigenvalues of the matrix A. Introduce the following value:

$$H(A) := \sum_{\lambda_i \in S(A)} \lg_2(\max\{1, |\lambda_i|\}). \tag{10}$$

Theorem 3. Consider the system (9) and suppose that Assumption 3 holds. Then, the topological entropy of the system (9) is equal to $H(A)$ where $H(A)$ is defined by (10).

The proof will be given in the full version of the paper.

The following corollary immediately follows from Theorem 3.

Corollary 1. Consider the system (9) and suppose that Assumption 3 holds. Then, the topological entropy of the system (9) is equal to 0 if and only if $|\lambda| \leq 1$ for any eigenvalue λ of the matrix A.

Remark 2. Theorem 3 together with Theorem 1 give an "almost" necessary and sufficient condition for observability of the system (9) in the communication channel class $\mathcal{C}_R$. Moreover, combining Theorem 3 with the results of (Matveev and Savkin, 2004*b*; Matveev and Savkin, 2004*a*), we obtain "almost" necessary and sufficient conditions for observability and stabilizability of linear systems via a noisy discrete channel in terms of inequalities between the classical Shannon's channel capacity and the topological entropy of the open-loop system.

Remark 3. Notice that, in fact, results similar to Theorem 3, but stated in different terms, were derived in (Nair and Evans, 2003; Nair *et al.*, 2004; Matveev and Savkin, 2004*d*). Also, Theorem 3 reminds the well-known result on topological entropy of algebraic automorphisms of torus; see e.g. (Adler and Weiss, 1967).

Now consider a linear uncertain discrete-time dynamical system of the form:

$$x(t+1) = [A + B\omega(t)]\, x(t), \quad x(1) \in \mathcal{X}_1 \tag{11}$$

where $t = 1, 2, 3, \ldots,$ $x(t) \in \mathbf{R}^n$ is the state, $\omega(t) \in \Omega$ is the uncertainty matrix, $\mathcal{X}_1 \subset \mathbf{R}^n$ is a given compact set, $\Omega \subset \mathbf{R}^{r \times n}$ is a given set, and A, B are given matrices of corresponding dimensions.

We suppose that the following assumptions hold.

Assumption 4. The matrix A has at least one eigenvalue λ outside of the unit circle: $|\lambda| > 1$.

Assumption 5. The pair (A, B) is reachable (see e.g. (Astrom and Wittenmark, 1997), p.94).

Assumption 6. The origin is an interior point of the set Ω: there exists a $\delta > 0$ such that

$$\|\omega\|_\infty < \delta \quad \Rightarrow \omega \in \Omega.$$

Here $\|\cdot\|_\infty$ is the induced matrix norm (5).

Now we are in a position to present the following corollary of Theorem 2.

Proposition 2. Consider the system (11). If Assumptions 3–6 hold, then for any coder-decoder pair of the form (3), (4) with any T, R

$$\sup_{t, x(\cdot)} \|x(t) - \hat{x}(t)\|_\infty = \infty,$$

where the supremum is taken over all times t and all solutions $x(\cdot)$ of the system (11).

Remark 4. Proposition 2 shows that any state estimator with bit rate constraints for a linear unstable system is not robust. For example, all estimators from (Tatikonda, 2000; Jain *et al.*, 2002) will produce infinite error under any small parametric perturbation of the matrix A.

5. OPTIMAL CONTROL VIA COMMUNICATION CHANNELS

In this section, we consider a linear discrete-time controlled system without uncertainty of the form:

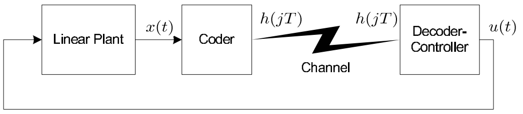

Fig. 2. Optimal control via digital communication channel

$$x(t+1) = Ax(t) + Bu(t), \qquad x(1) \in \mathcal{X}_1 \tag{12}$$

where $t = 1, 2, 3, \ldots$, $x(t) \in \mathbf{R}^n$ is the state, $u(t) \in \mathbf{R}^m$ is the control input, $\mathcal{X}_1 \subset \mathbf{R}^n$ is a given compact set, and A, B are given matrices of corresponding dimensions.

We consider the problem of optimal control of the linear system (12) via a digital communication channel with a bit-rate constraint. Our controller consists of two components. The first component is developed at the measurement location by taking the measured state $x(\cdot)$ and coding to the codeword $h(jT+1)$. This component will be called "coder". Then the codeword $h(jT+1)$ is transmitted via a limited capacity communication channel to the second component which is called "decoder-controller". The second component developed at a remote location takes the codeword $h(jT+1)$ and produces the sequence of control inputs $u(jT+1), \ldots, u((j+1)T-1), u((j+1)T)$. This situation is illustrated in Figure 2.

Our digital communication channel carries one discrete-valued symbol $h(jT+1)$ at time $jT+1$, selected from a coding alphabet $\mathcal{H}$ of size l. Here $T \geq 1$ is a given integer period, and $j = 0, 1, 2, \ldots$.

This restricted number l of codewords $h(jT+1)$ is determined by the transmission data rate of the channel. Let $R \geq 0$ be a given constant. We consider the class $\hat{\mathcal{C}}_R$ of such channels with any period T satisfying the transmission data rate constraint (2).

The coder and the decoder-controller are of the following form:

Coder:

$$h(jT+1) = \mathcal{F}_j\left(x(\cdot)|_1^{jT+1} \right); \tag{13}$$

Decoder-Controller:

$$\begin{pmatrix} u(jT+1) \\ \vdots \\ u((j+1)T-1) \\ u((j+1)T) \end{pmatrix} = \mathcal{U}_j\left(h_1, h_{T+1}, \ldots, h_{jT+1}\right). \tag{14}$$

We will consider the following quadratic cost function associated with the linear system (12):

$$J[x(\cdot), u(\cdot)] := \sum_{t=1}^{+\infty} [x(t)'C'Cx(t) + u(t)'Gu(t)] \tag{15}$$

where C and $G = G'$ are given matrices of corresponding dimensions.

We will need the following assumptions that are standard for linear quadratic optimal control problems.

Assumption 7. The pair (A, B) is stabilizable (see e.g. (Goodwin *et al.*, 2001)).

Assumption 8. The pair (A, C) has no unobservable nodes on the unit circle (see e.g. (Goodwin *et al.*, 2001)).

Assumption 9. The matrix G is positive definite.

In this section, we consider the following optimal control problem:

$$J[x(\cdot), u(\cdot)] \to \min. \tag{16}$$

If we do not have any limited capacity communication channel and the whole state $x(\cdot)$ is available to the controller, then the problem (12), (15), (16) is the standard linear quadratic optimal control problem and its solution is well-known (see e.g. (Goodwin *et al.*, 2001)). Under Assumptions 7–9, for any initial condition $x(1)$, the optimal control is given by

$$u(t) = Kx(t) \tag{17}$$

where

$$K = -(G + B'PB)^{-1}B'PA \tag{18}$$

and the square matrix P is a solution of the Discrete Time Algebraic Riccati equation

$$A\left(P - PB(G + B'PB)^{-1}B'P\right)A + C'C - P = 0 \tag{19}$$

such that the matrix $A + BK$ is stable (has all its eigenvalues inside the unit circle). Furthermore, the optimal value of the cost function is given by

$$J_{opt}[x(1)] = x(1)'Px(1). \tag{20}$$

Definition 7. The optimal control problem (12), (15), (16) is said to be solvable in the communication channel class $\hat{\mathcal{C}}_R$ if for any $\epsilon > 0$ there exists a period $T \geq 1$ and a coder-decoder-controller pair (13), (14) with a coding alphabet of size l satisfying the constraint (2) such that the following conditions hold.

(1) The closed-loop system (12), (13), (14) is stable in the following sense: for any $\epsilon_0 > 0$ there exists an integer $k \geq 1$ such that

$$\|x(t)\|_\infty < \epsilon_0 \qquad \forall t \geq k \tag{21}$$

for any solution $[x(\cdot), u(\cdot)]$ of the closed-loop system with initial condition $x(1) \in \mathcal{X}_1$.

(2) For any solution $[x(\cdot), u(\cdot)]$ of the closed-loop system with initial condition $x(1) \in \mathcal{X}_1$,

$$J[x(\cdot), u(\cdot)] \leq J_{opt}[x(1)] + \epsilon \qquad (22)$$

where $J_{opt}[x(1)]$ is given by (20).

Now we are in a position to present the main result of this section.

Theorem 4. Consider the system (12) and the cost function (15). Let $R \geq 0$ be a given constant and $H(A)$ be the value (10). Suppose that Assumptions 3, 7–9 are satisfied. Then the following two statements hold.

(1) If $R < H(A)$ then the optimal control problem (12), (15), (16) is not solvable in the communication channel class $\hat{\mathcal{C}}_R$.
(2) If $R > H(A)$ then the optimal control problem (12), (15), (16) is solvable in the communication channel class $\hat{\mathcal{C}}_R$.

The proof will be given in the full version of the paper.

REFERENCES

Adler, R. L., A. G. Konheim and M. H. McAndrew (1965). Topological entropy. *Transactions of the American Mathematical Society* **114**(2), 309–319.

Adler, R. L. and B. Weiss (1967). Entropy a complete metric invariant for automorphisms of torus. *Proceedings of The Natl. Acad. Sci. USA* **57**(6), 1573–1577.

Astrom, K. J. and B. Wittenmark (1997). *Computer-Controlled Systems.* Prentice Hall. Upper Saddle River, N.J.

Brockett, R. W. and D. Liberzon (2000). Quantized feedback stabilization of linear systems. *IEEE Transactions on Automatic Control* **45**(7), 1279–1289.

Goodwin, G. C., S. F. Graebe and M. E. Salgado (2001). *Control System Design.* Prentice Hall International, Inc.. Upper Saddle River, New Jersey.

Jain, R., T. Simsek and P. Varaiya (2002). Control under communication constraints. In: *Proceedings of the 41th IEEE Conference on Decision and Control.* Las Vegas, Nevada.

Kolmogorov, A. N. (1958). A new metric invariant of transient dynamical systems and automorphisms in lebesgue spaces. *Doklady Akademii Nauk SSSR* **119**(5), 861–864.

Kolmogorov, A. N. (1959). Entropy per unit time as a metric invariant of automorphisms. *Doklady Akademii Nauk SSSR* **124**(4), 754–755.

Matveev, A. S. and A. V. Savkin (2004*a*). An analogue of Shannon information theory for networked control systems. Stabilization via a noisy discrete channel. In: *Proceedings of the 43th IEEE Conference on Decision and Control.* Atlantis, Paradise Island, Bahamas.

Matveev, A. S. and A. V. Savkin (2004*b*). An analogue of Shannon information theory for networked control systems. State estimation via a noisy discrete channel. In: *Proceedings of the 43th IEEE Conference on Decision and Control.* Atlantis, Paradise Island, Bahamas.

Matveev, A. S. and A. V. Savkin (2004*c*). The problem of LQG optimal control via a limited capacity communication channel. *Systems and Control Letters* **53**, 51–64.

Matveev, A. S. and A. V. Savkin (2004*d*). Stabilization of multisensor networked control systems with communication constraints. In: *Proceedings of the 5th Asian Control Conference.* Melbourne, Australia.

Nair, G. N. and R. J. Evans (2002). Mean square stabilisability of stochastic linear systems with data rate constraints. In: *Proceedings of the 41th IEEE Conference on Decision and Control.* Las Vegas, Nevada.

Nair, G. N. and R. J. Evans (2003). Exponential stabilisability of finite-dimensional linear systems with limited data rates. *Automatica* **39**(4), 585–593.

Nair, G. N., R. J. Evans, I. M. Y. Mareels and W. Moran (2004). Topological feedback entropy for nonlinear systems. In: *Proceedings of the 5th Asian Control Conference.* Melbourne, Australia.

Pollicott, M. and M. Yuri (1998). *Dynamical Systems and Ergodic Theory.* Cambridge University Press. Cambridge.

Savkin, A. V. and I. R. Petersen (2003). Set-valued state estimation via a limited capacity communication channel. *IEEE Transactions on Automatic Control* **48**(4), 676–680.

Shannon, C. E. (1948). A mathematical theory of communication. *Bell Sys. Tech. Journ.* **27**, 379–424,623–656.

Tatikonda, S. C. (2000). *Control Under Communication Constraints. Ph.D. Thesis..* Massachusetts Institute of Technology.

Wong, W. S. and R. W. Brockett (1997). Systems with finite communication bandwidth constraints - part 1. state estimation problems. *IEEE Transactions on Automatic Control* **42**(9), 1294–1299.