FORMATION CONTROL OF AUTONOMOUS UNDERWATER VEHICLES

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Abstract: In this paper, we present a formation-keeping control of multiple autonomous underwater vehicles(UAVs) in 3D space. We use a so-called *Leader-Follower* formation approach, and our control task is to keep the relative position of each follower with a desired orientation to the leader. An Input/Output linearization is applied to the error system whose coordinates are the differences between a target point fixed with follower, and desired position defined in the leader's local coordinate system is realized. We will also prove the stability of zero-dynamics and give a condition for perfect formation. The efficiency of our control is demonstrated by numerical simulations.

Keywords: Autonomous Underwater Viheicle, Formation Control

1. INTRODUCTION

For maritime accidents, rescue activities and environment surveillance, researches for advanced autonomous underwater vehicles (AUVs) or unmanned underwater vehicles (UUVs) are very active (Y.Y.Nakamura, 1992) (O.J.Sordalen, 1993) (O.Egeland, 1994) (A.Pascoal, 1997) (G.Indiveri, 2000) (T.Ikeda, 2001). Besides such tasks it is hoped that the AUVs will be used for management of fishes and marine products in maritime engineering. Dynamics for AUVs is very complex due to effects of gravity, buoyancy, inertia force and other fluid dynamics, however, the fundamental movement of them can be represented by kinematic model with screws and fins (Y.Y.Nakamura, 1992). In the model there exist four controls, i.e., velocity control for lateral motion, and 3 angular velocity controls, and it can be assumed that they have two nonholonomic constraints with which instantaneous longitudinal and side step motions can not be created. In such a sense the AUVs can be approximately considered as an underactuated system. For such multiple AUVs to be effective for tasks and to be friendly for human operators, automatic formation control of a group of AUVs becomes very important.(J.Jongusuk, 2002) In this paper we consider a group of AUVs shown in Fig.1 and propose a formation control of the group. In the control algorithm we assume that a desired trajectory is given to a leader, and the followers tracks the leader in a specified formation. The stability and convergence of the formation will be proven and the validity of the method is shown by numerical simulations. Though the proposed method in this paper is derived based on a kinematic model, the method may be modified for dynamic models using a backstepping approach.

2. MODEL OF AUV

In this paper we consider AUVs shown in Fig.1 and it is assumed that they have 4 controls,i.e., linear velocity control of v_x, and angular velocity controls of $\omega_x, \omega_y, \omega_z$. Also it is assumed that they are governed two nonholonomic constrains such that they can not create instantaneous longitudinal and side step motion.

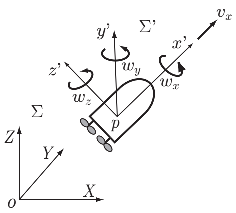

Fig. 1. AUV(Autonomous Underwater Vehicle)

In the figure Σ is a fixed coordinate system and Σ' is a moving coordinate system whose origin is attached to a center of mass of an AUV and its origin is defined as $\boldsymbol{p} := [p_x, p_y, p_z]^T$. $\boldsymbol{v} := [v_x, 0, 0]^T$ and $\boldsymbol{\omega} := [\omega_x, \omega_y, \omega_z]^T$ stand for linear velocity vector and angular velocity vector represented in the moving coordinate system Σ', respectively.

In the following discussion we assume the following assumptions:

(1) a group consists of a leader and a follower for simplicity.
(2) the leader and follower have the same kinematics.
(3) there is no obstacle in the working space.
(4) a desired trajectory for the leader is given and it is bounded and sufficiently smooth.
(5) linear velocities of the leader and follower in lateral motion are always positive in their own coordinate systems.
(6) the follower can obtain any information from the leader.

Required information of (6) in the assumptions are desired orientation in the fixed coordinate system, orientation in the moving coordinate system, and controls, which are all for the leader.

Under the assumptions the control problem here is that as in Fig.2 the follower tracks the leader in a specified formation for any initial formation.

2.1 Kinematics of AUV

In this subsection kinematics of an AUV is derived. Let assume that the orientation of an AUV is represented as a rotational matrix $R := [r_1, r_2, r_3] \in SO(3)$. The anguler velocity $\omega := [\omega_x, \omega_y, \omega_z]^T$ in the moving coordinate system Σ' satisfies

$$\dot{R} = RS(\omega)$$

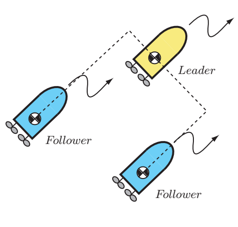

Fig. 2. Leader-Follower system

where $S(\omega) := \omega\times$, and the linear velocity $\dot{p}$ of the center of mass in a fixed coordinate system Σ can be represented as

$$\dot{\boldsymbol{p}} = \boldsymbol{R}\boldsymbol{v}, \tag{1}$$

where $\boldsymbol{v} = [v_x, 0, 0]^T$ is a linear velocity vector represented in Σ'. Since

$$\boldsymbol{R}^T \dot{\boldsymbol{p}} = \boldsymbol{v}, \tag{2}$$

the second and third equation in eq. (2) can be given as

$$\boldsymbol{r}_2^T \dot{\boldsymbol{p}} = 0, \qquad \boldsymbol{r}_3^T \dot{\boldsymbol{p}} = 0. \tag{3}$$

These equations show that velocity components in y' and z' directions are zero, which can be interpreted as two nonholonomic constraints. Using the above equations and eq. (2) and eq. (1), the kinematics of an AUV can be represented as

$$\begin{bmatrix} \dot{\boldsymbol{p}} \\ \dot{\boldsymbol{r}}_1 \\ \dot{\boldsymbol{r}}_2 \\ \dot{\boldsymbol{r}}_3 \end{bmatrix} = \left[\begin{array}{c|ccc} \boldsymbol{r}_1 & & \boldsymbol{0}_{3\times3} & \\ \hline & \boldsymbol{0}_{3\times1} & -\boldsymbol{r}_3 & \boldsymbol{r}_2 \\ \boldsymbol{0}_{9\times1} & \boldsymbol{r}_3 & \boldsymbol{0}_{3\times1} & -\boldsymbol{r}_1 \\ & -\boldsymbol{r}_2 & \boldsymbol{r}_1 & \boldsymbol{0}_{3\times1} \end{array}\right] \begin{bmatrix} v_x \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}. \tag{4}$$

In this model the state of the system is represented by 12 state variables (3 for position + 9 for rotation) and number of input is 4 where the variables are subject to 6 holonomic constraints given by

$$r_i^T r_i = 1, \quad r_i^T r_j = 0 \quad (i, j = 1, 2, 3, i \neq j),$$

and 2 nonholonomic constraints of eq. (3).

3. ERROR MODEL

In this section formation control problem is now formulated as a regulation problem based on an error model. The concept of the method is illustrated in Fig.3 where $\boldsymbol{p}_d := [p_{xd}, p_{yd}, p_{zd}]^T$ and

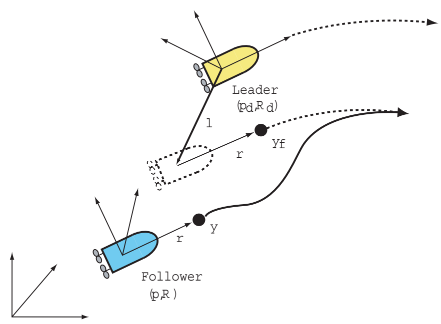

Fig. 3. Formation control of AUV

$\boldsymbol{R}_d := \{r_{ijd}\} \in SO(3)$ are the position of the moving coordinate and orientation of the leader w.r.t. the fixed coordinate system, respectively. It is assumed that desired linear and angular velocities, $\boldsymbol{v}_d := [v_{xd}, 0, 0]^T$ and $\boldsymbol{\omega}_d := [\omega_{xd}, \omega_{yd}, \omega_{zd}]^T$, for the leader in the moving coordinate system are given and it is assumed that $v_{xd} > 0$. From the assumption (2) it is assumed that the kinematics of the leader can be also represented as

$$\dot{\boldsymbol{p}}_d = \boldsymbol{R}_d \boldsymbol{v}_d, \tag{5}$$

$$\dot{\boldsymbol{R}}_d = \boldsymbol{R}_d \boldsymbol{S}(\boldsymbol{\omega}_d). \tag{6}$$

Furthermore, it is assumed that $\boldsymbol{l} = [l_x, l_y, l_z]^T$, which is called **formation vector**, specifies a desired relative position for the center of mass of the follower with respect to the center of mass of the leader as in Fig. 3. The control problem is that the position of the follower converges to the relative position and the orientation is coincided with that of the leader as $t \to \infty$, which can be represented as

$$\boldsymbol{p} \to \boldsymbol{p}_d + \boldsymbol{R}_d \boldsymbol{l}, \quad \boldsymbol{R} \to \boldsymbol{R}_d. \tag{7}$$

Please notice here that if the relative position is arbitrary, the orientation of the follower can not be coincided with that of the leader in general due to the nonholonomic constraints. The problem will be discussed later.

In order to derive an input-output linearization controller we define an output of the system called target position of the follower. The target position is fixed at $\boldsymbol{r}$ in the moving coordinate system of the follower and it is represented as

$$\boldsymbol{r} = \begin{bmatrix} r_x & 0 & 0 \end{bmatrix}^T. \tag{8}$$

According to the introduction of the target position for the follower, a reference position of the leader is defined as a sum of the target vector $\boldsymbol{r}$ and formation vector $\boldsymbol{l}$ in the moving coordinate system of the leader. If $\boldsymbol{y}$ and $\boldsymbol{y}_f$ are the target and reference position in the fixed coordinate system, they can be represented as

$$\boldsymbol{y} = \boldsymbol{p} + \boldsymbol{R}\boldsymbol{r}, \tag{9}$$

$$\boldsymbol{y}_f = \boldsymbol{p}_d + \boldsymbol{R}_d(\boldsymbol{l} + \boldsymbol{r}). \tag{10}$$

3.1 Error model of rotation

Error of the orientation is represented as a rotational matrix, $\tilde{\boldsymbol{R}} = \{\tilde{r}_{ij}\} \in SO(3)$, as

$$\tilde{\boldsymbol{R}} = \boldsymbol{R}_d^T \boldsymbol{R}, \tag{11}$$

and if $\tilde{\boldsymbol{R}} \to \boldsymbol{I}$, then $\boldsymbol{R} \to \boldsymbol{R}_d$. The time derivative of eq. (11) is given by

$$\begin{aligned} \dot{\tilde{\boldsymbol{R}}} &= \dot{\boldsymbol{R}}_d^T \boldsymbol{R} + \boldsymbol{R}_d^T \dot{\boldsymbol{R}} \\ &= -\boldsymbol{S}(\boldsymbol{\omega}_d)\tilde{\boldsymbol{R}} + \tilde{\boldsymbol{R}}\boldsymbol{S}(\boldsymbol{\omega}). \end{aligned} \tag{12}$$

By multiplying $\tilde{\boldsymbol{R}}^T$ to both sides of the above equation, we have

$$\tilde{\boldsymbol{R}}^T \dot{\tilde{\boldsymbol{R}}} = \boldsymbol{S}(\boldsymbol{\omega} - \tilde{\boldsymbol{R}}^T \boldsymbol{\omega}_d), \tag{13}$$

and if we introduce a new input $\boldsymbol{\omega}_c := [\omega_{xc}, \omega_{yc}, \omega_{zc}]^T$ given by

$$\boldsymbol{\omega}_c = \boldsymbol{\omega} - \tilde{\boldsymbol{R}}^T \boldsymbol{\omega}_d, \tag{14}$$

we have an error model for the rotation:

$$\dot{\tilde{\boldsymbol{R}}} = \tilde{\boldsymbol{R}}\boldsymbol{S}(\boldsymbol{\omega}_c). \tag{15}$$

3.2 Error model of position

When an error vector between the reference position and the target position, $\tilde{\boldsymbol{y}}_f := [\tilde{y}_{1f}, \tilde{y}_{2f}, \tilde{y}_{3f}]^T$, is defined as

$$\begin{aligned} \tilde{\boldsymbol{y}}_f &:= \boldsymbol{y} - \boldsymbol{y}_f \\ &= \boldsymbol{p} + \boldsymbol{R}\boldsymbol{r} - \boldsymbol{p}_d - \boldsymbol{R}_d(\boldsymbol{l} + \boldsymbol{r}), \end{aligned} \tag{16}$$

then the time derivative of it can be represented as

$$\begin{aligned} \dot{\tilde{\boldsymbol{y}}}_f &= \dot{\boldsymbol{p}} + \dot{\boldsymbol{R}}\boldsymbol{r} - \dot{\boldsymbol{p}}_d - \dot{\boldsymbol{R}}_d(\boldsymbol{l} + \boldsymbol{r}) \\ &= \boldsymbol{R}_d(\tilde{\boldsymbol{R}}(\boldsymbol{v} - \boldsymbol{S}(\boldsymbol{r})\boldsymbol{\omega}) - (\boldsymbol{v}_d - \boldsymbol{S}(\boldsymbol{l} + \boldsymbol{r})\boldsymbol{\omega}_d)) \end{aligned} \tag{17}$$

Since the target position is $\boldsymbol{r} = [r_x, 0, 0]^T$, using an input transformation of $\boldsymbol{u}_f := [v_x, r_x\omega_z, -r_x\omega_y]^T$, eq. (17) can be rewritten as

$$\dot{\tilde{\boldsymbol{y}}}_f = \boldsymbol{R}_d(\tilde{\boldsymbol{R}}\boldsymbol{u}_f - (\boldsymbol{v}_d - \boldsymbol{S}(\boldsymbol{l} + \boldsymbol{r})\boldsymbol{\omega}_d)). \tag{18}$$

From this equation it can be understood that the error system has 3 inputs, $(v_x, \omega_y, \omega_z)$. In the research eq. (18) is treated as an error model of the position tracking.

Based on the above derivations, error models for the problem are represented by eq. (15) and eq. (18) and the control problem can be represented as

$$\tilde{\boldsymbol{R}} \to \boldsymbol{I}, \quad \tilde{\boldsymbol{y}}_f \to \boldsymbol{0}_{3\times 1}. \tag{19}$$

4. FORMATION CONTROL

In this section a formation control law is derived based on eq. (15) and eq. (18) to satisfy eq. (19).

4.1 Control of position

If a square matrix $\Lambda_f := diag(\lambda_{1f}, \lambda_{2f}, \lambda_{3f})$ and $\boldsymbol{u}_f$ is defined as

$$\boldsymbol{u}_f = \tilde{\boldsymbol{R}}^T(\boldsymbol{v}_d - \boldsymbol{S}(\boldsymbol{l} + \boldsymbol{r})\boldsymbol{\omega}_d - \boldsymbol{R}_d^T \Lambda_f \tilde{\boldsymbol{y}}_f), \tag{20}$$

eq. (18) can be represented as

$$\dot{\tilde{\boldsymbol{y}}}_f = -\Lambda_f \tilde{\boldsymbol{y}}_f \tag{21}$$

and $\tilde{\boldsymbol{y}}_f$ converges to zero exponentially. By the definition of $\boldsymbol{u}_f$, the control input, $(v_x, \omega_y, \omega_z)$, can be calculated as

$$v_x = \tilde{\boldsymbol{r}}_1^T(\boldsymbol{u}_{fd} - \boldsymbol{R}_d^T \Lambda_f \tilde{\boldsymbol{y}}_f), \tag{22}$$

$$\omega_y = -\frac{1}{r_x}\tilde{\boldsymbol{r}}_3^T(\boldsymbol{u}_{fd} - \boldsymbol{R}_d^T \Lambda_f \tilde{\boldsymbol{y}}_f), \tag{23}$$

$$\omega_z = \frac{1}{r_x}\tilde{\boldsymbol{r}}_2^T(\boldsymbol{u}_{fd} - \boldsymbol{R}_d^T \Lambda_f \tilde{\boldsymbol{y}}_f). \tag{24}$$

Since the asymptotic convergence of $\tilde{\boldsymbol{y}}_f$ is independent of other internal variables, the stability of the whole system can be checked independently. (E.g., see a lemma in B.2 (A.Ishidori, 1995))

4.2 Zero dynamics

If eq. (24) and eq. (14) are used for (23), we have $\tilde{\boldsymbol{y}}_f \to 0$ and

$$\begin{bmatrix} \omega_{yc} \\ \omega_{zc} \end{bmatrix} = \frac{1}{r_x} \begin{bmatrix} -\tilde{\boldsymbol{r}}_3^T \\ \tilde{\boldsymbol{r}}_2^T \end{bmatrix} (\boldsymbol{v}_d - \boldsymbol{S}(\boldsymbol{l}+\boldsymbol{r})\boldsymbol{\omega}_d) - \begin{bmatrix} \tilde{\boldsymbol{r}}_2^T \\ \tilde{\boldsymbol{r}}_3^T \end{bmatrix} \boldsymbol{\omega}_d. \tag{25}$$

By substituting eq. (25) to eq. (15), and extracting the first column, we have

$$\begin{aligned} \dot{\tilde{\boldsymbol{r}}}_1 &= \tilde{\boldsymbol{r}}_2\omega_{zc} - \tilde{\boldsymbol{r}}_3\omega_{yc} \\ &= \frac{1}{r_x} \begin{bmatrix} -\tilde{\boldsymbol{r}}_3 & \tilde{\boldsymbol{r}}_2 \end{bmatrix} \Bigg(\begin{bmatrix} -\tilde{\boldsymbol{r}}_3^T \\ \tilde{\boldsymbol{r}}_2^T \end{bmatrix} (\boldsymbol{v}_d - \boldsymbol{S}(\boldsymbol{l}+\boldsymbol{r})\boldsymbol{\omega}_d) \\ &\qquad - r_x \begin{bmatrix} \tilde{\boldsymbol{r}}_2^T \\ \tilde{\boldsymbol{r}}_3^T \end{bmatrix} \boldsymbol{\omega}_d \Bigg). \end{aligned} \tag{26}$$

From properties of rotational matrices, we have

$$\begin{bmatrix} -\tilde{\boldsymbol{r}}_3 & \tilde{\boldsymbol{r}}_2 \end{bmatrix} \begin{bmatrix} -\tilde{\boldsymbol{r}}_3^T \\ \tilde{\boldsymbol{r}}_2^T \end{bmatrix} = \boldsymbol{I} - \tilde{\boldsymbol{r}}_1\tilde{\boldsymbol{r}}_1^T \tag{27}$$

$$\begin{bmatrix} -\tilde{\boldsymbol{r}}_3 & \tilde{\boldsymbol{r}}_2 \end{bmatrix} \begin{bmatrix} \tilde{\boldsymbol{r}}_2^T \\ \tilde{\boldsymbol{r}}_3^T \end{bmatrix} = -\boldsymbol{S}(\tilde{\boldsymbol{r}}_1) \tag{28}$$

and eq. (26) can be rewritten as

$$\dot{\tilde{\boldsymbol{r}}}_1 = \frac{1}{r_x} \Big((\boldsymbol{I} - \tilde{\boldsymbol{r}}_1\tilde{\boldsymbol{r}}_1^T)(\boldsymbol{v}_d - \boldsymbol{S}(\boldsymbol{l}+\boldsymbol{r})\boldsymbol{\omega}_d) + r_x\boldsymbol{S}(\tilde{\boldsymbol{r}}_1)\boldsymbol{\omega}_d \Big). \tag{29}$$

From the derivations above it can be understood that output zeroing of $\tilde{\boldsymbol{y}}_f$ produces a zero dynamics of $\tilde{\boldsymbol{r}}_1$ which is not affected by the control input.

4.3 Existence of equilibrium

Equilibrium state of the closed loop system of $\tilde{\boldsymbol{r}}_1$, $\tilde{\boldsymbol{r}}_{1e} := [\tilde{r}_{11e}, \tilde{r}_{21e}, \tilde{r}_{31e}]^T$, is given a solution of nonlinear algebraic equation:

$$(\boldsymbol{I} - \tilde{\boldsymbol{r}}_{1e}\tilde{\boldsymbol{r}}_{1e}^T)(\boldsymbol{v}_d - \boldsymbol{S}(\boldsymbol{l}+\boldsymbol{r})\boldsymbol{\omega}_d) + r_x\boldsymbol{S}(\tilde{\boldsymbol{r}}_{1e})\boldsymbol{\omega}_d = 0 \tag{30}$$

composed of desired parameters, $(v_{xd}, \boldsymbol{\omega}_d)$, for the leader and the formation vector $\boldsymbol{l}$. Since a necessary condition for the coincidence of the orientation is given by $\tilde{\boldsymbol{r}}_{1e} = [1, 0, 0]^T$, it is used in eq. (30) and we have

$$-l_z\omega_{xd} + l_x\omega_{zd} = 0 \tag{31}$$

$$l_y\omega_{xd} - l_x\omega_{yd} = 0, \tag{32}$$

which depends on the formation vector. The formation vector $\boldsymbol{l}$ satisfying the above condition is represented as

$$\boldsymbol{l} \in \ker \boldsymbol{D}(\boldsymbol{\omega}_d), \quad \boldsymbol{D}(\boldsymbol{\omega}_d) = \begin{bmatrix} \omega_{zd} & 0 & -\omega_{xd} \\ -\omega_{yd} & \omega_{xd} & 0 \end{bmatrix}. \tag{33}$$

By substituting the relation into the first column of eq. (30), we have

$$(1 - \tilde{r}_{11e})((1+\tilde{r}_{11e})v_{xd} + r_x\tilde{r}_{31e}\omega_{yd} - r_x\tilde{r}_{21e}\omega_{zd}) = 0, \tag{34}$$

and from the result it can be seen that $\tilde{\boldsymbol{r}}_{1e} = (1, 0, 0)$ is actually an equilibrium. However, we should note that if

$$(1 + \tilde{r}_{11e})v_{xd} + r_x\tilde{r}_{31e}\omega_{yd} - r_x\tilde{r}_{21e}\omega_{zd} = 0$$

is used with other conditions in (30), the resultant conditions become simltenious linear equations and the another solution exists besides $\tilde{\boldsymbol{r}}_{1e} = (1, 0, 0)$. In general such equilibrium is unstable locally.

4.4 Stability of zero dynamics

In order to investigate the stability of $\tilde{\boldsymbol{r}}_1 \to \tilde{\boldsymbol{r}}_{1e}$, we define a Lyapunov function candidate V_{1+}:

$$V_{1+} = \frac{1}{2}(\tilde{\boldsymbol{r}}_1 - \tilde{\boldsymbol{r}}_{1e})^T(\tilde{\boldsymbol{r}}_1 - \tilde{\boldsymbol{r}}_{1e}). \tag{35}$$

The time derivative of the function is given as

$$\begin{aligned} \dot{V}_{1+} &= (\tilde{\boldsymbol{r}}_1 - \tilde{\boldsymbol{r}}_{1e})^T\dot{\tilde{\boldsymbol{r}}}_1 \\ &= -\frac{1}{r_x}\Big(\tilde{\boldsymbol{r}}_{1e}^T(\boldsymbol{I} - \tilde{\boldsymbol{r}}_1\tilde{\boldsymbol{r}}_1^T)(\boldsymbol{v}_d - \boldsymbol{S}(\boldsymbol{l}+\boldsymbol{r})\boldsymbol{\omega}_d) \\ &\qquad - r_x\tilde{\boldsymbol{r}}_1^T\boldsymbol{S}(\tilde{\boldsymbol{r}}_{1e})\boldsymbol{\omega}_d \Big). \end{aligned} \tag{36}$$

Using eq. (30) we have

$$\dot{V}_{1+} = -\frac{1}{r_x}(1 - \tilde{\boldsymbol{r}}_{1e}^T\tilde{\boldsymbol{r}}_1)(\tilde{\boldsymbol{r}}_{1e}^T + \tilde{\boldsymbol{r}}_1^T)(\boldsymbol{v}_d - \boldsymbol{S}(\boldsymbol{l}+\boldsymbol{r})\boldsymbol{\omega}_d). \tag{37}$$

On the other hand, if we assume that we have $\boldsymbol{y}_f \to \boldsymbol{0}$ in eq. (21), since the control input v_x should be represented as

$$v_x = \tilde{\boldsymbol{r}}_1^T(\boldsymbol{v}_d - \boldsymbol{S}(\boldsymbol{l}+\boldsymbol{r})\boldsymbol{\omega}_d) \tag{38}$$

and the linear velocity of the center of mass at the equilibrium, v_{xe}, is

$$v_{xe} = \tilde{\boldsymbol{r}}_{1e}^T(\boldsymbol{v}_d - \boldsymbol{S}(\boldsymbol{l}+\boldsymbol{r})\boldsymbol{\omega}_d), \tag{39}$$

eq. (37) can be rewritten as

$$\dot{V}_{1+} = -\frac{1}{r_x}(1 - \tilde{\boldsymbol{r}}_{1e}^T\tilde{\boldsymbol{r}}_1)(v_{xe} + v_x). \tag{40}$$

Since $1 - \tilde{\boldsymbol{r}}_{1e}^T\tilde{\boldsymbol{r}}_1 \le 1$ and the assumption of $r_x > 0$, if

$$v_{xe} + v_x > 0, \tag{41}$$

then $\dot{V}_{1+}$ becomes negative semi-definite. Since only when $\tilde{\boldsymbol{r}}_1 = \tilde{\boldsymbol{r}}_{1e}$, we have $\dot{V}_{1+} = 0$ and it can be seen that $\tilde{\boldsymbol{r}}_1 \to \tilde{\boldsymbol{r}}_{1e}(t \to \infty)$ is attained if the solution,$\tilde{\boldsymbol{r}}_{1e}$, of eq. (30) exists subject to $\tilde{\boldsymbol{r}}_{1e}^T\tilde{\boldsymbol{r}}_{1e} = 1$, and w_d and v_{xd} are constant.

4.5 Determination of ω_x using Euler parameters

In the previous analysis under the condition of eq. (33), it is shown that (22)-(24) realize $\tilde{\boldsymbol{r}}_1 \to [1,0,0]^T$ and 3 inputs have been determined. In this section we consider to determine the remained control,ω_x, so that other error should be converged.

If we have $\tilde{r}_{23} \to 0$ or $\tilde{r}_{32} \to 0$, it is achieved that $\tilde{\boldsymbol{R}} \to \boldsymbol{I}_3$. From eq. (15), if ω_{xc} is defined as

$$\omega_{xc} = -\lambda_4 \frac{\tilde{r}_{23}}{\tilde{r}_{33}}, \quad \lambda_4 > 0 \tag{42}$$

we have the following convergence:

$$\dot{\tilde{r}}_{23} = -\lambda_4 \tilde{r}_{23}. \tag{43}$$

Based on the observation above, ω_x is determined using Euler parameters, $\tilde{\eta}, \tilde{\boldsymbol{\epsilon}} := [\tilde{\epsilon}_1, \tilde{\epsilon}_2, \tilde{\epsilon}_3]^T$ for the rotational error matrix of $\tilde{\boldsymbol{R}}$ to prevent a singularity of representation of rotations. Please notice that eq. (11) can be represented as :

$$\tilde{\eta} = \eta_d \eta + \boldsymbol{\epsilon_d^T} \boldsymbol{\epsilon} \tag{44}$$

$$\tilde{\boldsymbol{\epsilon}} = \eta_d \boldsymbol{\epsilon} - \eta \boldsymbol{\epsilon}_d - \boldsymbol{S}(\boldsymbol{\epsilon}_d)\boldsymbol{\epsilon} \tag{45}$$

where $(\eta, \boldsymbol{\epsilon})$ and $(\eta_d, \boldsymbol{\epsilon}_d)$ are Euler parameters to represent the orientation of the follower and the leader. For the time derivative of the error parameters we have

$$\begin{bmatrix} \dot{\tilde{\eta}} \\ \dot{\tilde{\boldsymbol{\epsilon}}} \end{bmatrix} = \frac{1}{2} \tilde{\boldsymbol{G}}^T \boldsymbol{\omega}_c, \tilde{\boldsymbol{G}} := \begin{bmatrix} -\tilde{\epsilon}_1 & \tilde{\eta} & \tilde{\epsilon}_3 & -\tilde{\epsilon}_2 \\ -\tilde{\epsilon}_2 & -\tilde{\epsilon}_3 & \tilde{\eta} & \tilde{\epsilon}_1 \\ -\tilde{\epsilon}_3 & \tilde{\epsilon}_2 & -\tilde{\epsilon}_1 & \tilde{\eta} \end{bmatrix}. \tag{46}$$

Furthermore, we have a transformation of $(\tilde{\eta}, \tilde{\boldsymbol{\epsilon}}) \to \tilde{\boldsymbol{R}}$, and the (1,1) element can be represented as

$$\tilde{r}_{11} = 2(\tilde{\eta}^2 + \tilde{\epsilon}_1^2) - 1 = 1 - 2(\tilde{\epsilon}_2^2 + \tilde{\epsilon}_3^2). \tag{47}$$

(See the details of Euler parameters and its properties in (F.Caccavale, 1999)).

From this equation we have an equivalence :

$$\tilde{\boldsymbol{r}}_1 \to [1,0,0]^T \Leftrightarrow (\tilde{\epsilon}_2, \tilde{\epsilon}_3) \to (0,0). \tag{48}$$

If a Lyapunov function candidate of $(\tilde{\eta}, \tilde{\epsilon}_1, \tilde{\epsilon}_2, \tilde{\epsilon}_3)$ is defined as

$$V_2 = (\tilde{\eta} - 1)^2 + \tilde{\epsilon}_1^2 + \tilde{\epsilon}_2^2 + \tilde{\epsilon}_3^2, \tag{49}$$

the time derivative of it is given as

$$\begin{aligned} \dot{V}_2 &= 2(\tilde{\eta}\dot{\tilde{\eta}} - \dot{\tilde{\eta}} + \tilde{\epsilon}_1\dot{\tilde{\epsilon}}_1 + \tilde{\epsilon}_2\dot{\tilde{\epsilon}}_2 + \tilde{\epsilon}_3\dot{\tilde{\epsilon}}_3) \\ &= \begin{bmatrix} \tilde{\epsilon}_1 & \tilde{\epsilon}_2 & \tilde{\epsilon}_3 \end{bmatrix} \boldsymbol{\omega}_c. \end{aligned} \tag{50}$$

Furthermore, from eq. (48) we have $\tilde{\boldsymbol{r}}_1 \to [1,0,0]^T$ and

$$\dot{V}_2 = \tilde{\epsilon}_1 \omega_{xc}. \tag{51}$$

If the control input ω_{xc} is defined as

$$\omega_{xc} = -\lambda_4 \tilde{\epsilon}_1, \tag{52}$$

we have

$$\dot{V}_2 = -\lambda_4 \tilde{\epsilon}_1^2 \le 0 \tag{53}$$

and $\tilde{\epsilon}_1 \to 0$ and we have

$$(\tilde{\eta}, \tilde{\epsilon}_1, \tilde{\epsilon}_2, \tilde{\epsilon}_3) \to (1,0,0,0) \Rightarrow \tilde{\boldsymbol{R}} \to \boldsymbol{I}_3. \tag{54}$$

The above equation means that the orientation of the follower converges to that of the leader, and the control objective of eq. (10) is realized. From the definition of eq. (52), the control input ω_x is calculated as

$$\omega_x = -\lambda_4 \tilde{\epsilon}_1 + \tilde{\boldsymbol{r}}_1^T \boldsymbol{\omega}_d. \tag{55}$$

5. MODIFICATION OF DESIRED ORIENTATION

If the desired orientation does not satisfy the condition of eq. (33), it is not ensured that $\tilde{\boldsymbol{r}}_1 \to [1,0,0]^T$ and in some cases oscillations is remained in the responses. In order to prevent the bad effects, we consider a method to modify the desired orientation $\boldsymbol{R}_d$ for the follower as

$$\boldsymbol{R}_r = \boldsymbol{R}_m \boldsymbol{R}_d \tag{56}$$

where $\boldsymbol{R}_r$ is a new desired orientation for the follower and $\boldsymbol{R}_m$ represents a modification.

Since $\boldsymbol{R}_m$ should be near $\boldsymbol{I}$ for the original tracking problem, and if $\boldsymbol{\omega}_r = [\omega_{xr}, \omega_{yr}, \omega_{zr}]^T$ is defined as

$$\dot{\boldsymbol{R}}_r = \boldsymbol{R}_r \boldsymbol{S}(\boldsymbol{\omega}_r), \tag{57}$$

it should satisfy the condition:

$$\boldsymbol{D}(\boldsymbol{\omega}_r)\boldsymbol{l} = \boldsymbol{0}. \tag{58}$$

In order to determine $\boldsymbol{\omega}_r$, we consider an optimization problem with a constraint as

$$J = \frac{1}{2} Trace(\boldsymbol{R}_m - \boldsymbol{I})^T (\boldsymbol{R}_m - \boldsymbol{I}) \tag{59}$$

$$sub. \quad to \quad \boldsymbol{D}(\boldsymbol{\omega}_r)\boldsymbol{l} = \boldsymbol{0}. \tag{60}$$

The details how to solve the problem are omitted due to space limitation, however, the basic idea is to find ω_r which makes $\dot{J}$ negative semidefinite. (M.Yamakita, 1996) Once the modification is determined, the control problem in eq. (7) is modified as

$$\boldsymbol{p} \to \boldsymbol{p}_d + \boldsymbol{R}_d \boldsymbol{l}, \quad \boldsymbol{R} \to \boldsymbol{R}_r. \tag{61}$$

6. NUMERICAL SIMULATIONS

In order to show the validity of the proposed method, numerical simulations are conducted where the desired motion of the leader is spiral as $(v_{xd}, \omega_{xd}, \omega_{yd}, \omega_{zd}) = (1,1,1,1)$. The condition in the simulations are given as follows.

(Common condition) Initial state of the follower $\boldsymbol{p}(0) = (-1,1,1)^T$, $\boldsymbol{R}(0) = \boldsymbol{I}$ Initial state of the leader $\boldsymbol{p}_d(0) = (0,0,0)^T$, $\boldsymbol{R}_d(0) = \boldsymbol{I}$

Target position vector $\boldsymbol{r} = (2,0,0)^T$

Simulation 1

When the formation vector $\boldsymbol{l}$ satisfies eq. (33), it is confirmed that both position and orientation converge to the

desired one without errors.
(conditions in Simulation 1) Formation vector $\boldsymbol{l} = (-1, -1, -1)$ **Simulation 2**
When the formation vector $\boldsymbol{l}$ does not satisfy the eq. (33) and the proposed modification is not used. It is observed that the formation control is not realized with oscillations.
(conditions in Simulation 2)
Formation vector $\boldsymbol{l} = (-1, -2, -3)$
Simulation 3
When the formation vector $\boldsymbol{l}$ does not satisfy the eq. (33). It is observed that if the modification of the orientation is introduced, the formation control is well done with some error though a big oscillation is remained without the modification. The performance will be improved by turning the parameter α or the criterion function.
(conditions in Simulation 3)
Formation vector $\boldsymbol{l} = (-1, -2, -3)$ Initial orientation $\boldsymbol{R}_r(0) = \boldsymbol{I}$ Weighting factor $\alpha = 0.3$

7. CONCLUDING REMARKS

In this paper a formation control method for AUVs in 3D space was proposed based on I/O linearization method where a follower tracks the leader keeping a specified relative position with a desired orientation. A condition of the perfect tracking is given.When the condition is not satisfied, some undesirable phenomena was observed and a modification of the fundamental control was also proposed. The validity of the proposed method were assured by numerical simulations. Since the proposed method was derived based on a kinematic model, the validity of the method for a dynamic model should be checked using a backstepping approach.

While the research is in progress, the second author has past. The other authors would like to pray for the repose of his soul. If there exists any fault in the research, all attribute to the third author.

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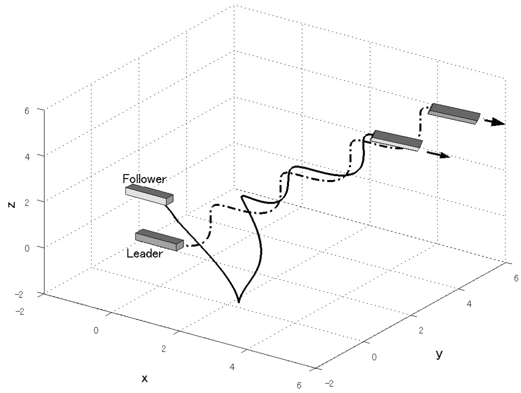

Fig. 4. Animation for Sim.1

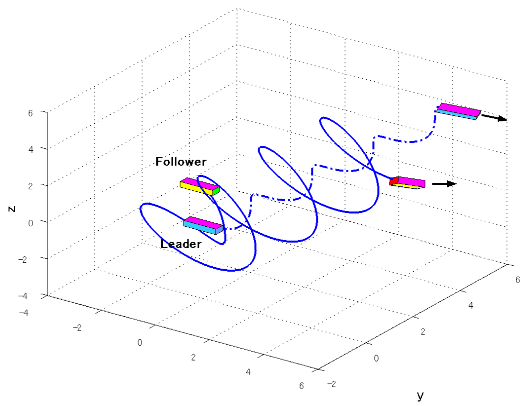

Fig. 5. Animation for Sim.2

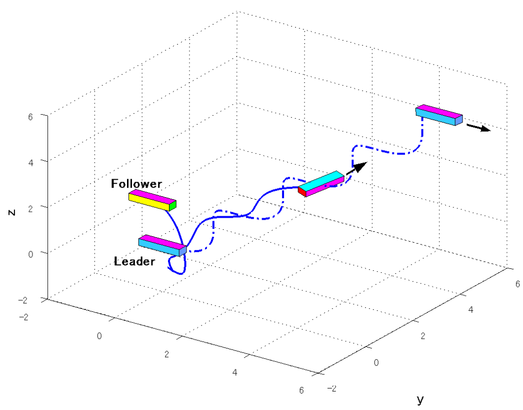

Fig. 6. Animation for Sim.3