H_2 CONTROL OF PREVIEW SYSTEMS

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Abstract: The H_2-optimal controller for systems with preview, in which the knowledge of external input is available in advance for the controller, is derived. The single input case is first considered and solved by transforming the problem into a non-standard LQR problem. It turns out that the extensions to multiple inputs and multiple preview times cases are straightforward. In every case considered, the controller consists of a static state feedback plus a finite impulse response block.

Keywords: preview systems, H_2-control

1. INTRODUCTION

In certain control problems, all or parts of the external input signals are known in advance. An example is a tracking problem where the tracked trajectory is known in advance. While exploiting this knowledge might improve the control system performance, most controller design techniques do not take it into account. Control systems that do exploit the advance knowledge of the input are commonly designated as preview control systems. Several results have been put forward to incorporate the advance knowledge of the external input signals into H_∞ and H_2 designs. The H_∞ preview control problem is considered in (Kojima and Ishijima, 2003), (Shaked and de Souza, 1995) while the closely related problem of H_∞ fixed-lag smoothing is treated in (Mirkin, 2003), (Theodor and Shaked, 1994). The H_2 fixed-lag smoothing problem has been solved in the 60s (see (Anderson and Moore, 1979) and references therein). Lately, there have been several results that treat the H_2 control problem of preview systems. In (Kojima and Ishijima, 1999), an LQ problem with stored disturbance is discussed, while in (Kojima, 2004) an H_2 preview control problem, which is equivalent to the single input control problem considered in this paper, is solved.

In this paper an alternative derivation of the one given in (Kojima, 2004) is provided. It is demonstrated that the techniques that are developed in (Moelja and Meinsma, 2004) for solving H_2 problem for systems with multiple i/o delays may also be applied to solve the H_2 preview problem. In addition, it is shown that the solution to the multiple inputs and the multiple preview times cases are straightforward extensions of the single-input solution.

The paper is organized as follows. After the introduction and some preliminaries, the single input H_2 preview control problem is considered. The result is then extended to the multiple inputs case and multiple preview times case in the subsequent sections. The paper is concluded with a numerical example and concluding remarks.

Preliminaries. Given a linear time invariant system (LTI) F, its impulse response is denoted by $F(t)$. The squared H_2-norm of a causal LTI system F is equal to

$$\|F\|_2^2 = \int_0^\infty \text{trace}\left[F(t)^T F(t)\right] dt. \qquad (1)$$

Suppose the input and output of the system F are respectively denoted by w and z, then the squared H_2-norm may also be defined as follows:

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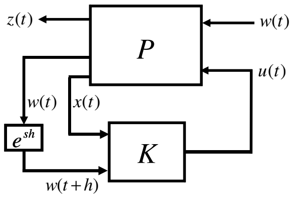
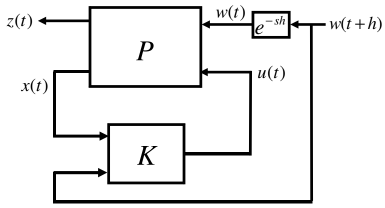


(a)



(b)

Fig. 1. The preview control setting (a) and its equivalence (b)

$$\|F\|_2^2 = \sum_{w=(0,\ldots,\delta(t),\ldots,0)} \int_0^\infty z(t)^T z(t)dt. \quad (2)$$

Throughout the paper, the unit step function is denoted as $\mathbb{1}(t)$.

2. PROBLEM FORMULATION

The preview control system configuration that is considered is shown in Figure 1(a). It is very similar to the standard full information control system, in which the controller uses the state x and the external input w as its inputs. The only difference is that the external signal w is available to the controller h time units in advance. This fact is represented in Figure 1(a) by the negative delay operator e^{sh}. To avoid employing a negative delay operator, the same effect may be achieved by delaying the external input fed to the plant, while the controller receives the non-delayed version. This setting is shown in Figure 1(b). The control problem itself is formally stated in what follows.

Consider the control system of Figure 1(b) where the dynamics of the plant $P(s)$ are governed by the state space equation:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_1 w(t) + B_2 u(t), \\ z(t) &= C_1 x(t) + D_2 u(t). \end{aligned} \quad (3)$$

and the system parameters satisfy the following standard assumptions:

A1 (C_1, A, B_2) is detectable and stabilizable;

A2 $\begin{bmatrix} A - j\omega I & B_2 \\ C_1 & D_2 \end{bmatrix}$ has full column rank $\forall \omega \in \mathbb{R}$.

In addition to the standard assumptions above, to simplify the formulas it is also assumed that

A3 $D_2^T D_2 = I$ and $C_1^T D_2 = 0$.

Assumption A3 will be relaxed later. The problem is to find a stabilizing control K such that the H_2-norm of the transfer function from w to z is minimized.

3. SINGLE-INPUT CASE

In this section the case where B_1 is a column vector, i.e. w is a scalar, is considered. To signify the difference, a lower case b_1 is used in place of B_1. The H_2-norm of the transfer function from w to z is equal to the L_2-norm of z provided that $w(t)$ is a delta function. Therefore, by setting

$$w(t) = \delta(t - h),$$

the H_2 optimization problem may be formulated as an LQR problem with a state jump at $t = h$. At this point, the original objective of designing a full information controller that takes $x(t)$ and $w(t+h)$ as inputs is temporarily set aside. Rather the attention is focused on finding the optimal u such that the L_2-norm of z is minimized given $w(t) = \delta(t - h)$. Later it shall be shown that the optimal control law may be implemented by a full information controller with preview as in Figure 1 and thus producing the desired optimal controller.

Given that $B_1 = b_1$ and $w(t) = \delta(t - h)$, the state space equation (3) becomes

$$\begin{aligned} \dot{x}(t) &= Ax(t) + b_1\delta(t - h) + B_2 u(t), \\ z(t) &= C_1 x(t) + D_2 u(t), \quad x(0) = 0, \end{aligned} \quad (4)$$

and our objective is

$$\begin{aligned} \min_u J(x_0, u) &= \min_u \int_0^\infty \|C_1 x(t) + D_2 u(t)\|_2^2 \, dt \\ &= \min_u \int_0^\infty x(t)^T Q x(t) + u(t)^T u(t) dt \end{aligned} \quad (5)$$

where

$$Q = C_1^T C_1. \quad (6)$$

The delta function input at $t = h$ in (4) raises the state such that $x(h^+) = x(h^-) + b_1$, so that (4) may be rewritten as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_2 u(t), \\ x(0) &= 0, \ x(h^+) = x(h^-) + b_1, \\ z(t) &= C_1 x(t) + D_2 u(t). \end{aligned} \quad (7)$$

The state space equation (7) together with the criterion function (5) constitute an LQR problem. The only difference of the LQR problem (7,5) and a standard LQR problem is the state jump at $t = h$. One way to circumvent the problem is to use the technique from (Moelja and Meinsma, 2004) of dividing the optimization time horizon into two regions with $t = h$ as the boundary so that the state jump can be considered as a boundary condition. It turns out that the optimal control problem in each time region may be solved essentially independent from the other.

Lemma 1. Consider the LQR problem corresponding to the state space equation (7) and the objective (5). Let M be the stabilizing solution of the familiar LQR Riccati equation

$$Q + A^T M + MA - MB_2 B_2^T M = 0. \quad (8)$$

Define

$$u_{2,\mathrm{opt}}(t) = -B_2^T M x(t) \quad (9)$$

and let $u_{1,\text{opt}}$ be the solution of the LQR problem corresponding to the state-space equation

$$\dot{x}(t) = Ax(t) + B_2 u_1(t), \quad x(0) = 0, \qquad (10)$$

with the objective

$$u_{1,\text{opt}}(t) = \arg\min_{u_1} [(x(h) + b_1)^T M(x(h) + b_1) + \int_0^h \left(x^T Q x + u_1^T u_1\right) dt]. \qquad (11)$$

Then the solution of the LQR problem (7,5) is given by

$$u_{\text{opt}}(t) = [\mathbb{1}(t) - \mathbb{1}(t-h)]u_{1,\text{opt}}(t) + \mathbb{1}(t-h)u_{2,\text{opt}}(t) \qquad (12)$$

and the optimal cost is given by

$$\min_{u_1}\Big(\int_0^h \left(x^T Q x + u_1^T u_1\right) dt + [x(h) + b_1)^T M(x(h) + b_1]\Big). \qquad (13)$$

Proof. Consider the state space equation (7). Assume temporarily that the optimal state at $t = h^-$, denoted by $x_{\text{opt}}(h^-)$, is known. It follows that

$$x_{\text{opt}}(h^+) = x_{\text{opt}}(h^-) + b_1.$$

For the time region $t \in [h^+, \infty]$, the equation (7) becomes

$$\dot{x} = Ax + B_2 u, \quad x(h^+) = x_{\text{opt}}(h^-) + b_1, \qquad (14)$$

while the cost over this this time region is given by

$$J_{[h^+,\infty]} = \int_h^\infty \left(x(t)^T Q x(t) + u(t)^T u(t)\right) dt. \qquad (15)$$

The problem of minimizing (15) given (14) is a standard infinite horizon LQR problem, the solution of which is the state feedback

$$u_{\text{opt}}(t) = B_2^T M x(t), \; t \in [h^+, \infty], \qquad (16)$$

while the optimal cost is

$$\begin{aligned} J_{[h^+,\infty],opt} &= x(h^+)^T M x(h^+) \\ &= [x_{\text{opt}}(h^-) + b_1]^T M [x_{\text{opt}}(h^-) + b_1], \end{aligned} \qquad (17)$$

where M is the solution of the Riccati equation (8). Hence, it is proved that for $t \in [h^+, \infty]$ the optimal input is indeed given by the state feedback (9). It is also clear that the optimal cost contribution over $t = [h^+, \infty]$, which is given by (17), depends solely on $x_{\text{opt}}(h^-)$. It follows that the infinite horizon LQR problem of minimizing (5) is equivalent to minimizing the finite horizon cost function

$$\min_{u}\Big(\int_0^h \left(x^T Q x + u^T u\right) dt + [x(h) + b_1]^T M [x(h) + b_1]\Big),$$

from which the optimal input for $t \in [0, h^-]$ may be obtained. ■

Lemma 1 gives a partial solution to the LQR problem (7,5). It is now ascertained that for $t \in [h, \infty]$ the optimal input is a state feedback given by (9). What is left is to solve the finite horizon LQR problem (10,11). The solution is summarized in the following lemma. The derivation is based on the Pontryagin minimum principle (see for example Appendix C of (Anderson and Moore, 1989)).

Lemma 2. Consider the LQR problem corresponding to the state space equation (10) with the objective (11) where M is the stabilizing solution of the Riccati equation (8). Then the optimal input of the LQR problem (10,11) is given by

$$u_{1,\text{opt}}(t) = -B_2^T M x(t) - B_2^T e^{-A_p^T (t-h)} M b_1, \qquad (18)$$

with $A_p = A - B_2 B_2^T M$.

Proof. We begin by applying the minimum principle to the optimal control problem (10,11). It may be shown (see for example Appendix C of (Anderson and Moore, 1989)), that the optimal input is given by

$$u_{1,\text{opt}}(t) = B_2^T p(t), \qquad (19)$$

where the co-state p and the optimal state x satisfy the following equation:

$$\begin{bmatrix} \dot{x} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} A & B_2 B_2^T \\ Q & -A^T \end{bmatrix} \begin{bmatrix} x \\ p \end{bmatrix} \qquad (20)$$

with the boundary condition

$$x(0) = 0, \quad p(h) = -Mx(h) - Mb_1. \qquad (21)$$

Notice that except for the boundary condition, the equations are similar to the standard case where $b_1 = 0$. Furthermore, using similar arguments as in the standard case, it may be shown that the differential equation (20,21) has a unique solution.

To obtain the solution of the differential equation (20,21), define the state transformation

$$q(t) = Mx(t) + p(t). \qquad (22)$$

With the state transformation and keeping in mind that M is the solution of the Riccati equation (8), the differential equation (20) is simplified to

$$\begin{bmatrix} \dot{x} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} A_p & B_2 B_2^T \\ 0 & -A_p^T \end{bmatrix} \begin{bmatrix} x \\ q \end{bmatrix} \qquad (23)$$

with the boundary condition

$$x(0) = 0, \quad q(h) = -Mb_1. \qquad (24)$$

It follows that the trajectory of $q(t)$ is given by

$$q(t) = e^{-A_p^T t} q(0). \qquad (25)$$

The initial condition $q(0)$ may be computed by setting $t = h$ in (25) and substituting the boundary condition (24), resulting in:

$$q(0) = -e^{A_p^T h} M b_1, \qquad (26)$$

so that the complete expression for $q(t)$ is obtained:

$$q(t) = -e^{-A_p^T (t-h)} M b_1. \qquad (27)$$

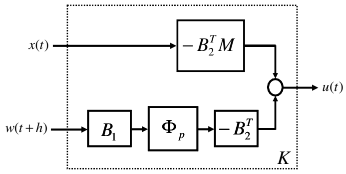


Fig. 2. The optimal controller, single preview time case

Using (22) and (27), $p(t)$ may be computed:

$$p(t) = -Mx(t) - e^{-A_p^T(t-h)}Mb_1. \qquad (28)$$

The optimal input $u_{1,\text{opt}}(t) = B_2^T p(t)$ is then given by

$$u_{1,\text{opt}}(t) = -B_2^T Mx(t) - B_2^T e^{-A_p^T(t-h)}Mb_1.$$

■

The optimal controller

Lemma 1 combined with Lemma 2 provides a complete solution to the infinite horizon LQR problem (7,5). By Lemma 1 and Lemma 2, the optimal u is given by

$$\begin{aligned} u_{\text{opt}}(t) &= [\mathbb{1}(t) - \mathbb{1}(t-h)]u_{1,\text{opt}}(t) + \mathbb{1}(t-h)u_{2,\text{opt}}(t) \\ &= -[\mathbb{1}(t) - \mathbb{1}(t-h)]B_2^T(Mx(t) + e^{-A_p^T(t-h)}Mb_1) \\ &\quad - \mathbb{1}(t-h)B_2^T Mx(t) \\ &= -B_2^T Mx(t) - [\mathbb{1}(t) - \mathbb{1}(t-h)]B_2^T e^{-A_p^T(t-h)}Mb_1 \end{aligned} \qquad (29)$$

This is the unique optimal u for the control system of Figure 1 when $w(t) = \delta(t-h)$. Hence, if we manage to find a full information controller with preview that also produces the same input if we set $w(t) = \delta(t-h)$, then we automatically obtain the desired H_2-optimal controller. In the following theorem, the optimal controller is derived.

Theorem 3. Consider the control system of Figure 1(b) where the plant's dynamics are governed by (3). Suppose that w is scalar, i.e. $B_1 = b_1$ has a single column. Then the optimal controller that minimizes the H_2-norm of the transfer function from w to z is the controller in Figure 2, where Φ_p has the following impulse response:

$$\Phi_p(t) = [\mathbb{1}(t) - \mathbb{1}(t-h)]e^{-A_p^T(t-h)}M. \qquad (30)$$

Here M is the stabilizing solution of the Riccati equation (8), while $A_p = A - MB_2B_2^T$. Notice that Φ_p has a finite impulse response with support on $[0, h]$.

Proof. It may be verified that the controller in Figure 2 generates the optimal u given by (29) when driven by $w(t+h) = \delta(t)$. ■

The optimal H_2-norm

The squared optimal H_2-norm is equal to the optimal cost function (11), which is derived in what follows. It follows from (19,20) that

$$\frac{d}{dt}(p^T x) = p^T\dot{x} + x^T\dot{p} = u_{1,\text{opt}}^T u_{1,\text{opt}} + x^T Qx. \qquad (31)$$

Taking the integral of both sides of (31), the optimal value of the integral term in (11) is

$$\begin{aligned} &\int_0^h \left(x^T Qx + u_{1,\text{opt}}^T u_{1,\text{opt}}\right) dt \\ &= p(h)^T x(h) - p(0)^T x(0) = p(h)^T x(h). \end{aligned} \qquad (32)$$

The expression of $p(t)$ is readily available in (28), while the expression for $x(t)$ may be computed from the differential equation (23) and the initial condition (24,26). It is given by

$$\begin{aligned} x(t) &= \Sigma_{11}(t)x(0) + \Sigma_{12}(t)q(0) \\ &= -\Sigma_{12}(t)e^{A_p^T h}Mb_1, \end{aligned} \qquad (33)$$

where

$$\Sigma(t) = \begin{bmatrix} \Sigma_{11}(t) & \Sigma_{12}(t) \\ \Sigma_{21}(t) & \Sigma_{22}(t) \end{bmatrix} = e^{St}, \qquad (34)$$

with

$$S = \begin{bmatrix} A_p & B_2B_2^T \\ 0 & -A_p^T \end{bmatrix}. \qquad (35)$$

Plugging (32) into (11) and using (28,33) to simplify the expression, the following is obtained:

$$\begin{aligned} &\min_{u_1}[(x(h) + b_1)^T M(x(h) + b_1) \\ &\quad + \int_0^h \left(x^T Qx + u_1^T u_1\right) dt] \\ &= b_1^T Mb_1 - b_1^T M\Sigma_{12}(h)e^{A_p^T h}Mb_1, \end{aligned} \qquad (36)$$

where M is the solution of the Riccati equation (8), $\Sigma(t)$ is given by (34), and $A_p = A - B_2B_2^T M$. The formula (36) may be further simplified by finding a simpler expression for Σ_{12}. Let X be the solution of the Lyapunov equation:

$$A_p X + XA_p^T + B_2B_2^T = 0. \qquad (37)$$

Since A_p is Hurwitz, the Lyapunov equation has a unique solution. By defining

$$W = \begin{bmatrix} I & -X \\ 0 & I \end{bmatrix}, \qquad (38)$$

it is straightforward to compute

$$WSW^{-1} = \begin{bmatrix} A_p & 0 \\ 0 & -A_p^T \end{bmatrix}. \qquad (39)$$

Using (39), it may be shown that

$$e^{St} = \begin{bmatrix} e^{A_p t} & Xe^{-A_p^T t} - e^{A_p t}X \\ 0 & e^{-A_p^T t} \end{bmatrix}, \qquad (40)$$

implying that

$$\Sigma_{12}(t) = Xe^{-A_p^T t} - e^{A_p t}X. \qquad (41)$$

Plugging (41) into (36) results in a simplified expression of the optimal cost which is equal to the squared optimal H_2 norm:

$$J_{\text{opt}}(h) = b_1^T Mb_1 - b_1^T M(X - e^{A_p h}Xe^{A_p^T h})Mb_1. \qquad (42)$$

Compared to the formula given in (Kojima, 2004), which involves solving a differential Riccati equation, the formula (42) appears simpler and only requires solving the Lyapunov equation (37) and computing the exponential of a matrix.

Effect of the preview time h on the H_2 performance

It may be shown that the first derivative of the optimal squared H_2-norm with respect to h is

$$\partial J_{\text{opt}}(h)/\partial h = -b_1^T M e^{A_p h} B_2 B_2^T e^{A_p^T h} M b_1 \leq 0. \tag{43}$$

Evidently, the squared optimal H_2-norm as a function of h is non-increasing. In particular, it may be shown that if (C_1, A, B_2) is observable and controllable, $J_{\text{opt}}(h)$ is strictly decreasing. Thus, as the preview time increases, the performance increases as well. Moreover, the first term in the right hand side of (42) is the optimal squared H_2-norm for $h = 0$ (i.e. no preview), so that the second term may be viewed as the performance gain due to the previewed input. The minimum achievable H_2-norm is obtained if we set $h = \infty$ (i.e. infinite preview), which gives

$$J_{\text{opt},h=\infty} = b_1^T (M - MXM) b_1. \tag{44}$$

4. MULTIPLE INPUTS CASE

The optimal controller for the multiple inputs case is a straightforward extension of the single-input result. It turns out that the controller has exactly the same structure as in the single-input case.

Corollary 4. Consider the control system of Figure 1(b) where the plant's dynamics are governed by (3). The optimal controller that minimizes the H_2-norm of the transfer function from w to z is the controller in Figure 2. Here M is the stabilizing solution of the Riccati equation (8), while the impulse response of Φ_p is given by (30). Moreover the squared optimal H_2-norm is given by

$$\text{trace}(B_1^T M B_1 + B_1^T M (X - e^{A_p h} X e^{A_p^T h}) M B_1), \tag{45}$$

where $A_p = A - B_2 B_2^T M$, and X is the solution of the Lyapunov equation (37).

Proof. According to definition (2), the squared H_2-norm of the control system of Figure 1(b) may be computed by conducting n_w experiments, with n_w the dimension of w, as described in what follows. For the k-th experiment, the k-th element of w is set to the delayed delta function $\delta(t - h)$ while the other elements are set to zero. The squared H_2-norm of the closed loop system is then obtained by summing up the squared L_2-norm of the output z for all n_w experiments. Since for each experiment only one element of w is active, for each experiment the control system may be recast into one with scalar external input w. Hence, the single-input results of the previous section applies. Using Theorem 3, it is straightforward to prove that for each experiment, the controller of

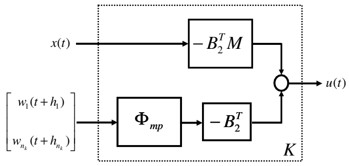


Fig. 3. The optimal controller, multiple preview times case

Figure 2 generates the optimal input. This implies that the controller is the optimal controller. The squared optimal H_2-norm is obtained by summing up the optimal cost for all n_w experiments, which individually may be computed using (42). ■

5. MULTIPLE PREVIEW TIMES EXTENSION

Consider the case where the preview time for each component of the external input w may be different. It turns out that the arguments on which the proof of Corollary 4 is based may also be applied. The derivation of the optimal controller is outlined in what follows.

Suppose that the k-th component of w and its corresponding preview time are denoted as w_k and h_k, respectively. As in the multiple input case, n_w experiments are conducted, where in the k-th experiment the k-th element of w is set to the delayed delta function $\delta(t - h_k)$ while others are set to zero. For each experiment the problem also reduces to a single-input problem. The only difference with the previous section is that here the preview time is different for each experiment. Nevertheless, the results from Section 3 still apply.

Using Theorem 3, it is straightforward to ascertain that the controller of Figure 3 generates the optimal input for each experiment. There, the block Φ_{mp} is a system with finite impulse response. The k-th column of its impulse response, denoted by $\Phi_{mp,k}(t)$, is given by

$$\Phi_{mp,k}(t) = e^{-A_p^T (t-h_k)} M b_{1,k} (\mathbb{1}(t) - \mathbb{1}(t - h_k)) \tag{46}$$

where M is the solution of the Riccati equation (8), $A_p = A - B_2 B_2^T M$, and $b_{1,k}$ denotes the k-th column of the matrix B_1.

6. RELAXING ASSUMPTION A3

Assumption A3 allows us to formulate the LQR problem (4,5). The assumption may be relaxed using the well-known method of input substitution. The method works by introducing the state feedback

$$u(t) = R^{-\frac{1}{2}} v(t) - R^{-1} D_2^T C_1 x(t) \tag{47}$$

in (3), where $R = D_2^T D_2$ and v is the new input. With this change of the input, the state equation becomes

$$\dot{x}(t) = \bar{A} x(t) + b_1 \delta(t - h) + \bar{B}_2 v(t),\ x(0) = x_0, \tag{48}$$

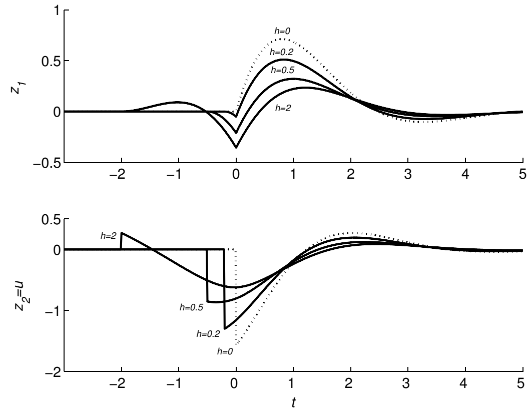


Fig. 4. The output z, top: z_1, bottom: $z_2 = u$

while the cost criterion is given by

$$\min_v \int_0^\infty x(t)^T \bar{Q} x(t) + v(t)^T v(t) dt \qquad (49)$$

where $\bar{Q} = C_1^T(I - D_2 R^{-1} D_2^T)C$, $\bar{A} = (A - B_2 R^{-1} D_2^T C_1)$, and $\bar{B}_2 = B_2 R^{-\frac{1}{2}}$. The resulting LQR problem is of the same form as the problem (4,5) and therefore may be solved using the technique from the previous sections.

7. NUMERICAL EXAMPLE

In this section we solve a single input example which is already converted to an LQR problem of the form (4,5). The example is taken from (Kojima, 2004) and described in what follows. The state space equation is given by

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} w(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t) \\ z(t) &= \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \\ w(t) &= \delta(t-h),\ x(0) = 0. \end{aligned} \qquad (50)$$

The objective is to minimize the L_2-norm of $z(t)$ given $w(t) = \delta(t - h)$. Using formulas from Section 3, the solution may be computed. The output $z = [z_1, z_2]^T$ given $w(t) = \delta(t-h)$ is shown in Figure 4 for various values of the preview time. Note that in this figure, the signals are shifted h time units to the left to recover the original setting in which the controller is fed with advance version of the external input from the design setting in which the external input fed to the plant is delayed while the controller is supplied with the non-delayed version. Notice that in this case the optimal input is equal to the second component of the output z. It is observed that using preview, the controller compensates for changes in the external input before the changes occur. The optimal L_2-norm of $z(t)$ given $w(t) = \delta(t - h)$, which is equal to the optimal H_2-norm of the transfer function from w to z, may be computed using (42). It is shown as a function of the preview time h in Figure 5. As expected, it decreases as the preview time increases. The optimal value for infinite preview time may be computed using (44).

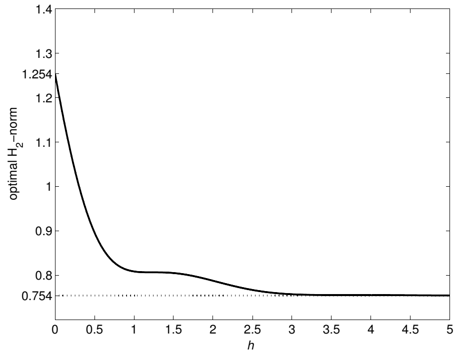


Fig. 5. The optimal H_2-norm.

8. CONCLUDING REMARKS

In this paper, the H_2 control problem of preview systems is considered. The single input case is solved and extensions for multiple inputs case and multiple preview times case are provided. The results show that by providing the external input in advance to the controller, the H_2 performance of the control system may be improved.

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