STUDY ON FULL DECOUPLING PROBLEM OF LINEAR PERIODIC SYSTEMS

P. Zhang [*,1] **S. X. Ding** [*,1] **T. Jeinsch** [**]

[*] *Institute for Automatic Control and Complex Systems, University of Duisburg-Essen, 47057 Duisburg, Germany*
[**] *IAV GmbH, Philipp-Mueller-Str. 12, 23966 Wismar, Germany*

Abstract: This paper studies the fault detection problem of linear periodic systems. The aim is to design residual generators, which deliver a residual signal fully decoupled from unknown disturbances. First, a periodic parity relation based full decoupling residual generator with a periodic varying parity vector is designed. Then, a periodic observer based full decoupling residual generator is obtained. Examples are given to illustrate the achieved results.

Keywords: Fault detection; Linear periodic systems; Full decoupling; Parity space approach; Observer

1. PROBLEM FORMULATION

During the last three decades, model based fault detection (FD) technology has attracted much attention (Gertler, 1998; Chen and Patton, 1999; Frank *et al.*, 2000; Patton *et al.*, 2000). It is well recognized that model-based fault detection problem is indeed an output estimation problem. In the context of linear time-invariant (LTI) systems, a number of approaches have been proposed for the design of FD systems. Under certain conditions, the fault indicating signal, usually called residual, can be fully decoupled from the unknown disturbances. To this aim, methods like eigenstructure assignment, parity space, unknown input observer etc. have been developed (Gertler, 1998; Chen and Patton, 1999; Patton *et al.*, 2000).

Periodic systems are the simplest class of linear systems next to LTI systems and exist in different areas (Bittanti and Colaneri, 1999; Souza and Trofino, 2000). Periodic systems have also been used to describe multirate sampled-data systems, nonlinear systems linearized along a periodic regime (Bittanti and Colaneri, 1999), and, more recently, networked control systems with periodic communication pattern (Rehbinder and Sanfridson, 2004). Our study is motivated not only by the continuous theoretical development in periodic control and filtering (Xie and Souza, 1993; Bittanti and Colaneri, 1999; Lampe and Rossenwasser, 2004) but also by the increasing applications of periodic control in practice like helicopter vibration control (Arcara *et al.*, 2000), satellite attitude control (Lovera *et al.*, 2002) as well as wind turbine (Stol, 2003). Extension of the FD technique to periodic systems will improve the safety and reliability of such applications.

This paper studies the full decoupling problem of linear discrete-time periodic systems described by

$$\begin{aligned} x(k+1) &= A_k x(k) + B_k u(k) + E_k^d d(k) + E_k^f f(k) \\ y(k) &= C_k x(k) + D_k u(k) + F_k^d d(k) + F_k^f f(k) \end{aligned} \qquad (1)$$

where $x \in \mathbf{R}^n$ denotes the state vector, $u \in \mathbf{R}^p$ the control input vector, $y \in \mathbf{R}^m$ the measured output vector, $d \in \mathbf{R}^{k_d}$ the disturbance vector, and $f \in \mathbf{R}^{k_f}$ the vector of faults to be detected, $A_k, B_k, C_k, D_k, E_k^d, E_k^f, F_k^d, F_k^f$ are known real bounded periodic matrix functions of period

[1] supported by the EU grant IST-2001-32122 IFATIS

θ and with appropriate dimensions, namely, $\forall k$,

$$\begin{bmatrix} A_{k+\theta} & B_{k+\theta} & E^d_{k+\theta} & E^f_{k+\theta} \\ C_{k+\theta} & D_{k+\theta} & F^d_{k+\theta} & F^f_{k+\theta} \end{bmatrix} = \begin{bmatrix} A_k & B_k & E^d_k & E^f_k \\ C_k & D_k & F^d_k & F^f_k \end{bmatrix}$$

The aim is to design a residual generator, so that the residual r satisfies

(i) $\lim_{k\to\infty} r(k) = 0$, if $f = 0$ and no matter what the control and disturbance inputs are;
(ii) r deviates from zero as long as $f \neq 0$.

This work follows the pioneering work of Fadali et al. (Fadali and Gummuluri, 2001) and Varga (Varga, 2004). In the scheme of (Fadali and Gummuluri, 2001), first a periodic observer is designed, then a bank of periodic FIR filters are designed based on lifted reformulation of the system over one period to achieve full disturbance rejection. A computational approach is proposed in (Varga, 2004), which realizes full disturbance decoupling by computing a stable left annihilator for the periodic system.

The first approach of this paper is motivated by the observation that parity space approach treats each time instant independently. It is shown that the full decoupling problem of periodic systems can be solved through a group of independent linear equations. A periodic relation based residual generator with a periodic varying parity vector can thus be easily obtained.

In the second part of the paper, it is shown that if given a parity vector, then a periodic observer based residual generator can be readily constructed. Moreover, if the periodic parity vector realises a full decoupling, so does the resulting periodic observer based residual generator. The freedom in the observer gain can be used to meet other FDI performance specifications.

2. PERIODIC PARITY SPACE APPROACH

The essence of parity space approach is to derive the so-called parity relations (Chow and Willsky, 1984). It is widely accepted due to simple computation and straightforward implementation. In this section, we shall show that the parity space approach can be easily extended to periodic systems.

At time instant k, consider the input-output relation of periodic system (1) during the moving horizon $[k-s, k]$, where s is a fixed integer and represents the length of the horizon. Write k into

$$k = j\theta + i + s$$

with $j = 0, 1, 2, \cdots$, and $i = 0, \cdots, \theta-1$. A parity relation is obtained as

$$\begin{aligned} Y(k) = {} & H_{o,i}x(k-s) + H_{u,i}U(k) \\ & + H_{d,i}D(k) + H_{f,i}F(k) \end{aligned} \quad (2)$$

where

$$Y(k) = \begin{bmatrix} y'(k-s) & y'(k-s+1) & \cdots & y'(k) \end{bmatrix}'$$
$$U(k) = \begin{bmatrix} u'(k-s) & u'(k-s+1) & \cdots & u'(k) \end{bmatrix}'$$
$$D(k) = \begin{bmatrix} d'(k-s) & d'(k-s+1) & \cdots & d'(k) \end{bmatrix}'$$
$$F(k) = \begin{bmatrix} f'(k-s) & f'(k-s+1) & \cdots & f'(k) \end{bmatrix}'$$
$$H_{o,i} = \begin{bmatrix} C_i \\ C_{i+1}A_i \\ \vdots \\ C_{i+s}A_{i+s-1}\cdots A_{i+1}A_i \end{bmatrix} \quad (3)$$
$$H_{u,i} = \begin{bmatrix} D_i & O & \cdots & O \\ C_{i+1}B_i & D_{i+1} & \ddots & \vdots \\ \vdots & & \ddots & O \\ C_{i+s}\cdots A_{i+1}B_i & \cdots & & D_{i+s} \end{bmatrix}$$
$$H_{d,i} = \begin{bmatrix} F^d_i & O & \cdots & O \\ C_{i+1}E^d_i & F^d_{i+1} & \ddots & \vdots \\ \vdots & & \ddots & O \\ C_{i+s}\cdots A_{i+1}E^d_i & \cdots & & F^d_{i+s} \end{bmatrix}$$
$$H_{f,i} = \begin{bmatrix} F^f_i & O & \cdots & O \\ C_{i+1}E^f_i & F^f_{i+1} & \ddots & \vdots \\ \vdots & & \ddots & O \\ C_{i+s}\cdots A_{i+1}E^f_i & \cdots & & F^f_{i+s} \end{bmatrix}$$

Due to the periodicity of system matrices, the matrices $H_{o,i}, H_{u,i}, H_{d,i}, H_{f,i}$ in parity relation (2) is θ-periodic with respect to i. Based on parity relation (2), a residual generator can be constructed as

$$r(k) = v_i(Y(k) - H_{u,i}U(k)) \quad (4)$$

where r is the so-called residual signal, row vectors v_i, $i = 0, 1, \cdots, \theta-1$, are design parameters called parity vector. If v_i are selected in such a way that

$$v_i \begin{bmatrix} H_{o,i} & H_{d,i} \end{bmatrix} = 0, \quad v_i H_{f,i} \neq 0 \quad (5)$$

for each $i = 0, 1, \cdots, \theta-1$, then

$$r(k) = v_i H_{f,i} F(k)$$

The residual will be influenced neither by the initial state $x(k-s)$ nor by the disturbance vector d or the control input vector u. As a result, the conditions (i)-(ii) are satisfied and a full decoupling is realized. The existence condition for solution of (5) is $\forall i$,

$$rank \begin{bmatrix} H_{o,i} & H_{d,i} & H_{f,i} \end{bmatrix} > rank \begin{bmatrix} H_{o,i} & H_{d,i} \end{bmatrix} \quad (6)$$

The relation between (6) and the condition presented in (Varga, 2004), which is based on transfer function matrix of stacked lifted reformulations, is worthy of further exploration.

It is worth noting that (5) is a group of *independent* linear equations and can be easily solved. To illustrate it, let us consider the following example.

Example 1 Consider a periodic system of period $\theta = 2$ described by (1) with (Fadali and Gummuluri, 2001)

$$A_0 = \begin{bmatrix} 0.25 & 0.25 & 0.1 & -0.1 \\ 0.5 & 0.1 & 0.1 & 0.5 \\ 0.5 & -0.2 & 0.2 & 0.25 \\ 0.1 & 0 & 0.25 & 0.1 \end{bmatrix}, B_0 = \begin{bmatrix} 0.5 \\ 0.1 \\ 0.1 \\ 0.25 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 0.1 & 0.2 & 0.1 & -0.1 \\ -0.1 & 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0.1 & 0.25 \\ 0 & 0.1 & 0.1 & 0.25 \end{bmatrix}, B_1 = \begin{bmatrix} 0.1 \\ 0.5 \\ 0.1 \\ 0.5 \end{bmatrix}$$

$$E_0^d = \begin{bmatrix} 1.3 \\ 1.8 \\ 1.6 \\ 0.32 \end{bmatrix}, E_0^f = \begin{bmatrix} 0.1 \\ -1 \\ 0.2 \\ 0.1 \end{bmatrix}, E_1^d = \begin{bmatrix} 3.2 \\ 2 \\ -1 \\ -2 \end{bmatrix}$$

$$E_1^f = \begin{bmatrix} 0.1 \\ -1 \\ 0.2 \\ 0.1 \end{bmatrix}, C_0 = \begin{bmatrix} 0.25 & 0.1 & 0.2 & 0.1 \\ -0.1 & 0.5 & 0.2 & 0.5 \\ 0.25 & 0.5 & -0.1 & 0.1 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 0.1 & 0.25 & 0.1 & -0.1 \\ 0.25 & 0.1 & 0.2 & 0.1 \\ 0.1 & 0.25 & -0.2 & 0.5 \end{bmatrix}$$

$$F_0^d = O, F_0^f = O, F_1^d = O, F_1^f = O$$

Let $s = 1$. It is easily computed that

$$H_{o,0} = \begin{bmatrix} 0.25 & 0.1 & 0.2 & 0.1 \\ -0.1 & 0.5 & 0.2 & 0.5 \\ 0.25 & 0.5 & -0.1 & 0.1 \\ 0.19 & 0.03 & 0.03 & 0.13 \\ 0.2225 & 0.0325 & 0.1 & 0.085 \\ 0.1 & 0.09 & 0.12 & 0.115 \end{bmatrix}$$

$$H_{o,1} = \begin{bmatrix} 0.1 & 0.25 & 0.1 & -0.1 \\ 0.25 & 0.1 & 0.2 & 0.1 \\ 0.1 & 0.25 & -0.2 & 0.5 \\ 0.115 & 0.21 & 0.055 & 0.1 \\ 0.04 & 0.38 & 0.06 & 0.435 \\ -0.075 & 0.26 & 0.025 & 0.225 \end{bmatrix}$$

$$H_{d,0} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0.708 & 0 \\ 0.857 & 0 \\ 0.42 & 0 \end{bmatrix}, \ H_{d,1} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0.6 & 0 \\ -0.52 & 0 \\ 1.7 & 0 \end{bmatrix}$$

$$H_{f,0} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -0.23 & 0 \\ -0.025 & 0 \\ -0.23 & 0 \end{bmatrix}, H_{f,1} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -0.025 & 0 \\ -0.42 & 0 \\ -0.485 & 0 \end{bmatrix}$$

$$H_{u,0} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0.06 & 0 \\ 0.18 & 0 \\ 0.18 & 0 \end{bmatrix}, \ H_{u,1} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0.145 & 0 \\ 0.51 & 0 \\ 0.315 & 0 \end{bmatrix}$$

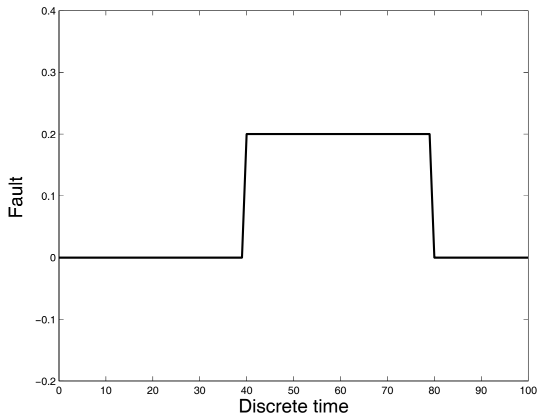

Fig. 1. The fault signal

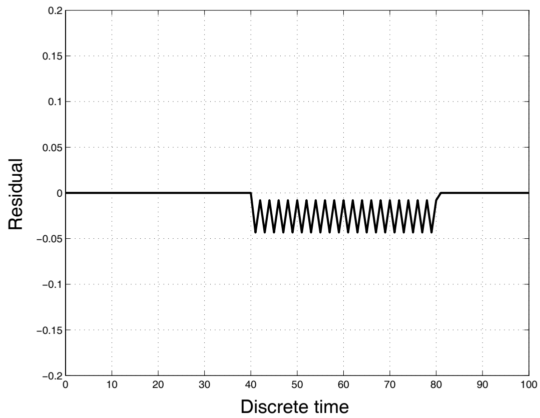

Fig. 2. The residual signal generated by residual generator (8)

To decouple the residual from the disturbances, solve $v_0 \begin{bmatrix} H_{o,0} & H_{d,0} \end{bmatrix} = 0$, $v_0 H_{f,0} \neq 0$ for v_0 and $v_1 \begin{bmatrix} H_{o,1} & H_{d,1} \end{bmatrix} = 0$, $v_1 H_{f,1} \neq 0$ for v_1. It yields

$$\begin{aligned} v_0 &= [-0.0631 \ -0.1348 \ 0.0314 \\ &\qquad 0.2316 \ -0.5703 \ 0.7733] \\ v_1 &= [0.3535 \ 0.2589 \ 0.1962 \\ &\qquad -0.8290 \ -0.1421 \ 0.2491] \end{aligned} \quad (7)$$

As a result, the residual generator is

$$\begin{aligned} r(k) &= v_0 \left(\begin{bmatrix} y(k-1) \\ y(k) \end{bmatrix} - H_{u,0} \begin{bmatrix} u(k-1) \\ u(k) \end{bmatrix} \right) \\ &\text{if } k = 2j+1, \\ r(k) &= v_1 \left(\begin{bmatrix} y(k-1) \\ y(k) \end{bmatrix} - H_{u,1} \begin{bmatrix} u(k-1) \\ u(k) \end{bmatrix} \right) \\ &\text{if } k = 2j+2 \end{aligned} \quad (8)$$

In the simulation, it is assumed that the control input is step signal (step time at 0) of amplitude 1, the disturbance $d(k) = \sin(0.01\pi k)$, the fault appears at the $40th$ discrete time as shown in Fig.1. The residual signal is shown in Fig.2. It is seen that the residual signal r is not influenced by u, d and changes only if $f \neq 0$.

3. PERIODIC OBSERVER BASED FULL DECOUPLING RESIDUAL GENERATOR

In this section, we present one approach to design periodic observer based full decoupling residual generators for periodic system (1).

To the aim of fault detection, an observer based residual generator is constructed as

$$\begin{aligned} z(k+1) &= G_k z(k) + H_k u(k) + L_k y(k) \\ r(k) &= W_k z(k) + Q_k u(k) + V_k y(k) \end{aligned} \quad (9)$$

with $z \in \mathbf{R}^s$. The goal is to design the θ-periodic matrices G_k, H_k, L_k, W_k, Q_k and V_k, so that the conditions (i)-(ii) are fulfilled. In the fault-free case, the dynamics of residual generator (9) is governed by

$$\begin{aligned} e(k+1) &= G_k e(k) + (H_k - T_{k+1}B_k + L_k D_k)u(k) \\ &\quad + (G_k T_k + L_k C_k - T_{k+1}A_k)x(k) \\ &\quad + (L_k F_k^d - T_{k+1}E_k^d)d(k) \\ r(k) &= W_k e(k) + (W_k T_k + V_k C_k)x(k) \\ &\quad + (Q_k + V_k D_k)u(k) + V_k F_k^d d(k) \end{aligned} \quad (10)$$

where $e(k) = z(k) - T_k x(k)$. If the equations

$$T_{k+1}A_k - G_k T_k = L_k C_k \quad (11)$$

$$W_k T_k + V_k C_k = 0 \quad (12)$$

$$H_k = T_{k+1}B_k - L_k D_k \quad (13)$$

$$Q_k = -V_k D_k \quad (14)$$

hold for any $k = 0, 1, \cdots, \theta - 1$, then the residual dynamics (10) reduces to

$$\begin{aligned} e(k+1) &= G_k e(k) + (L_k F_k^d - T_{k+1}E_k^d)d(k) \\ r(k) &= W_k e(k) + V_k F_k^d d(k) \end{aligned} \quad (15)$$

and meets the basic requirement that $\forall u$,

$$\lim_{k\to\infty} r(k) = 0, \quad \text{if } d = 0, f = 0 \quad (16)$$

as long as the stability of (15) is guaranteed.

Equations (11)-(14) are an extension of the well-known Luenberger condition in periodic systems. If matrices G_k, L_k, T_k, W_k, V_k satisfying (11)-(12) are found, then H_k, Q_k follow readily from (13)-(14). However, it is not easy to solve difference equation (11). Inspired by the fact that in the LTI case there is a one to one relationship between observer based and parity relation based residual generators (Ding *et al.*, 1998), the question now is whether we can construct a periodic observer from a periodic parity vector. The answer is positive, as shown by theorem 1.

Theorem 1 Assume that

$$v_k = \begin{bmatrix} v_{k,0} & v_{k,1} & \cdots & v_{k,s} \end{bmatrix}, \ k = 0, 1, \cdots, \theta - 1$$

satisfy $v_k H_{o,k} = 0$. Then

$$G_k = \begin{bmatrix} 0 & \cdots & 0 & g_{k,1} \\ 1 & \ddots & \vdots & g_{k,2} \\ \vdots & \ddots & 0 & \vdots \\ 0 & \cdots & 1 & g_{k,s} \end{bmatrix} \quad (17)$$

$$L_k = -\begin{bmatrix} v_{k,0} \\ v_{k-1,1} \\ \vdots \\ v_{k-s+1,s-1} \end{bmatrix} - \begin{bmatrix} g_{k,1} \\ g_{k,2} \\ \vdots \\ g_{k,s} \end{bmatrix} v_{k-s,s}$$

$$T_k = \begin{bmatrix} v_{k-1,1} & \cdots & v_{k-1,s-1} & v_{k-1,s} \\ v_{k-2,2} & \cdots & v_{k-2,s} & 0 \\ \vdots & & \vdots & \vdots \\ v_{k-s,s} & \cdots & 0 & 0 \end{bmatrix} \times \begin{bmatrix} C_k \\ C_{k+1}A_k \\ \vdots \\ C_{k+s-1}A_{k+s-2}\cdots A_k \end{bmatrix}$$

$$W_k = \begin{bmatrix} 0 & 0 & \cdots & 0 & -1 \end{bmatrix}$$

$$V_k = v_{k-s,s}$$

solve equations (11)-(12).

Proof: To prove equation (11), note that

$$v_k H_{o,k} = 0 \Leftrightarrow$$

$$\begin{bmatrix} v_{k,0} & v_{k,1} & \cdots & v_{k,s} \end{bmatrix} \begin{bmatrix} C_k \\ C_{k+1}A_k \\ \vdots \\ C_{k+s}A_{k+s-1}\cdots A_{k+1}A_k \end{bmatrix} = 0$$

The first row of $T_{k+1}A_k - G_k T_k$ is thus

$$\begin{bmatrix} v_{k,1} & \cdots & v_{k,s} \end{bmatrix} \begin{bmatrix} C_{k+1} \\ \vdots \\ C_{k+s}A_{k+s-1}\cdots A_{k+1} \end{bmatrix} A_k - g_{k,1}v_{k-s,s}C_k = -v_{k,0}C_k - g_{k,1}v_{k-s,s}C_k$$

which is equal to the first row of $L_k C_k$. The $(j+1)$-th $(j = 1, \cdots, s-1)$ row of $T_{k+1}A_k - G_k T_k$ is

$$\begin{aligned} &\begin{bmatrix} v_{k-j,j+1} & \cdots & v_{k-j,s} \end{bmatrix} \\ &\times \begin{bmatrix} C_{k+1} \\ \vdots \\ C_{k+s-j}A_{k+s-j-1}\cdots A_{k+1} \end{bmatrix} A_k \\ &- \begin{bmatrix} v_{k-j,j} & v_{k-j,j+1} & \cdots & v_{k-j,s} \end{bmatrix} \\ &\times \begin{bmatrix} C_k \\ C_{k+1}A_k \\ \vdots \\ C_{k+s-j}A_{k+s-j-1}\cdots A_k \end{bmatrix} \\ &- g_{k,j+1}v_{k-s,s}C_k \\ &= -v_{k-j,j}C_k - g_{k,j+1}v_{k-s,s}C_k \end{aligned}$$

which is equal to the $(j+1)$-th row of $L_k C_k$. Equation (11) is proven. Since

$$W_k T_k + V_k C_k = -v_{k-s,s}C_k + v_{k-s,s}C_k = 0,$$

equation (12) holds. ■

Theorem 1 points out that, given a periodic varying vector belonging to the parity space, a periodic observer based residual generator satisfying (11)-(12) and thus (16) can be found.

Note that $g_k = \left[g_{k,1} \ \cdots \ g_{k,s} \right]'$, $k = 0, 1, \cdots, \theta - 1$, appearing in matrices G_k, L_k are free design parameters and should be selected in such a way that residual dynamics (15) is stable. It means that the characteristic multipliers of system (15), i.e. the eigenvalues of $G_{\theta-1} \cdots G_1 G_0$, should be located inside the unit circle. Indeed, residual dynamics (15) can be equivalently written as

$$e(k+1) = \bar{G}_k e(k) + (\bar{L}_k F_k^d - T_{k+1} E_k^d) d(k) - g_k r(k)$$
$$r(k) = W_k e(k) + V_k F_k^d d(k)$$

with $\bar{G}_k, \bar{L}_k$ independent of g_k as follows

$$\bar{G}_k = \begin{bmatrix} 0 & \cdots & 0 & 0 \\ 1 & \ddots & \vdots & 0 \\ \vdots & \ddots & 0 & \vdots \\ 0 & \cdots & 1 & 0 \end{bmatrix}, \ \bar{L}_k = - \begin{bmatrix} v_{k,0} \\ v_{k-1,1} \\ \vdots \\ v_{k-s+1,s-1} \end{bmatrix}$$

g_k can thus be interpreted as the gain vector of the implicit feedback in the residual generator. This freedom could be used to meet further specifications on the residual dynamics. A simple choice of g_k could be $g_k = 0$, $k = 0, \cdots, \theta - 1$. In this case, all characteristic multipliers are at the origin.

If besides (11)-(14), the following two equations

$$T_{k+1} E_k^d - L_k F_k^d = 0 \quad (18)$$
$$V_k F_k^d = 0 \quad (19)$$

are also satisfied, then the full decoupling can be achieved by periodic observer based residual generator (9) since now the residual dynamics is described by

$$e(k+1) = G_k e(k), \ r(k) = W_k e(k) \quad (20)$$

The following theorem provides a way to solve (11)-(14), (18), (19).

Theorem 2 Assume that

$$v_k = \left[v_{k,0} \ v_{k,1} \ \cdots \ v_{k,s} \right], k = 0, 1, \cdots, \theta - 1$$

satisfy (5). Then G_k, L_k, T_k, W_k, V_k given by (17) solve (11), (12), (18), (19).

Proof: Because $v_k H_{d,k} = 0$, $k = 0, \cdots, \theta - 1$, multiplying v_k with each column of $H_{d,k}$ yields

$$v_{k,s} F_{k+s}^d = 0, \quad (21)$$

$$v_{k,j} F_{k+j}^d + \sum_{l=j+1}^{s} v_{k,l} C_{k+l} \Phi_{k+l,k+j+1} E_{k+j}^d = 0, \quad j = 0, 1, \cdots, s-1 \quad (22)$$

where

$$\Phi_{k_2,k_1} = \begin{cases} I, & \text{if } k_2 = k_1 \\ A_{k_2-1} A_{k_2-2} \cdots A_{k_1}, & \text{if } k_2 > k_1 \end{cases}$$

Equation (19) follows directly from (21) since $V_k = v_{k-s,s}$. Equation (22) can be re-written as

$$v_{k-j,j} F_k^d + \sum_{l=j+1}^{s} v_{k-j,l} C_{k-j+l} \Phi_{k-j+l,k+1} E_k^d = 0$$

by substituting k by $k - j$. Thus the $(j+1)$-th $(j = 0, 1, \cdots, s-1)$ row of $T_{k+1} E_k^d - L_k F_k^d$, i.e.

$$\left[v_{k-j,j+1} \ \cdots \ v_{k-j,s} \right] \times \begin{bmatrix} C_{k+1} \\ C_{k+2} A_{k+1} \\ \vdots \\ C_{k+s-j} A_{k+s-j-1} \cdots A_{k+1} \end{bmatrix} E_k^d + v_{k-j,j} F_k^d + g_{k,j+1} v_{k-s,s} F_k^d,$$

is equal to 0. Equation (18) is proven. ■

Theorem 2 shows that if the periodic parity vector $v_0, v_1, \cdots, v_{\theta-1}$ realises a full decoupling, so does the periodic observer based residual generator (9) with coefficients (17). Thus a periodic full decoupling observer based residual generator can be obtained from a periodic full decoupling parity vector. It is interesting to note that

- the order of the periodic observer is equal to the order of the parity relation,
- matrices L_k, V_k, T_k in periodic observer-based residual generator (9) at *each* time are related to the periodic parity vector *over one period*, which indicates the importance of correct information of period time θ.

In summary, the proposed procedure to design a periodic observer-based full decoupling residual generator is as follows:

- Set the value of s.
- Construct matrices $H_{o,i}, H_{d,i}, H_{f,i}$, $i = 0, \cdots, \theta - 1$ as (3).
- Solve (5) for vectors $v_0, \cdots, v_{\theta-1}$.
- Partition v_i as $v_i = \left[v_{i,0} \ v_{i,1} \ \cdots \ v_{i,s} \right]$ with $v_{i,j} \in \mathbf{R}^{1 \times m}$, $i = 0, \cdots, \theta - 1$, $j = 0, \cdots, s$.
- get matrices G_k, L_k, T_k, W_k, V_k according to (17) with $g_0, \cdots, g_{\theta-1}$ guaranteeing the stability of residual dynamics.
- Compute H_k, Q_k from (13), (14).

On the other side, if a periodic observer-based residual generator (9) with G_k, L_k, W_k of the form

$$G_k = \begin{bmatrix} 0 & \cdots & 0 & g_{k,1} \\ 1 & \ddots & \vdots & g_{k,2} \\ \vdots & \ddots & 0 & \vdots \\ 0 & \cdots & 1 & g_{k,s} \end{bmatrix}, \ L_k = \begin{bmatrix} L_{k,1} \\ L_{k,2} \\ \vdots \\ L_{k,s} \end{bmatrix}$$
$$W_k = \left[0 \ \cdots \ 0 \ -1 \right]$$

is given, then the vector

$$v_k = \left[L_{k,1} + g_{k,1} V_k \ \ L_{k+1,2} + g_{k+1,2} V_{k+1} \ \cdots \ L_{k+s-1,s} + g_{k+s-1,s} V_{k+s-1} \ \ -V_{k+s} \right] \quad (23)$$

with $k = 0, 1, \cdots, \theta - 1$ is a periodic parity vector satisfying $v_k H_{o,k} = 0$. Furthermore, if the given residual generator (9) is full decoupled from the disturbances, then vector (23) satisfies (5) and also realises a full decoupling.

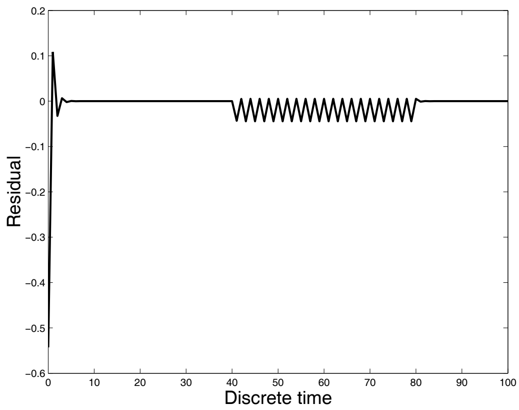

Fig. 3. The residual signal generated by residual generator (24)

Example 2 Consider the same periodic system as in example 1. From the periodic full decoupling parity vector got in (7), a periodic observer-based residual generator can be obtained as

$$
\begin{aligned}
z(k+1) &= G_k z(k) + H_k u(k) + L_k y(k) \\
r(k) &= W_k z(k) + Q_k u(k) + V_k y(k) \qquad (24) \\
G_0 &= -0.2,\ G_1 = -0.3,\ H_0 = 0.0504 \\
H_1 &= -0.1142,\ W_0 = -1,\ W_1 = -1 \\
L_0 &= \left[\, -0.1027\ 0.1064\ 0.0184 \,\right] \\
L_1 &= \left[\, -0.2840\ -0.4300\ 0.0358 \,\right] \\
V_0 &= \left[\, -0.8290\ -0.1421\ 0.2491 \,\right], Q_0 = 0 \\
V_1 &= \left[\, 0.2316\ -0.5703\ 0.7733 \,\right], Q_1 = 0
\end{aligned}
$$

The transformation matrices are

$$
\begin{aligned}
T_0 &= \left[\, -0.1308\ -0.0294\ -0.2191\ -0.1290 \,\right] \\
T_1 &= \left[\, -0.0421\ 0.1942\ -0.2456\ 0.3064 \,\right]
\end{aligned}
$$

It is worth noting that the periodic observer is only of first order. By changing the value of s, the order of the periodic observer could be adjusted. Under the same simulation conditions as in Example 1, the simulation result is shown in Fig. 3. It is seen that the residual signal is not influenced by u, d and the influence of initial estimation error disappears after several time points, which verify the theoretical results.

4. CONCLUSION

In this paper, approaches to design full decoupling residual generators for periodic systems have been presented. The periodic parity space approach needs solving only a group of independent linear algebraic equations. Then, by exploring the relationship between periodic parity vector and periodic observer based residual generators, a periodic observer based full decoupling residual generator is obtained. A study of the FD system design for periodic systems with parametric disturbances and faults is being carried out.

5. REFERENCES

Arcara, P., S. Bittanti and M. Lovera (2000). Periodic control of helicopter rotors for attenuation of vibrations in forward flight. *IEEE Transactions on Control Systems Technology* **8(6)**, 883–894.

Bittanti, S. and P. Colaneri (1999). *Periodic Control.* Chap. Vol. 16, pp. 2–16. Webster J (ed). John Wiley Encyclopaedia on Electrical and Electronic Engineering.

Chen, J. and R. J. Patton (1999). *Robust Model-Based Fault Diagnosis for Dynamic Systems.* Kluwer Academic Publishers. Boston.

Chow, E. Y. and A. S. Willsky (1984). Analytical redundancy and the design of robust failure detection systems. *IEEE Transactions on Automatic Control* **29**, 603–614.

Ding, S. X., E. L. Ding and T. Jeinsch (1998). A numerical approach to optimization of FDI systems. In: *Proc. Of the 37th IEEE CDC.* Tampa USA. pp. 1137–1142.

Fadali, M. S. and S. Gummuluri (2001). Robust observer-based fault detection for periodic systems. In: *Proc. 2001 ACC.* pp. 464–469.

Frank, P. M., S. X. Ding and B. Koeppen-Seliger (2000). Current developments in the theory of FDI. In: *Proc. Of IFAC Symposium SAFE-PROCESS'2000.* pp. 16–27.

Gertler, J.J. (1998). *Fault Detection and Diagnosis in Engineering Systems.* Marcel Dekker.

Lampe, B. P. and E. N. Rossenwasser (2004). Closed formulae for the L2-norm of linear continuous-time periodic systems. In: *Proc. PSYCO.* Yokohama, Japan. pp. 231–236.

Lovera, M., E. De Marchi and S. Bittanti (2002). Periodic attitude control techniques for small satellites with magnetic actuators. *IEEE Transactions on Control Systems Technology* **10(1)**, 90–95.

Patton, R. J., P. M. Frank and R. N. Clark (Eds.) (2000). *Issues of Fault Diagnosis for Dynamic Systems.* Springer.

Rehbinder, H. and M. Sanfridson (2004). Scheduling of a limited communication channel for optimal control. *Automatica* **40**, 491–500.

Souza, C. E. De and A. Trofino (2000). An LMI approach to stabilization of linear discrete-time periodic systems. *International Journal of Control* **73**, 696–703.

Stol, K.A. (2003). Time-varying control of wind turbines. In: *Proc. 2003 ACC.* p. 3796.

Varga, A. (2004). Design of fault detection filters for periodic systems. In: *Proc. 43rd IEEE CDC.* Atlantis, Bahamas.

Xie, L.H. and C.E. De Souza (1993). H_∞ state estimation for linear periodic systems. *IEEE Transactions on Automatic Control* **38(11)**, 1704–1707.