OUTPUT TRACKING WITH CONSTRAINED INPUTS VIA ADAPTIVE RECURRENT NEURAL CONTROL

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Abstract: This paper extends previous results to the output tracking problem of nonlinear systems with unmodelled dynamics and constrained inputs. A recurrent high order neural network is used to identify the unknown system dynamics and a learning law is obtained using the Lyapunov methodology. A stabilizing control law for the output tracking error dynamics is developed using the Lyapunov methodology and the Sontag control law for nonlinear systems with constrained inputs.

Keywords: Recurrent neural networks, output trajectory tracking, adaptive control, constrained inputs, Lyapunov function, inverse optimal control.

1. INTRODUCTION

In many control applications, the process presents highly nonlinear behavior, uncertainties, unknown disturbances and bounded inputs. All these phenomena are required to be considered for control analysis and synthesis. The problem of designing robust controllers for nonlinear systems with uncertainties, which guarantee stability and trajectory tracking, has received an increasing attention lately. The presence of constrained inputs limits the ability compensate the effects of unmodeled dynamics and external disturbances. These effects are reflected on the loss of stability, undesired oscillations and other adverse effects. There are several results on linear control systems with input constrains, (Hu and Lin, 2001). For nonlinear systems, control with constrained inputs is restricted by requiring to know the system model. Some algorithms allow the presence of uncertainties satisfying the matching condition (El-Farra and Christofides, 2001). In (El-Farra and Christofides, 2001), a control law, based on the Sontag formula (Lin and Sontag, 1991) with constrained inputs, is developed and applied to a chemical reactor. To relax the restriction of requiring knowledge of the system model, identification via recurrent neural networks arises as a potential solution (Hokimyan et. al., 2001), (Sanchez, et. al.,2003).

Since the seminal paper (Narendra and Parthasarathy, 1990), there has been a continuously increasing interest in applying neural networks to identification and control of nonlinear systems. Lately, the use of recurrent neural networks is being developed, which allows more efficient modeling of the underlying dynamical systems. Recent books, as (Rovithakis and Christodoulou, 2000), have reviewed the application of dynamic neural networks for nonlinear system identification and control. In (Rovithakis and Christodoulou, 2000), adaptive identification and control by means of on-line learning is analyzed; the stability of the closed-loop system is established based on the Lyapunov

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function method. In (Sanchez and Ricalde, 2003), the problem of trajectory tracking, for nonlinear systems in presence of constrained inputs and uncertainties with application in chaos control and synchronization, is considered.

In this paper, we extend our previous results (Sanchez and Ricalde, 2003), to nonlinear systems with less inputs than states. The output trajectory tracking problem with constrained inputs is solved with adaptive control scheme composed by a recurrent neural identifier, which is used to build an on-line model for the unknown plant, and a control law to force the unknown plant to track the output reference trajectory. An update law for the recurrent high order neural network is proposed via the Lyapunov methodology. A robust learning law to avoid the parameter drift in the presence of modelling error is also proposed. The control law is synthesized using the Lyapunov methodology and a modification of the Sontag control law for stabilizing systems with constrained inputs and uncertain terms. This control law explicitly depends on the input constraints. Boundedness of the tracking error is proven and an estimation of the closed loop stability region is given in order to determine the available bounds of the uncertainties and the desired tracking error. The proposed scheme is validated for the output trajectory tracking of a nonlinear oscillator.

2. RECURRENT HIGHER-ORDER NEURAL NETWORKS

Artificial Recurrent Neural Networks are mostly based on the Hopfield model (Hopfield, 1984). These networks are considered as good candidates for nonlinear systems applications which deal with uncertainties and are attractive due to their easy implementation, robustness and capacity to adjust their parameters on line.

In (Kosmatopoulos, et. al.), recurrent higher-order neural networks (RHONN) are defined as

$$\dot{x}_i = -a_i x_i + \sum_{k=1}^{L} w_{ik} \prod_{j \in I_k} y_j^{d_j(k)}, \qquad i = 1, ..., n \tag{1}$$

where x_i is the ith neuron state, L is the number of higher-order connections, $\{I_1, I_2, ..., I_L\}$ is a collection of non-ordered subsets of $\{1, 2, ..., m+n\}$, $a_i > 0$, w_{ik} are the adjustable weights of the neural network, $d_j(k)$ are nonnegative integers, and y is a vector defined by $y = [y_1, .., y_n, y_{n+1}, ..., y_{n+m}]^\top = [S(x_1), ..., S(x_n), S(u_1), ..., S(u_m)]^\top$,with $u = [u_1, u_2, ..., u_m]^\top$ being the input to the neural network, and $S(\cdot)$ a smooth sigmoid function formulated by $S(x) = \frac{1}{1+\exp(-\beta x)} + \varepsilon$. For the sigmoid function, β is a positive constant and ε is a small positive real number. Hence, $S(x) \in [\varepsilon, \varepsilon + 1]$.

As can be seen, (1) allows the inclusion of higher-order terms.

By defining a vector $z(x, u) = [z_1(x, u), ..., z_L(x, u)]^\top = \left[\prod_{j \in I_1} y_j^{d_j(1)}, ..., \prod_{j \in I_L} y_j^{d_j(L)}\right]^\top$, (1) can be rewritten as

$$\dot{x}_i = -a_i x_i + w_i^\top z_i(x, u), \qquad i = 1, ..., n \tag{2}$$

where $w_i = [w_{i,1} ... w_{i,L}]^\top$.

In this paper, we consider the following RHONN,

$$\dot{x}_i = -a_i x_i + w_i^\top z_i(x) + w_{gi} z_{gi}(x) u_i, \quad i = 1, ..., n \tag{3}$$

Reformulating (3) in matrix form yields

$$\dot{x} = Ax + Wz(x) + W_g z_g(x) u \tag{4}$$

where $x \in \Re^n$, $W \in \Re^{n \times L}, W_g \in \Re^{n \times L}$, $z(x) \in \Re^L, z_g(x) \in \Re^{L \times p}$, $u \in \Re^p$, and $A = -\lambda I, \lambda > 0$.

3. CONTROL PROBLEM FORMULATION.

We consider the nonlinear system

$$\dot{x}_p = f_p(x_p) + g_p(x_p) sat(u) \tag{5}$$

with output $y = h(x_p) \in \Re$, where $x_p \in \Re^n$, and $sat(u) \in \Re$ is the standard saturation nonlinearity. The vector functions f_p, g_p are assumed to be unknown, but with full state measurement. The system (5) is modeled by a RHONN in order to estimate the system dynamical model. The control goal is to force (5) to track the reference system given by

$$\dot{x}_r = f_r(x_r, u_r), \qquad x_r \in \Re \tag{6}$$

4. PLANT IDENTIFICATION

Consider the unknown nonlinear plant

$$\dot{x}_p = F_p(x_p, u) \triangleq f_p(x_p) + g_p(x_p) sat(u) \tag{7}$$

where x_p, $f_p \in \Re^n$, $g_p \in \Re^n$, $u \in \Re$.

Taking into account that f_p is unknown, one can model (7) by a recurrent neural network as in (4). Hence we propose the following recurrent neural model for the unknown plant

$$\dot{\chi} = -\lambda \chi + Wz(x_p) + w_{per} + W_g z(x_p) sat(u) \tag{8}$$

where $\lambda > 0, \chi \in \Re^n, u \in \Re$; $W \in \Re^{n \times p}, W_g \in \Re^{n \times 1}$, $z(x) \in \Re$, x_p is the state to identify, χ is the neural network state, and u is the applied input to the system. W, W_g are the neural network adapted weights, $z(x_p), z_g(x_p)$ are sigmoid functions and the term $w_{per} \in \Re^n$ represents the modelling error which is assumed to be bounded.

Assumption 1. For every $w_{ij} \in \mathsf{W}$, the system (8) is bounded for every bounded state x_p.

To derive an adaptation law, which minimizes the identification error, we consider the case of no modelling error. We assume that there exist unknown but constant weights W^* such that the plant is completely described by the neural network

$$\dot{x}_p = -\lambda x_p + W^* z(x_p) + W_g^* z_g(x) sat(u) \qquad (9)$$

where all the elements are as described earlier.

Now, we proceed to analyze the error between the identifier and the plant

$$e_i = \chi - x_p \qquad (10)$$

The identification error dynamics is given by

$$\begin{aligned} \dot{e}_i &= \dot{\chi} - \dot{x}_p \\ \dot{e}_i &= -\lambda e + \tilde{W} z(x_p) + \tilde{W}_g z_g(x_p) u \end{aligned} \qquad (11)$$

where $\tilde{W} = W - W^*$, $\tilde{W}_g = W_g - W_g^*$.

To perform the stability analysis of (11), we consider the Lyapunov function candidate

$$V = \frac{1}{2}\|e_i\|^2 + \frac{1}{2\gamma} tr\left\{\tilde{W}^\top \tilde{W}\right\} + \frac{1}{2\gamma_g} tr\left\{\tilde{W}_g^\top \tilde{W}_g\right\} \qquad (12)$$

where $\gamma, \gamma_g > 0$. Differentiating (12) along the solutions of (9) , we obtain

$$\begin{aligned} \dot{V} = -\lambda\|e_i\|^2 + e_i^T \tilde{W} z(x_p) + e_i^T \tilde{W}_g z_g(x_p) u \\ +\frac{1}{\gamma} tr\left\{\dot{\tilde{W}}^\top \tilde{W}\right\} + \frac{1}{\gamma_g} tr\left\{\dot{\tilde{W}}_g^\top \tilde{W}_g\right\} \end{aligned} \qquad (13)$$

The asymptotic stability of the identification error is achieved if we select the weight adaptation laws

$$\begin{aligned} tr\left\{\dot{\tilde{W}}^\top \tilde{W}\right\} &= -\gamma e^\top \tilde{W} z(x_p) \\ tr\left\{\dot{\tilde{W}}_g^\top \tilde{W}_g\right\} &= -\gamma_g^\top e \tilde{W}_g z(x_p) u \end{aligned} \qquad (14)$$

Substituting the adaptation law in (13) gives

$$\dot{V} = -\lambda\|e_i\|^2 \leq 0 \qquad (15)$$

which is semidefinite negative. We now apply the Barbalat's lemma (Khalil, 1996). Since $V(t) > 0, \forall e_i, \tilde{W} \neq 0$ and $\dot{V}(t) \leq 0$, $V(t)$ is bounded. Hence, $\|e_i\|$ is bounded on $[0, T]$, the maximal interval of existence of the solution for any given initial state. $V(t)$ is nonincreasing and bounded from below by zero, and converges as $t \to \infty$. From (15) e_i and $\tilde{W}$ are bounded on $[0, T]$, the maximal interval of existence of the solution for any given initial state. This implies that $T = \infty$. We conclude $e_i \to 0$ as $t \to \infty$. Then $\lim_{t\to\infty} W \to W_\infty$ and $\lim_{t\to\infty} \tilde{W}_g \to \tilde{W}_\infty$, where W_∞ and $\tilde{W}_\infty$ are constant values.

The assumption of no modelling error is seldom satisfied. Hence, the adjusted weight parameters could drift to infinity. To avoid the parameter drift, the following robust learning law for the neural network weights is proposed as in (Rovithakis and Christodoulou, 2000):

$$\dot{\hat{W}} = \begin{cases} -\gamma e_i z(x_{p_j}) & \text{if } |w_i| < w_m \\ -\gamma e_i z(x_{p_j}) - \sigma\gamma w_i & \text{if } |w_i| \geq w_m \end{cases} \qquad (16)$$

$$\dot{\hat{W}}_g = \begin{cases} -\gamma_g e_i z(x_{p_j}) u_i & \text{if } |w_i| < w_{gm} \\ -\gamma_g e_i z(x_{p_j}) u_i - \sigma\gamma_g w_i & \text{if } |w_i| \geq w_{gm} \end{cases}$$

where σ is a positive constant and w_m, w_{gm} are the upper bounds for the neural network weights. The robust learning law does not affect the stability of the identification error. For a detailed demonstration, see (Rovithakis and Christodoulou, 2000).

5. TRAJECTORY TRACKING ANALYSIS

Consider the nonlinear system with constrained input (7), which we model by the neural network

$$\begin{aligned} \dot{\chi} &= -\lambda\chi + W z(x_p) + w_{per} + W_g z_g(x_p) sat(u) \\ y &= h(\chi) \end{aligned} \qquad (17)$$

where we assume that the modeling error is bounded. In the following, for simplicity, we will use u instead of $sat(u)$. We will design a robust controller that satisfies $|u| \leq u_{\max}$ and guarantees boundedness of the tracking error between the plant and the reference signal generated by

$$\dot{x}_{ref} = f_{ref}(x_{ref}), \qquad x_{ref} \in \Re \qquad (18)$$

The system (8) is converted in a partially linear system by the change of coordinates

$$T(\chi) = \begin{bmatrix} \xi \\ \varsigma \end{bmatrix} = \begin{bmatrix} \xi_1 & \xi_2 & \dots & \xi_r & \varsigma_1 & \dots & \varsigma_{n-r} \end{bmatrix}^T$$

$$= \begin{bmatrix} h(\chi) & L_f h(\chi) & \dots & L_f^{r-1} h(\chi) & \psi_1(\chi) & \dots & \psi_{n-r}(\chi) \end{bmatrix}^T$$

where r is the relative degree of (7) , such that the system (17) is converted to

$$\begin{aligned} \dot{\xi}_1 &= \xi_2 \\ &\vdots \\ \dot{\xi}_{r-1} &= \xi_2 \\ \dot{\xi}_r &= L_f^r h\left(\psi^{-1}(\xi,\varsigma)\right) + L_w L_f^{r-1}\left(T^{-1}(\xi,\varsigma)\right) w_{per} \\ &\quad + L_g L_f^{r-1} h\left(\psi^{-1}(\xi,\varsigma)\right) u \\ \dot{\varsigma}_1 &= \Psi_1(\xi,\varsigma) \\ &\vdots \\ \dot{\varsigma}_{n-r} &= \Psi_{n-r}(\xi,\varsigma) \\ y &= \xi_1 \end{aligned} \qquad (19)$$

Let us consider the tracking error defined as

$$e_t := \xi_1 - x_{ref} \tag{20}$$

The time derivative of the tracking error is

$$\begin{aligned}\dot{e}_t &= \dot{\xi}_1 - \dot{x}_{ref} = -\lambda\chi + Wz(x_p) + w_{per} \\ &+ W_g z(x_p)u - f_r(x_r, u_r)\end{aligned} \tag{21}$$

Now let us define $e_t^T = [e_{t1}, e_{t2}, ..., e_{tr}]$

$$\begin{aligned}e_{t1} &= \xi_1 - x_{ref} \\ e_{t2} &= \dot{\xi}_1 - \dot{x}_{ref} \\ &\vdots \\ e_{tr} &= \xi_1^{r-1} - x_{ref}^{r-1}\end{aligned} \tag{22}$$

From (19) and (22) , we obtain the tracking error dynamic system

$$\begin{aligned}\dot{e}_{t1} &= e_{t2} \\ \dot{e}_{t2} &= e_{t3} \\ &\vdots \\ \dot{e}_{tr} &= L_f^r h\left(\psi^{-1}(\xi, \varsigma)\right) + L_w L_f^{r-1}\left(T^{-1}(\xi, \varsigma)\right) w_{per} \\ &- x_{ref}^r + L_g L_f^{r-1} h\left(\psi^{-1}(\xi, \varsigma)\right) u\end{aligned} \tag{23}$$

$$\begin{aligned}\dot{e}_t &= f_e(e_t) + w(e_t) w_{per} + g_e(e_t) u \\ f_e(e_t) &= L_f^r h\left(\psi^{-1}(\xi, \varsigma)\right) - x_{ref}^r \\ w(e_t) &= L_w L_f^{r-1}\left(T^{-1}(\xi, \varsigma)\right) \\ g_e(e_t) &= L_g L_f^{r-1} h\left(\psi^{-1}(\xi, \varsigma)\right)\end{aligned} \tag{24}$$

The tracking problem can be analyzed as a stabilization problem for the error dynamics (24).

5.1 Tracking Error Stabilization

To perform the stability analysis for the system, the following Lyapunov function is formulated:

$$\begin{aligned}V &= \frac{1}{2}\|e_i\|^2 + \frac{1}{2}e_t^T P e_t + \frac{1}{2\gamma} tr\left\{\tilde{W}^\top \tilde{W}\right\} \\ &+ \frac{1}{2\gamma_g} tr\left\{\tilde{W}_g^\top \tilde{W}_g\right\}\end{aligned} \tag{25}$$

where P is a positive definite matrix which satisfies the Ricatti inequality

$$A^T P + PA - Pbb^T P < 0$$

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \in R^{r \times r}, b = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} \in R^r$$

Its time derivative, along the trajectories of (24), is

$$\begin{aligned}\dot{V} &= -\lambda\|e_i\|^2 + e_i^T \tilde{W} z(x_p) + e_i^T \tilde{W}_g z_g(x_p) u \\ &+ L_{f_e} V + L_{w_e} V w_{per} + L_{g_e} V u \\ &+ \frac{1}{\gamma} tr\left\{\dot{\tilde{W}}^\top \tilde{W}\right\} + \frac{1}{\gamma_g} tr\left\{\dot{\tilde{W}}_g^\top \tilde{W}_g\right\}\end{aligned} \tag{26}$$

Replacing the learning law (14) in (26) we obtain

$$\dot{V} = -\lambda\|e_i\|^2 + L_{f_e} V + L_{w_e} V w_{per} + L_{g_e} V u \tag{27}$$

Furthermore, we assume that the uncertain term $(L_w V) w_{per}$ is bounded by above as

$$(L_w V) w_{per} \leq |e_t| w_b$$

In order to stabilize the tracking error dynamics, let us consider the following modification of the Sontag control law (El-Farra and Christofides, 2001),(Sanchez and Ricalde, 2003),

$$\begin{aligned}u &= -\tfrac{1}{2} R^{-1}(e_t, \hat{W}) L_g V \\ &= -\frac{L_{f_e} V + \eta \sum_{k=1}^{q} w_{bk} |L_{wk_e} L_f h(x)| \left(\frac{\left|2b^T P e_t\right|^2}{\left|2b^T P e_t\right| + \phi}\right)}{(L_g V)^2 \left[1 + \sqrt{1 + (u_{\max} L_g V)^2}\right]} L_g V \\ &- \frac{\sqrt{(L_{f_e} V + \eta w_b |L_{w_e} L_f h(x)|)^2 + (u_{\max} L_g V)^4}}{(L_g V)^2 \left[1 + \sqrt{1 + (u_{\max} L_g V)^2}\right]} L_g V\end{aligned} \tag{28}$$

where η, ϕ are adjustable parameters.

Replacing the control law (28) in (27) and taking into account the bound for $(L_w V) w_{per}$, we obtain

$$\begin{aligned}\dot{V} &\leq -\lambda\|e_i\|^2 - \frac{\sqrt{\left(L_{f_t} V + \eta w_b |L_{w_e} V|\right)^2 + (u_{\max} L_g V)^4}}{\left[1 + \sqrt{1 + (u_{\max} L_g V)^2}\right]} \\ &+ \frac{w_b \left|L_{w_e} L_f^{r-1} h(x)\right| \left|2b^T P e_t\right| \left(\phi - \left(\phi - (\eta - 1)\left|2b^T P e_t\right|\right)\right)}{\left|2b^T P e_t\right| + \phi \left[1 + \sqrt{1 + (u_{\max} L_g V)^2}\right]} \\ &+ \frac{\left(L_{f_t} V + w_b |L_{w_e} V|\right) \sqrt{1 + (u_{\max} L_g V)^2}}{\left[1 + \sqrt{1 + (u_{\max} L_g V)^2}\right]}\end{aligned} \tag{29}$$

It is easy to verify that when $\left|2b^T P e_t\right| > \frac{\phi}{\chi - 1}$, the second term is strictly negative; hence, we proceed to study the case when $|e| \leq \frac{\phi}{\chi - 1}$. First, we consider that the modelling error term is a disturbance which satisfies a growth bound of the form

$$\left|L_{w_e} L_f^{r-1} h(x)\right| \leq \delta \left|2b^T P e_t\right| + \mu$$

Then, we can obtain the following bound using an analog procedure as in (El-Farra and Christofides, 2001),

$$\begin{aligned}&\frac{w_b \left|L_{w_e} L_f^{r-1} h(x)\right| \left|2b^T P e_t\right| \left(\phi - \left(\phi - (\eta - 1)\left|2b^T P e_t\right|\right)\right)}{\left|2b^T P e_t\right| + \phi \left[1 + \sqrt{1 + (u_{\max} L_g V)^2}\right]} \\ &\leq \beta(\phi) \qquad \forall \qquad |e_t| \leq \frac{\phi}{\eta - 1}\end{aligned}$$

Substituting $\beta(\phi)$ in (29) we obtain

$$\begin{aligned}\dot{V} \leq & -\lambda \|e_i\|^2 + \beta(\phi) \\ & + \frac{(L_{f_t}V + w_b |L_{w_e}V|)\sqrt{1+(u_{\max}L_gV)^2}}{\left[1+\sqrt{1+(u_{\max}L_gV)^2}\right]} \\ & - \frac{\sqrt{(L_{f_t}V + \eta w_b |L_{w_e}V|)^2 + (u_{\max}L_gV)^4}}{\left[1+\sqrt{1+(u_{\max}L_gV)^2}\right]}\end{aligned} \tag{30}$$

Now, to determine the sign of the last two terms, we consider two cases,

Case 1. $L_fV + \eta w_b |L_{w_e}V| \leq 0$, substituting this inequality in (30) yields

$$\begin{aligned}\dot{V} \leq & -\lambda \left(\|e_i\|^2 + \|e_t\|^2\right) + \beta(\phi) \\ & - \frac{\sqrt{(L_{f_t}V + \eta w_b |L_{w_e}V|)^2 + (u_{\max}L_gV)^4}}{\left[1+\sqrt{1+(u_{\max}L_gV)^2}\right]}\end{aligned} \tag{31}$$

Case 2. $0 < L_fV + \eta w_b |e_t| \leq u_{\max} |L_gV|$. For this case, we consider the inequality

$$(L_fV + \eta w_b |L_{w_e}V|)^2 \leq (u_{\max}L_gV)^2$$

Replacing this bound in (30), we obtain

$$\begin{aligned}\dot{V} \leq & -\lambda \left(\|e_i\|^2 + \|e_t\|^2\right) + \beta(\phi) \\ & + \frac{(1-\chi) w_b |L_{w_e}V| \sqrt{1+(u_{\max}L_gV)^2}}{\left[1+\sqrt{1+(u_{\max}L_gV)^2}\right]}\end{aligned} \tag{32}$$

where we select $\eta > 1$ such that the last term on the right hand is strictly negative.

From (31) and (32), we deduce that exists a class K function α such that

$$\dot{V} \leq -\lambda \left(\|e_i\|^2 + \|e_t\|^2\right) - \alpha(|e_t|) + \beta(\phi) \tag{33}$$

Then, by an appropriate selection of ϕ we can make $\beta(\phi)$ small enough such that $\beta(\phi) \leq \frac{1}{2}\alpha(|e_t|)$ in order to obtain

$$\dot{V} \leq -\lambda \left(\|e_i\|^2 + \|e_t\|^2\right) - \frac{1}{2}\alpha(|e_i|, |e_t|) \tag{34}$$

Therefore, whenever the inequality $L_fV + \eta w_b |e_t| \leq u_{\max} |L_gV|$ holds, the trajectories of (22) will approach an ultimate bound if

$$\lambda \left(\|e_i\|^2 + \|e_t\|^2\right) + \alpha(|e_i|, |e_t|) > \beta(\phi)$$

Then, the designer can choose the parameters ϕ and η in order to obtain a suitable tracking error.

Since $\dot{V}$ is a semidefinite negative function, by the Barbalat's lemma (Khalil, 1996) we have

$$-\lambda \left(\|e_i\|^2 + \|e_t\|^2\right) - \alpha(|e_t|) + \beta(\phi) \to 0 \quad \text{as} \quad t \to \infty \tag{35}$$

The left side of (35) is a class K function, then

$$e_i \to 0, \; e_t \to 0, \; \beta(\phi) \to 0$$

When $e_i \to 0$, from the weight adaptation law 14 we have $\dot{w}_{ij} \to 0$. Then $\lim_{t\to\infty} \hat{W} \to \hat{W}_\infty$, $\lim_{t\to\infty} \tilde{W} \to \tilde{W}_\infty$ where $\hat{W}_\infty$ and $\hat{W}_\infty$ are constant values.

Remark 1. Since the control law explicitly depends of $u_{\max}$, inequality $L_fV + w_b |e_t| \leq u_{\max} |L_gV|$, it can be used to reduce the available maximum input as e_t decreases in order to smooth the applied control. Then, $u_{\max}$ can be expressed as

$$u_{\max} = \begin{cases} u_{\max} |e_t| & \text{if } 0 \leq |e_t| < e_t^* \\ u_{\max} & \text{if } |e_t| \geq e_t^* \end{cases} \tag{36}$$

where e_t^* is an admissible tracking error value which can be obtained via simulations.

Theorem. Consider the unknown nonlinear system with constrained input (7), which is modeled by the recurrent high order neural network (8), the on-line learning law (14) and the control law (28) with parameters defined in (27), then for $L_{f_t}V + w_b |L_{w_e}V| \leq u_{\max} |L_gV|$, the control law (28) guarantees an ultimate bound of the tracking error.

6. APPLICATION EXAMPLE

In order to demonstrate the applicability of the proposed adaptive control scheme with constrained inputs, the following example is tested. We consider a sinusoid reference tracking. The unknown system to control is the Van Der Pol one generated by

$$\begin{aligned}\dot{x}_{p1} &= x_{p2} \\ \dot{x}_{p2} &= \left(0.5 - x_{p1}^2\right) x_{p2} - x_{p1} + 0.5\cos(1.1t) + sat(u)\end{aligned} \tag{37}$$

with $y_p = x_{p_1}$ and $x_p(0) = \begin{pmatrix} 1.5 & 0 \end{pmatrix}$. We want this system output to track the following reference signal $x_r = 1.5 + \sin(t/4)$ for a maximum input $u_{\max} = 20$. For simulations, the following recurrent neural network is used:

$$\begin{aligned}\dot{\chi} &= -\lambda\chi + Wz(x_p) + bsat(u) \\ y(\chi) &= \chi_1\end{aligned} \tag{38}$$

with $\lambda = 15$, $b = \begin{bmatrix} 0 & 1 \end{bmatrix}^T$ and the high order sigmoid vector defined as $z(x_p) =$

$$\begin{aligned}&[\, \tanh(x_{p1}) \;\; \tanh(x_{p2}) \;\; \tanh(x_{p1})\tanh(x_{p2}) \\ &\tanh^2(x_{p1}) \;\; \tanh^2(x_{p2}) \;\; \tanh^2(x_{p1})\tanh^2(x_{p2}) \\ &\tanh^3(x_{p1}) \;\; \tanh^3(x_{p2}) \,]\end{aligned} \tag{39}$$

To avoid parameter drift, we use the robust adaptation law (16) with $\gamma = 250, w_{\max} = 100, \sigma = 50$. The coordinate change is set as

$$\left(\xi_1 \ \xi_2\right) = \left(\chi_1 \ -\lambda\chi_1 + \sum_{i=1}^{8} w_{1i} z_i \left(x_p\right)\right)$$

For the Lyapunov function we select $c = 0.9$.For the control law, we select $w_b = 0.2$, $\eta = 2$ and $\phi = 0.02$. Fig. 1 displays the time evolution for the output of the plant and the neural network. The control law is applied at $t = 20$ sec. We modify the input bounds according to (36) with $e_t^* = 0.15$. As can be seen, the control law achieves the desired tracking performance even in presence of uncertainties due to the modelling error and input constraints. The applied input is displayed in Fig. 2.

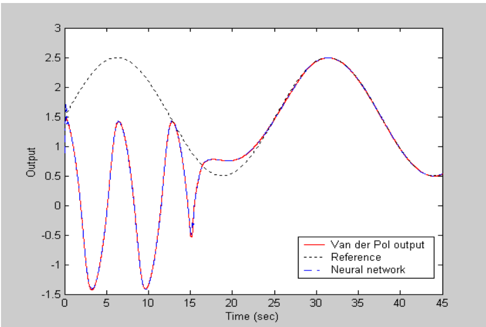

Fig. 1. State evolution of the system output, reference signal and neural network output.

7. CONCLUSIONS

An adaptive control structure based on a recurrent neural network for output tracking of unknown nonlinear systems with constrained inputs was developed. This structure is composed of a neural network identifier, which builds an online model of the unknown plant, which is the base to compute the time derivatives of the output, and a control law for trajectory tracking with constrained inputs is developed using the Sontag law and the Lyapunov methodology. Stability of the identification and tracking error and optimality

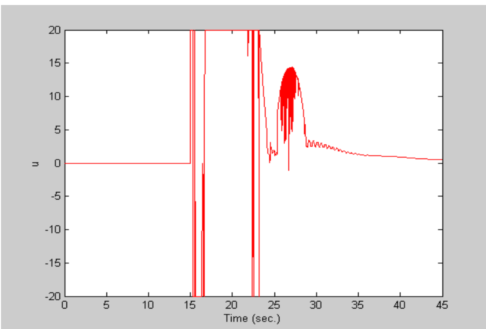

Fig. 2. Applied input

analysis is developed via Lyapunov methodology. The applicability of the proposed structure was tested via simulations, by the output tracking of the VanderPol forced oscillator.

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