A GRAPH OF STATE CLASSES FOR FUZZY TIME PETRI NETS

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Abstract: The goal of this paper is to extend the state graph class concept of a time Petri net to the fuzzy state graph class concept of a Fuzzy Time Petri net (FTPN). The fuzzy state class graph is defined by focusing on: i) the fuzzy time domains associated to the state classes; ii) the update of the fuzzy time intervals after the firing of a transition; iii) the computation of Possibility and Necessity measures of transition firing.

Keywords: Time Petri net, possibility theory, fuzzy modeling, state space

1. INTRODUCTION

The technological evolution of these last years, mainly in computer sciences and communication networks, induced the development of distributed systems in many areas of the society, and particularly in the industrial area. These systems are complex ones and their design requires formal description models in order to check properties and to evaluate performances. Important characteristics of the behavior of distributed systems are parallelism, choice, transmission, synchronization, resource sharing, etc.

Petri nets are a good model for representing the main characteristics of distributed system and for making a qualitative analysis of these aspects. However, in many systems, some informations are ill-known (conditions, time duration, etc.).

In relation to the temporal aspect, some extensions as Time Petri Net (TPN) (Merlin and Farber 1976) and Fuzzy Time Petri Net (FTPN) (Cardoso, 1998) have been defined. A TPN associates to each transition a firing interval, and in a FTPN this interval is a fuzzy one. The dynamic behavior of a TPN can be represented by a state class graph (Berthomieu and Menasche, 1983) which preserves the Linear Time Logic properties. The goal of this paper is to extend the state graph class concept of a TPN to the fuzzy state graph class of a FTPN.

The paper is organized as follows: section 2 introduces the basic models of our work (Petri nets and possibility theory). In section 3 we define the fuzzy state class graph and we present the results obtained in the analysis of a real application (a data transfer protocol) in section 4.

2. TEMPORAL ILL-KNOWN INFORMATION AND PETRI NETS

2.1. *Temporal ill-known information*

Possibility theory can be used for the representation and the management of imprecision and uncertainty in temporal knowledge. It is the simplest theory to manage incomplete information and it contrasts with the usual probabilistic encoding of knowledge, which must be numerical and relies on an additivity assumption. Moreover, classical probability theory is unable to model ignorance (and partial ignorance) in a natural way (Dubois and Prade, 1989).

The available knowledge about a date **a** is represented by means of a possibility distribution $\pi_a(\tau)$. It is a fuzzy number represented by a trapezoid A = [a_1, a_2, a_3, a_4]. In the following, these two notations will be indistinctly used to represent a date **a**: $\pi_a(\tau)$ or A (the fuzzy set that delimits **a**). The greater $\pi_a(\tau)$, the greater the possibility that **a** is equal to τ. In the time interval [a_1, a_4] (called *support*), $0 < \pi_a \le 1$. In the time interval [a_2, a_3] (called *core*), $\pi_a = 1$. Outside the interval [a_1, a_4], $\pi_a(\tau) = 0$. We consider *normalized* possibility distribution: $\exists\ \tau,\ \pi_a(\tau)=1$. There are three particular cases: a) triangular form, $a_2 = a_3$; b) imprecise case, $a_1 = a_2$ and $a_3 = a_4$; c) precise case, $a_1 = a_2 = a_3 = a_4$.

Given a possibility distribution $\pi_a(\tau)$,(Dubois and Prade, 1988 define the measures of possibility Π(B) and necessity N(B) that the date **a** belongs to a crisp set B of X:

$$\Pi(B) = \sup_{x \in B} \pi_a(x) \quad \text{and} \quad N(B) = \inf_{x \notin B}(1-\pi_a(x)) = 1 - \Pi(\bar{B})$$

where $\bar{B}$ is the complement of B. If Π(B)=0 it is impossible that **a** belongs to B; if Π(B)=1, it is possible, but it depends on the value of N(B). If N(B)=1, it is certain. These measures are related by $\Pi(B) = 1 - N(\bar{B})$. So, if N(B) > 0 it implies that Π(B) =1 and if Π(B) < 1 it implies that N(B) =0.

Given two fuzzy dates **a** and **b** the fuzzy subtraction A ⊖ B is given by (Dubois and Prade, 1989):

$$[a_1\ a_2\ a_3\ a_4] \ominus [b_1\ b_2\ b_3\ b_4] = [a_1\text{-}b_4\ \ a_2\text{-}b_3\ \ a_3\text{-}b_2\ \ a_4\text{-}b_1] \quad (1)$$

The time instants possibly/necessarily before or after a date **a** (represented by π_a) are given by the fuzzy sets in Table 1 and shown in Fig. 1 and Fig. 2:

Table 1 Fuzzy sets Possib/Nec before/after a date a

	Possibly	Necessarily
before **a**	(-∞ A]	(-∞ A [
after **a**	[A +∞)	] A +∞)

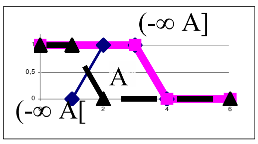

Fig. 1. Possib/Nec before **a**

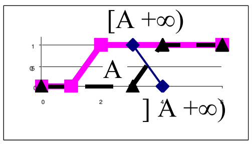

Fig. 2. Possib/Nec after **a**

Given two possibility distributions associated with dates **a** and **b**, $\pi_a(\tau)$ and $\pi_b(\tau)$, (Dubois and Prade, 1989) defines the possibility Π(**a** ≤ **b**) and the necessity N(**a** ≤ **b**) that date **a** be before **b** date :

$$\Pi(a \le b) = \sup_{x \le y}(\min(\pi_a(x), \pi_b(y)) \quad (2)$$

$$N(a \le b) = 1 - \sup_{x > y}(\min(\pi_a(x), \pi_b(y)) \quad (3)$$

We can also write Π(**a** ≤ **b**) = max ([A +∞) ∩ (-∞ B]) and N(**a** ≤ **b**) = max (]A +∞) ∩ (-∞ B[).

2.2. Time Petri Nets

A Time Petri Net (Merlin and Farber 1976) is a triple <N, Mo, I^0> where: PN =<P, T, Pre, Post> is a Petri net, Mo is the initial marking, $I^0 : T \rightarrow (Q+ \cup 0) * (Q+ \cup \infty)$. The function I^0, the static temporal interval, associates to each transition t_i a time interval [α_i, β_i] that represents the set of possible firing dates since the enabling time (α_i is the lower bound, β_i is the upper bound).

So, beside the concept of *enabled* (by the marking), there is the concept of *fireable*. An enabled transition is fireable at least at the lower bound α_i and at last at the upper bound β_i. Note that the date where the transition is fired is not specified. The case where $\alpha_i = 0$ and $\beta_i = \infty$ corresponds to a classical Petri net.

Transition firing:
The *firing* of a transition t_i in a TPN from a state S = (M, I) at a date θ_i, leads to a state S' = (M', I') where I' is such that (I is the dynamic time interval):

1. if t_j is newly enabled (enabled by M', but not by M), $I'(t_j) = I^0(t_j)$;
2. for the transitions that are not *enabled* I'=Ø ;
3. if t_j *remains* enabled, $I'(t_j)$=[max (0, $\alpha_j - \theta_i$), $\beta_j - \theta_i$].

As a transition t_i of a TPN is *fireable* at an instant θ_i (during its firing interval). After t_i firing, there is an infinity of states with the same marking but with different time firing domain. The concept of *state class* and the definition of a State Class Graph (SCG) were introduced by Berthomieu and Menasche, 1983.

2.3. State Class Graph

A *state class* is a set of states with the same marking and whose time firing domain is the union of the time firing domain of its states. The TPN adopts the *strong semantics* (if several transitions are fireable, one of them must be fired before the upper bound of the other ones). The parallelism is managed by *interleaving* (transitions are fired sequentially, defining a total order). In the following we present a simplified way to construct a SCG using interval theory ((Cardoso and Valette, 2005) defines another graph of classes using temporal constraints network).

Two important points in the construction of SCG are the *firing interval* of a transition and the *time domain* of transitions that remain enabled after the firing of t_i.

The firing of a transition leads to a new state class, whose *time domain* is composed by:

- the *time interval* of all transitions enabled at this marking and
- the *time constraints* among couples of transitions that remain enabled (the time memory since the previous class where both transitions were enabled).

In the graphical representation of a SCG (Fig. 3) a node is associated with: i) a marking, ii) a *time domain* (the *time interval* and the *time constraints)*. The arc linking two state classes is labeled by the *firing interval* of the transition that provokes this state change.

a. Time constraints

Let $I^j(t_i) = [\alpha_i, \beta_i]$ be the dynamic temporal interval of t_i in a class C_j and let

$$[t_k - t_i]_0 = I^0(t_k) - I^0(t_i) \cap [0,\infty) = [\alpha_k - \beta_i, \beta_k - \alpha_i]_0 \cap [0,\infty) \quad (4)$$

be the temporal constraint in the initial class C_0. We consider $[t_k - t_i]_{-\infty} \subseteq (-\infty, \infty)$. The temporal constraint between enabled transitions t_i and t_k at class C_j, $j>0$, is given by:

$$[t_k - t_i]_j = (I^j(t_k) - I^j(t_i)) \cap [t_k - t_i]_{j-1} \cap [0, \infty) \quad (5)$$

If $[t_k–t_i]_j$ is not empty for all t_k, t_i is *fireable* (it can actually be fired), elsewhere it cannot be fired from C_j.

If $[t_k - t_i]_{j-1} \subseteq I^j(t_k) - I^j(t_i)$ it means that the temporal constraint depends only on the time intervals at the class C_j. In this case, this restriction is redundant and the only temporal information kept in the class is the interval of *enabled* transitions.

b. Firing interval

Let $t_i = [\alpha_i, \beta_i]$ be the time interval of t_i, and $sUB = \min_i(\beta_i)$ be the **s**maller **U**pper **B**ound of all *fireable* transitions. Due to the strong semantics, a fireable transition t_i can be fired from a class C_j during the *firing interval*

$$D^j(t_i) = [\alpha_i, sUB^j] \quad (6)$$

The firing of t_i leads to a new class C_{j+1}.

c. Time interval updating

The update of the *time interval* of a transition depends whether there are time constraints or not in a previous class. Consider that transitions t_i and t_k are enabled in a given class C_j, and that the firing of t_i would lead to a new class C_{j+1}. The new *time interval* of t_k in the class C_{j+1} is

$$I^{j+1}(t_k) = [t_k - t_i]_j \cap [I^j(t_k) - D^j(t_i)] \cap [0, \infty) \quad (7)$$

The intersection with $[0, \infty)$ is done in order to consider only positive instants of time. If the temporal restriction $[t_k - t_i]_j$ is redundant ((5), the time interval depends only on $I^j(t_k)$ and $D^j(t_i)$ and (7 leads back to:

$$I^{j+1}(t_k) = (I^j(t_k) - D^j(t_i)) \cap [0, \infty) = ([\alpha_k, \beta_k] - [\alpha_i, sUB^j]) \cap [0, \infty) \quad (8)$$

The intersection with $[0, \infty)$ is equivalent of doing $[\max(0, \alpha_k\text{-}sUB)\ \ \beta_k\text{-}\alpha_i]$.

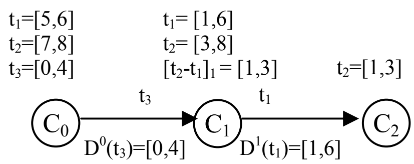

Fig. 3. State Class graph of Example 1

Example 1

Let us consider a TPN with three parallel transitions t_1, t_2 and t_3 enabled at the initial class C_0 with *time intervals* $I^0(t_1) = [5, 6]$, $I^0(t_2) = [7, 8]$ and $I^0(t_3) = [0, 4]$. Applying (4) we obtain $[t_2 - t_3]_0 = [3,8]$ and $[t_1 - t_3]_0 = [1,6]$, so only t_3 can be fired ($[t_2 - t_1]_0 = [1,3]$ but $[t_3 - t_1]_0 = \varnothing$). As these constraints are redundant, the time domain of C_0 is made up only by the time interval (see the SCG in Fig. 3). Using (6) we obtain $D^0(t_3) = [0,4]$ and the firing of t_3 leads to a new class C_1 where t_1 and t_2 remain enabled. Using (8) the new *time intervals* for t_1 and t_2 at class C_1 are $I^1(t_1) = [1, 6]$ and $I^1(t_2) = [3, 8]$. Using (5), $[t_2 - t_1]_1 = [1, 3]$ and $[t_1 - t_2]_1 = \varnothing$ so only t_1 is fireable from C_1. In fact, as in class C_0 t_1 must be fired before t_2, this restriction must also be met in class C_1. As $sUB^1 = 6$, the firing interval of t_1 from C_1 is $D^1(t_1) = [1. 6]$. After t_1 firing, the new time interval of t_2 at class C_2, using (7), is $I^2(t_2) = [t_2 - t_1]_1 \cap [I^1(t_2) - D^1(t_1)] = [1, 3]$.

3. TIME FUZZY STATE CLASS GRAPH

3.1. Fuzzy Time Petri nets

The Fuzzy Time Petri net (FTPN) (Cardoso, 1998) has the same structure that a TPN but the static time interval associated with a transition is a fuzzy one, I_f. Let be $I : X \rightarrow [0,1]$ a finite set of *fuzzy intervals* defined over $X = [0, +\infty)$. A FTPN = $<N, Mo, I_f>$ is a triple where: PN is a Petri net, Mo is the initial marking and $I_f : T \rightarrow I$ is a fuzzy time static interval that associates with each transition t_i a fuzzy interval $I_f = [a_i\ b_i\ c_i\ d_i]$ that represents the set of possible firing dates of t_i. Note that if $a_i=b_i$ and $c_i=d_i$, the FTPN is equivalent to a TPN with $I(t_i) = [a_i, c_i]$. $I_f(t_i)$ is the fuzzy set that delimits the possibility distribution of the firing date of t_i, $\pi_{If(ti)} : X \rightarrow [0,1]$. For the sake of simplicity the fuzzy time interval of a transition t_i is noted by the name of the transition: $I_f(t_i) = t_i$ and so the notation $\pi_{If(t_i)}$ is replaced by π_{t_i}:

$t_i \in T, \pi_{t_i} : T \rightarrow [X \rightarrow [0,1]]$

We must to point out that a transition is fired at a date belonging to its fuzzy time interval which we do not know beforehand.

3.2. Fuzzy transition firing

The dynamic behavior of a FTPN (resulting from the transition firing) is similar to the one of a TPN, and so the construction of the fuzzy state class graph is based on the SCG (section 2.3). The main difference, besides that the intervals are fuzzy instead of imprecise, is that the arcs are labeled by the possibility and necessity measures.

As in the case of a TPN, we must calculate the fuzzy *firing interval* of a transition and update the fuzzy *time domain* of transitions that remain enabled after the transition firing.

a. Time constraints

Let $I_f(t_i) = [a_i\ b_i\ c_i\ d_i]$ be the temporal interval of t_i in a class C_j, $\ominus$ the fuzzy subtraction and

$$[t_k \ominus t_i]_0 = I^0_f(t_k) \ominus I^0_f(t_i) \cap [0, \infty)$$

be the temporal constraint at the initial class C_0. The temporal constraint between enabled transitions t_i and t_k at class C_j, $j>0$, is given by:

$$[t_k \ominus t_i]_j = (I^j_f(t_k) \ominus I^j_f(t_i)) \cap [t_k \ominus t_i]_{j-1} \cap [0, \infty) \quad (9)$$

that is (5) extended to the fuzzy case. If $[t_k \ominus t_i]_j$ is not empty for all t_k at C_j, t_i is *fireable* (it can actually be fired). As in (5), if $[t_k \ominus t_i]_{j-1} \subseteq I_f^j(t_k) \ominus I_f^j(t_i)$ the temporal constraint is redundant.

b. Firing interval

Let be T_i the fuzzy interval that delimits π_{ti}. For all enabled transitions $t_i \in T_c = \{t_1, \dots, t_k\}$ at a class C_j, the fuzzy firing interval $D_f^j(t_i)$ during which a transition t_i can be fired from C_j is given by:

$$Df^j(t_i) = Ti \cap [t_i, +\infty) \bigcap_{j,\, j\neq i} (-\infty, t_j]\ ,\ \forall\ t_j \in Tc\ ,\ t_j \neq t_i \quad (10)$$

where $sUB = \min_i(d_i)$ is the smaller upper bound of the support of all enabled transitions. $D_f^j(t_i)$ must be normalized (if it is not already):

$$Df^j(t_i) = Df^j(t_i)/\max(D^j(t_i)) \quad (11)$$

meaning that an event that is initially defined as possible (core non empty) must stay possible during updating. In the following we consider only normalized intervals. The firing of t_i leads to a new class C_{j+1}.

Example 2

Let us consider two parallel transitions t_1 and t_2 in a FTPN, with $I_f^0(t_1) = [0\ 2\ 2\ 6]$, and $I_f^0(t_2) = [4\ 5\ 7\ 9]$ (Fig. 4). As $[t_1 \ominus t_2]_0$ and $[t_2 \ominus t_1]_0$ are not empty, both transitions are fireable. Using (10), the fuzzy firing interval of t_1 is $D_f^0(t_1) = [0\ 2\ 2\ 6]$ (Fig. 6), on the arc (C_0,C_1) in Fig. 5. The Fig. 7 shows $D_f^0(t_2) = [4\ 4.4\ 4.4\ 6]$ (little black triangle) obtained using (10) and after normalization (big gray triangle) using (11) on the arc (C_0,C_3) in Fig. 5.

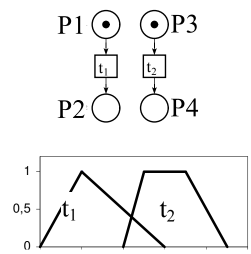

Fig. 4. $I^0_f(t_1)$ and $I^0_f(t_2)$

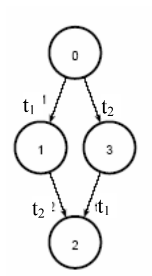

Fig. 5. FSCG

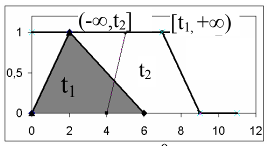

Fig. 6. $D_f^0(t_1)$

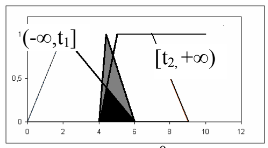

Fig. 7. D_f^0 (t2)

c. Time interval updating

The firing of a transition t_i from a class C_j leads to a new class C_{j+1} with a new time domain. In the same way that in TPN, there are two cases. The concept of time constraint is the same both in TPN and FTPN. We extend the imprecise case of (7) to the fuzzy case and the new *fuzzy time interval* in the class C_{j+1} (considering only positives instants of time) after the firing of t_i during the interval $D_f^j(t_i)$ is:

$$I_f^{j+1}(t_k) = [t_k \ominus t_i]_j \cap [I_f^j(t_k) \ominus D_f^j(t_i)] \cap [0, \infty) \quad (12)$$

If $[t_k \ominus t_i]_j$ is redundant, (12) leads back to:

$$I_f^{j+1}(t_k) = (I_f^j(t_k) \ominus D_f^j(t_i)) \cap [0, +\infty) \quad (13)$$

that is the fuzzy extension of (8) for a TPN. $I_f^{j+1}(t_k)$ must be also normalized.

Let us extend the Example 1 for a FTPN using $I_f^0(t_1) = [5\ 5\ 6\ 6]$, $I_f^0(t_2) = [7\ 7\ 8\ 8]$ et $I_f^0(t_3) = [0\ 1\ 3\ 4]$ at C_0. Only t_3 is fireable. After the firing of t_3 from C_0 with $D_f^0(t_3) = I_f^0(t_3)$ using (10), t_1 and t_2 remain enabled at the new class C_1. Applying (13) we have $I_f^1(t_1) = [1\ 2\ 5\ 6]$ and $I_f^1(t_2) = [3\ 4\ 7\ 8]$ at class C_1; using (9), the time constraint at this class is $[t_2 \ominus t_1]_1 =$

[1 1 3 3] and so only t_1 is fireable. The firing of t_1 during the interval D_f^1 (t_1)=[1 2 5 6] leads to C_2 with $I_f^2(t_2)$=[1 1 3 3] (12).

3.3. Possibility and Necessity measures of transition firing

If there are several fireable transitions we must determine, for each transition t_i, the possibility and the necessity that it can be fired before the others. Extending (2) to a set of enabled transitions t_j:

$$\Pi(t_i \le \bigcap_{j,j\neq i} t_j) = \min \bigcap_{j,j\neq i} \{\sup_{x \le y}(\min(\pi_{ti}(x), \pi_{tj}(y))\}$$

It corresponds also to $\Pi(t_i \le t_j\})$ = sup $D_f^j(t_i)$, where $D_f^j(t_i)$ is the fuzzy set before normalization.

The necessity that t_i has to be fired before the others extending (3 for several enabled transition is:

$$N(t_i \le \bigcap_{j,j\neq i} t_j) = \min\left\{\bigcap_{j,j\neq i} \sup\,]ti, tj[\right\} = \min \bigcap_{j,j\neq i} \sup\left\{]ti, +\infty) \bigcap (-\infty, tj\, [\right\}$$

For the sake of simplicity, we can note $\Pi(t_i)$ and $N(t_i)$. In Example **2**, the possibility labeling arc (C^0, C^1) in Fig. 5 is $\Pi(t_1 \le t_2)$ = sup $D_f^0(t_1)$ = 1 and the necessity is $N(t_1 \le t_2)$ = 0,6; the labels for arc (C^0, C^3) are $\Pi(t_2 \le t_1)$ = sup D_f^0 (t_2) = 0.4 and $N(t_2 \le t_1)$ = 0.

4. APPLICATION RESULTS

Let us consider an unidirectional protocol of data transfer (Juanole *et al*, 2003) modeled by the Petri net in Fig. 8. The data storing is indicated by the dotted ellipse in the figure; the production is modeled by places p1 and p_2 and transition t_1; the consummation by t_5 and p_6 and the transmission by t_2. We want to know if an overwrite, due to the earlier arriving of a new message whereas the precedent was not yet consumed, can occur in this system (represented by transition t_4).

The underlined Petri net (the structure without time specifications) is unbounded and so it is not possible to know if the overwrite is done. The temporal analysis using a TPN or a FTPN allows obtaining some more information.

The following time specification are considered for the FTPN: $\pi_{t1}(\tau)$ = [4 5 5 6] , $\pi_{t2}(\tau)$ = [2 2 3 3] ; $\pi_{t3}(\tau)$ = $\pi_{t4}(t)$ = [0 0 0 0], $\pi_{t5}(\tau)$ = [0 1 3 4]. The FSCG obtained from the FTPN is shown Fig. 9. The details of all classes C^j (with the marking and fuzzy time domain) as well the fuzzy firing interval $D^j(t_i)$ and the possibility and necessity measures that label the arc (C^j, C^{j+1}) are presented in Table 2. We can see at the FSCG that a message can be overwrite (transition t_4 can be fired). From class C_5, t_4 can be fired during the fuzzy firing interval $D^5(t_4)$=[0 0 0 0], with $\Pi(t_4)$=1 and $N(t_4)$=0. Transition t_5 can also be fired during $D^5(t_5)$=[0 0 0 0], with $\Pi(t_5)$=1 and $N(t_5)$=0. It means that both events are equally possible: this information is the same that would be obtained by a TPN. Let us now consider class C_{10}, from where t_2 and t_5 can be fired. We have $D^{10}(t_2)$=[2 2 2 3] with $\Pi(t_2)$ =0.33 and $N(t_2)$=0; $D^{10}(t_5)$=[0 0 0 3] with $\Pi(t_5)$=1 and $N(t_5)$ =0.67. In this case, $N(t_5) > N(t_2)$ and the firing of t_5 is more necessary that the one of t_2.

Let us consider classes C_8, C_9, C_{10} and C_{11} in relation to, respectively, classes C_5, C_3, C_4 and C_6 (see the appendix). They have:
- the same marking,
- the *support* of the fuzzy time intervals are the same, even if the core are different for some transitions (e.g. t_1 and t_2 for C_6 and C_{11}),
- for all output arcs of classes C_8, C_9, C_{10} and C_{11}, the *support* of the fuzzy firing interval is equal, respectively, to the support of the ones leaving classes C_5, C_3, C_4 and C_6.

We call the classes C_5 and C_8 support-equivalent, or *s-equiv* for short. The following classes are *s-equiv:* (C_2, C_{12}, C_{14}), (C_3, C_9, C_{13}, C_{15}), (C_4, C_{10}), (C_6, C_{11}), (C_7, C_{16}, C_{17}). Folding the *s- equiv* classes leads to a graph were only the *support* of the fuzzy intervals are considered ($\pi \in \{0,1\}$). All possibility and necessity measures on the arcs are Π = 1 and N=0. It is in fact the particular case of imprecision and the FTPN leads back to a TPN with the following time specification: $I^0(t_1)$= [4,6], $I^0(t_2)$ = [2,3], $I^0(t_3)$ = $I^0(t_4)$ = [0,0], $I^0(t_5)$= [0,4]. The folded FSCG with the *s-equiv* classes is equal to the SCG generated by this TPN using tool TINA (Berthomieu *et all*, 2004). The TPN is a particular case of a a FTPN when the interval of all transitions are imprecise rather than fuzzy.

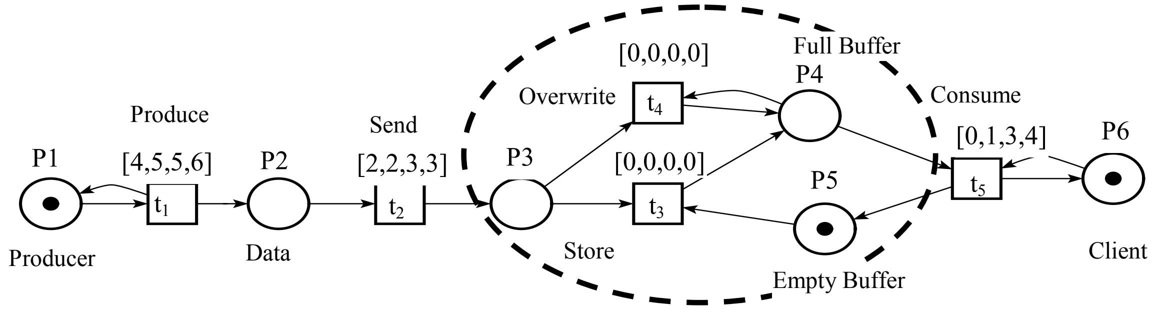

Fig. 8. Fuzzy Time Petri net model of a protocol

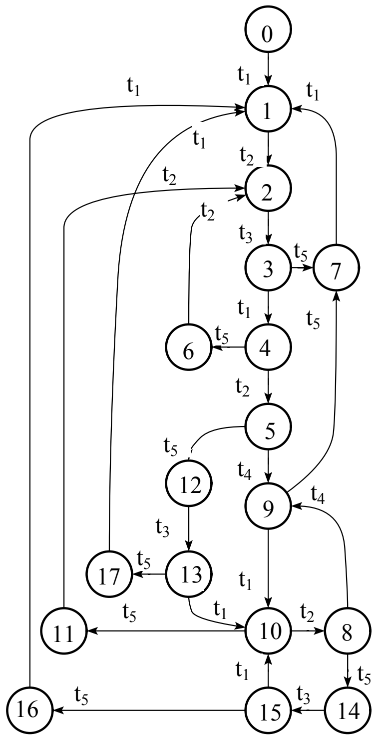

Fig. 9. Fuzzy State Class Graph

5. CONCLUSION

This paper presents a methodology to construct the Fuzzy State Class Graph (FSCG) of a Fuzzy Time Petri Net, which is inspired and extends the State Class Graph of Time Petri net (Berthomieu and Menasche, 1983). Possibility theory allows to clearly distinguish the statements "A is possible" and "A is certain". It can be interpreted as a representation of ordinal uncertainty based on linear ordering (Dubois, and Prade, 1988). So, the advantage of the FTPN in relation to the TPN is that it allows an ordering between the possibility/necessity of the firing of a transition when several are fireable. In fact, in a SCG, all firing transitions are equally possible, besides in a FSCG the possibility and necessity values of firing can be different. By the way, if only the support of the time intervals (and so the firing intervals) are considered, we obtain a TPN and the FSCG becomes a SCG.

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Table 2 The classes of the FSCG for Fig. 8

Classes with marked places and time interval of fireable transitions

C0, p1 p5 p6, t_1=[4 5 5 6]	**C9**, p1 p4 p6 ,t_1=[1 3 3 4],t_5=[0 1 3 4]
C1, p1 p2 p5 p6, t_1=[4 5 5 6], t_2=[2 2 3 3]	**C10**, p1 p2 p4 p6, t_1=[4 5 5 6],t_2=[2 2 3 3],t_5=[0 0 0 3]
C2, p1 p3 p5 p6, t_1=[1 2 3 4], t_3=[0 0 0 0]	**C11**, p1 p2 p5 p6, t_1=[1 5 5 6], t_2=[0 2 3 3] $t_1 - t_2$=[1 2 3 4]
C3, p1 p4 p6, t_1=[1 2 3 4], t_5=[0 1 3 4]	**C12**, p1 p3 p5 p6, t_1=[1 3.25 3.25 4], t_3=[0 0 0 0]
C4, p1 p2 p4 p6,t_1=[4 5 5 6], t_2=[2 2 3 3], t_5=[0 0 1 3]	**C13**, p1 p4 p6, t_1=[1 3.25 3.25 4], t_5=[0 1 3 4]
C5, p1 p3 p4 p6, t_1=[1 3 3 4], t_4=[0 0 0 0], t_5=[0 0 0 1] $t_1 - t_5$ =[1 4 5 6]	**C14**, p1 p3 p5 p6, t_1=[1 3.4 3.4 4], t_3=[0 0 0 0]
C6, p1 p2 p5 p6, t_1=[1 4 5 6], t_2=[0 1 3 3] $t_1 - t_2$=[1 2 3 4]	**C15**, p1 p4 p6, t_1=[1 3.4 3.4 4], t_5=[0 1 3 4]
C7, p1 p5 p6, t_1=[0 0 2 4]	**C16**, p1 p5 p6, t_1=[0 0.4 2.4 4]
C8, p1 p3 p4 p6, t_1=[1 3 3 4], t_4=[0 0 0 0],t_5=[0 0 0 1] t_1-t_5=[4 5 5 6] – [0 0 0 3]=[1 5 5 6]	**C17**, p1 p5 p6, t_1=[0 0.25 2.25 4]

Firing interval $D^j(t_i)$ with possibility/necessarily measures

$D^0(t_1)$ = [4 5 5 6], N = 1	$D^5(t_4)$ = [0 0 0 0], Π = 1, N = 0	$D^9(t_5)$ = [0 1 3 4], Π = 1	$D^{14}(t_3)$ = [0 0 0 0], N = 1
$D^1(t_2)$ = [2 2 3 3], N = 1	$D^5(t_5)$ = [0 0 0 0], Π = 1	$D^{10}(t_2)$ = [2 2 2 3], Π = 1/3	$D^{15}(t_1)$ = [0 3.12 3.12 4], Π = 0.961
$D^2(t_3)$ = [0 0 0 0], N = 1	$D^6(t_2)$ = [0 1 3 3],N = 1	$D^{10}(t_5)$ = [0 0 0 3], N = 2/3	$D^{15}(t_5)$ = [0 1 3 4], N = 0.039
$D^3(t_1)$ = [1 2 3 4], Π = 1, N = 0	$D^7(t_1)$ = [0 0 2 4], N = 1	$D^{11}(t_2)$ = [0 2 3 3], N = 1	$D^{16}(t_1)$ = [0 0.4 2.4 4], N = 1
$D^3(t_5)$ = [0 1 3 4], Π = 1, N = 0	$D^8(t_4)$ = [0 0 0 0], Π = 1, N = 0	$D^{12}(t_3)$ = [0 0 0 0], N = 1	$D^{17}(t_1)$ = [0 0.25 2.25 4],N = 1
$D^4(t_2)$ = [2 2 2 3], Π = 0.5	$D^8(t_5)$ = [0 0 0 0], Π = 1, N = 0	$D^{13}(t_1)$ = [0 3.07 3.07 4], Π = 0.923	
$D^4(t_5)$ = [0 0 1 3], N = 0.5	$D^9(t_1)$ = [1 3 3 4], Π = 1	$D^{13}(t_5)$ = [0 1 3 4], N = 0.077	