ON THE EXISTENCE OF PETRI NET CONTROLLER FOR DISCRETE EVENT SYSTEMS UNDER PARTIAL OBSERVATION

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Abstract: This paper addresses the existence of a maximally permissive PN controller for the forbidden state problem of bounded Petri nets (PN) under partial observation. Based on a Ramadge-Wonham theory, the refined controlled observer reachability graph of the controlled PN is determined and represents the most permissive behavior with liveness requirement and uncontrollable / unobservable transitions. The theory of regions is then used to generate a set of control places, if such controller exists, to be added to the plant PN model. Necessary and sufficient conditions for the existence of pure and impure control places are then presented.

Keywords: Supervisory Control, Petri Nets, Unobservable Transitions, Forbidden State Problems.

1. INTRODUCTION

Previously, we proposed a general approach for synthesizing a set of control places that optimally solve a forbidden state problem of any bounded Petri net (PN) when such PN controller exists (Achour, et al. 2004). Indeed, it considers a general set of forbidden states in addition to liveness requirement and the presence of uncontrollable / unobservable transitions.

This paper addresses the existence of optimal PN controller for control places design problem for bounded Petri nets under partial observation. The goal of the supervision is to avoid a set of forbidden states. The control synthesis method is based on the theory of regions which is a technique for Petri net synthesis from automata models. Plant models under consideration are generalized bounded Petri nets. The Petri net controller is optimal in the sense of maximum permissiveness under the restriction of the liveness of the controlled system. Supervisory control of partially observable Petri net plants has been addressed in (Moody and Antsaklis, 1999; Stremersch, 2000; Moody, et al., 1996) using linear algebra approaches which are extensions of the optimal place-invariant approach proposed in (Yamalidou, et al., 1996) for enforcing linear constraints for totally controllable and observable Petri net plants. The most serious drawback of these approaches is that the optimality of the control policy cannot be guaranteed. An exception is the linear algebra approach proposed in (Darondeau and Xie, 2003) for enforcing linear constraints of firing count vector and/or markings of partially controllable and partially observable marked graphs. Note that the liveness of the controlled system cannot be enforced with these approaches. We follow, in this paper, the approach of (Ghaffari, et al., 2003) to address the forbidden state-transition problems of generalized Petri net model with uncontrollable and unobservable transitions.

This paper is organized as follows. Section 2 introduces the observation problem, and presents Observer Reachability Graph, Refined Observer Reachability Graph and Refined Controlled Observer Reachability Graph that we use for determination of the desired behaviour of the optimally controlled plant. Necessary and sufficient conditions for the existence of pure and impure control places are presented in section 3. Two particular cases for which optimal PN controller does not exist are presented in Section 4.

2. DESIRED CONTROLLED BEHAVIOR

The supervisory control problem considered in this paper concerns a bounded Petri net with uncontrollable and/or unobservable transitions and starting from a given initial marking. Each transition is either controllable and observable, observable but not controllable, or unobservable and uncontrollable. The goal of the supervisory control is the avoidance of a set of forbidden states and the enforcement of the liveness or nonblockingness that requires the reachability of some marked states. In this paper, we restrict to the case with the initial state as the unique marked state. The liveness requirement is then equivalent to the reversibility.

2.1 Observer Reachability Graph

In a Petri net N with unobservable transitions, the system state represented by the current marking is not fully known.

Example 1: Consider the Petri net of figure 1. From the initial marking $(1, 0, 0, 0)^t$ and after the firing of the transition t_1, the real state of the plant becomes unknown. In fact t_2 and t_3 are unobservable and from the supervisor point of view, the token can be in p_2 or in p_3 or in p_4.

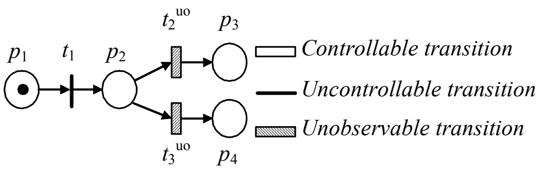

Fig. 1. Petri net model with unobservable transitions

A reachability graph $RG(N, M_0)$ is often used to model the behaviour of a bounded Petri net (N, M_0) and for its supervisory control. With unobservable transitions, the plant state marking is only partially known and the reachability graph cannot be used for the supervisory control. For this reason, we transform a reachability graph into an observer reachability graph, denoted by $RG^O(N, M_0)$, to represent all possible sequences or language of observable transitions. Algorithm 1 which is based on the framework presented in (Cassandras and Lafortune, 1999) is used for this transformation. The construction of this observer reachability graph is based on the assumption that the initial marking M_0 is known and on the following definitions.

Definition 1: $R^{uo}(x)$ denotes the set of indistinguishable states with respect to a Petri net marking x, and represents the set of states reachable from x by firing only unobservable transitions. Formally, $R^{uo}(x) = \{ x' \in RG(N, M_0) \mid \exists\ \sigma \in (T^{uo})^*$ and $x[\sigma\rangle x' \}$ where T^{uo} is the set of unobservable transitions. For a set S of states, $R^{uo}(S) = \bigcup_{x \in S} R^{uo}(x)$.

From the above example, it is clear that the state of the observer can only be modified by observable transitions and hence each state of the observer represents a set of plant states. In this paper, we call the state of the observer macro-state and plant states micro-states. More precisely, each macro-state y corresponds to a set of indistinguishable micro-states and vice versa. As a result, the initial macro-state y_0 of the observer corresponds to the set of micro-states $R^{uo}(M_0)$ that are states reachable from M_0 before the firing of unobservable transitions.

$RG^O(N, M_0)$ is constructed as follows.

Definition 2: A transition t is said firable at a macro-state y if t is firable from at least one micro-state x, i.e. $\exists\ x \in y$ such that $x[t>$ where the notation $x[t>$ indicates that t is firable at x .

Definition 3: The firing of any observable transition t from a macro-state y leads to another macro-state y' defined as follows:

$$y' = \bigcup_{\substack{x \in y \\ x[t\rangle x'}} R^{uo}(x')$$

which corresponds to micro-states reachable from a micro-state x in y by first firing the observable transition t followed by unobservable transitions.

Two macro-states y and y' are considered as the same macro-state if they correspond to the same set of micro-states.

The construction of the observer reachability graph $RG^O(N, M_0)$ of $RG(N, M_0)$ is summarized in the following algorithm:

Algorithm 1:
1. Define the initial macro-state $y_0 = R^{uo}(M_0)$
2. For each unexplored macro-state y and for each observable transition t firable at y, determine the macro-state y' obtained from y by firing t.
3. If y' exists, add an arc labelled t from y to y'; otherwise, create the new macro-state y' in the observer reachability graph and add arc (y, y') with label t.
4. If there exists at least one unexplored macro-state, go to step 2. Otherwise, stop.

Let us use algorithm 1 to derive the observer reachability graph of the Petri net example of figure 1. The reachability graph is given in Figure 2.a. Initial macro-state y_0 is $y_0 = R^{uo}(x_0) = \{x_0\}$. Transition t_1 is the unique observable transition firable from y_0 and leads to a macro-stat y_1 defined as follows:

$$y_1 = \bigcup_{\substack{x \in y_0 \\ x[t_1\rangle x'}} R^{uo}(x') = R^{uo}(x_1) = \{x_1, x_2, x_3\}.$$

No observable transitions are firable from $y_1=\{x_1, x_2, x_3\}$ and the construction is finished. The observer reachability graph $RG^O(N, M_0)$ is shown in figure 2.

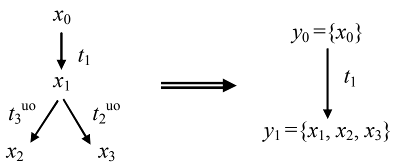

Fig. 2. Reachability graph and observer reachability graph of the Petri net of figure 1

If the firing of an observable transition $t°$ at a macro-state y leads to another macro-state y', then y' is said reachable from y and is denoted as $y[t^o>y'$.

Remark: Firing an observable transition $t°$ at a macro-state y leads to one macro-state even if $t°$ is firable at more than one micro-state in y. This implies that the reachability relation of the observer reachability graph is an aggregate representation of different micro-state-transition relations in the reachability graph.

To summarize, firing an unobservable transition t^{uo} at a micro-state x_i leads to a micro-state x_j but does not change the macro-state y_k. On the other hand, firing an observable transition t^o at a macro-state y leads to another macro-state y'.

2.2 Refined Observer Reachability Graph

We now introduce the refined observer reachability graph $RG^{RO}(N, M_0)$ which is a state graph obtained by refinement of the observer reachability graph $RG^O(N, M_0)$ to introduce unobservable transitions between micro-states. More precisely, each node representing a macro-state y is represented by several nodes each representing a micro-state of y and unobservable transitions between these micro-states are represented. Further firings of an observable transition $t°$ from different micro-states of y are represented separately. This graph is equivalent to the parallel composition of the $RG^O(N, M_0)$ and $RG(N, M_0)$. The refined observer reachability graph $RG^{RO}(N, M_0)$ related to the example 1 is given in figure 3.

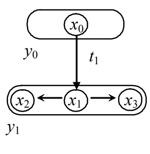

Fig. 3. Refined observer reachability graph $RG^{RO}(N, M_0)$

2.3 Refined Controlled Observer Reachability Graph

This section deals with desired controlled behavior represented by refined controlled observer reachability graph $RG^{RCO}(N, M_0)$. The $RG^{RO}(N, M_0)$ determination is based on forbidden states, liveness requirement and by taking in to account unobservable/uncontrollable transitions.

The starting point of this computation is the refined observer reachability graph $RG^{RO}(N, M_0)$.

Definition 4: A macro-state is said forbidden if at least one of its micro-states is forbidden. Let Y_F be the set of forbidden macro-states.

Definition 5: A macro-state is said dangerous if it leads to forbidden macro-state by firing uncontrollable transitions. Let Y_D be the set of dangerous macro-states. These states can be determined on the refined observer reachability graph $RG^{RO}(N, M_0)$.

Definition 6: The Live Sub-Graph (LSG) is a sub-graph of $RG^{RO}(N, M_0)$ containing all macro-states y such that all micro-states x in y can be led to a micro-state representing the initial marking.

Definition 7: A macro-state is said blocking if it contains at least one micro-state that uncontrollably leads outside the Live Sub-Graph (LSG) of the refined observer reachability graph. Let Y_B be the set of blocking macro-states. In other terms, if a macro-state contains a blocking micro-state then the macro-state if blocking too.

The computation of the maximally permissive behavior consists in iteratively determining and then removing all forbidden/dangerous/blocking macro-states till convergence to a LSG containing no forbidden/dangerous/blocking macro-states. The maximally permissive behavior is represented by the Live Sub-Graph LSG of controlled system, since only the dangerous and blocking macro-states are eliminated is enumerated in (Achour, *et al*. 2004).

Definition 8: The sub-graph of observable reachability graph $RG^O(N, M_0)$ obtained by removing all macro-states not belonging to Refined Controlled Observer Reachability Graph $RG^{RCO}(N, M_0)$ is called Controlled Observable Reachability Graph denoted by $RG^{CO}(N, M_0)$.

Clearly, $RG^{CO}(N, M_0)$ represents the observable maximally permissive behavior of the controlled plant and will be used for the determination of control places. Let Y_L be the set of all macro-states of $RG^{CO}(N, M_0)$ and Ω the set of all macro-state-transitions leading outside $RG^{CO}(N, M_0)$. Formaly, the set of state-transitions the controller has to disable is $\Omega = \{(y \xrightarrow{t} y') \mid y \in Y_L \wedge y' \notin Y_L\}$. Each state-transition to be disable by the controller is called an event separate instance.

3. CONTROL PLACES DESIGN

Forbidden state problems and forbidden state-transition problems (Ghaffari, *et al.* 2003) of bounded Petri nets (N, M_0) may be solved by the addition of control places p_c defined by $(M_0(p_c), C^+(p_c, .), C^-(p_c, .))$ where $M_0(p_c)$ is its initial marking and $C^+(p_c, .)$ (resp. $C^-(p_c, .)$) is its weighing vector of arcs from transitions of N to p_c (resp. from p_c to transitions of N). Clearly, due to observability constraints, such arcs are allowed only between p_c and observable transitions.

3.1 Preliminaries and some basic relations

Consider the plant Petri net model (N, M_0) and its controlled observer reachability graph $RG^{CO}(N, M_0)$. Each control place p and for any observable transition t from any macro-state M in $RG^{CO}(N, M_0)$:

$$M'(p) = M(p) + C(p,t), \quad \forall (M \xrightarrow{t} M') \in RG^{RCO}(N, M_0) \qquad (1)$$

where M_0 is the initial macro-state, $C(p, .)$ the incidence vector of p and M' the new macro-state or equivalently the destination node of arc t.

Consider now any non-oriented cycle γ of the refined controlled observer reachability graph. Applying the state equation (1) to nodes in γ and summing them up gives the following *Cycle equation*.

$$\sum_{t \in T} C(p,t).\vec{\gamma}[t] = 0, \quad \forall \gamma \in S \qquad (2)$$

where $\vec{\gamma}[t]$ denotes the algebraic sum of all occurrences of t in γ and S the set of non-oriented cycles of the graph.

$$M(p) = M_0(p) + C(p,.)\vec{\Gamma}_M,$$

where $\vec{\Gamma}_M$ is the algebraic counting vector of Γ_M. The reachability of any macro-state M in $RG^{CO}(N, M_0)$ implies that:

$$M_0(p) + C(p,.)\vec{\Gamma}_M \geq 0, \quad \forall M \in RG^{RCO}(N, M_0) \qquad (3)$$

which will be called the *Reachability condition*.
Lemma 1: Any control place p_c of the controlled net satisfies both relations (2) and (3).

3.2 Pure control places

In order to obtain exactly the desired behavior, for each event separation instance $(M \xrightarrow{t} M')$ in Ω such that M is a reachable macro-state of $RG^{CO}(N, M_0)$ and t is a controllable transition to be disable by the controller at M, t should be disabled by some control place p_c. Since p_c is pure, t is disabled at M if and only if:

$$M_0(p_c) + C(p_c,.)\vec{\Gamma}_M + C(p_c,t) < 0, \quad \forall (M \xrightarrow{t} M') \in \Omega \qquad (4)$$

Relation (4) will be called *event separation condition.*

Theorem 1: *A desired behavior $RG^{CO}(N, M_0)$, can be realized by adding pure control places to (N, M_0) if and only if there exists a solution $(M_0(p_c), C(p_c, .))$ satisfying conditions (2), (3) and (4) for each event separation instance in Ω.*

Proof: ($\Leftarrow$) Consider the controlled net $(Nc,M_0) = (N,M_0) \cup \{p_c\}$. Since all places in $\{p_c\}$ satisfy equations (2) and (3), all transitions in RG^{CO} are firable and the reachability graph obtained by firing labeling transitions in RG^{CO} is still RG^{CO} which imply that $RG^{CO} \subseteq R(Nc,M_0)$. Further, each event separation instance in Ω is solved by a control place p_{ci}, i.e. event separation condition (4) holds with the place p_{ci}. This implies that all transitions in Ω are disabled in the controlled net (Nc,M_0). To conclude, $R(Nc,M_0) = RG^{CO}$.

($\Rightarrow$) Assume that the reachability graph of the controlled system $R(Nc,M0)$ is the maximally permissive behavior. Each place of (Nc,M_0) satisfies necessarily equations (2) and (3), and for each instance in Ω, t is necessarily prevented by some place p_c, thus satisfying relation (4).

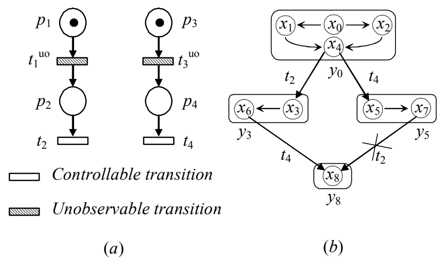

Fig. 4. A forbidden state-transition problem 1

Pure control places cannot solve all the forbidden state-transition problems. Consider for this purpose the Petri net model of figure 4.a. Control specification is not to fire t_2 after t_4, i.e. to forbid firing of t_2 from states such that p_3 and p_4 are both unmarked. The refined observer Reachability Graph is given in Figure 2.b and the refined controlled observer reachability Graph without imposing the liveness requirement is obtained by removing the arc t_2 outgoing y_5. . The event separation instance to solve is $(y_5 \xrightarrow{t_2} y_8)$. Let us use the theory of regions to determine a pure control place p_c solving this problem. As $RG^{RCO}(N, M_0)$ does not contain any cycle, p_c satisfies the following system;

Reachability conditions of admissible macro-states:

(1.*i*) $y_0 : M_0(p_c) \geq 0$

(1.*ii*) $y_3 : M_0(p_c) + C(p_c,t_2) \geq 0$

(1.*iii*) $y_5 : M_0(p_c) + C(p_c,t_4) \geq 0$

(1.*iv*) $y_8 : M_0(p_c) + C(p_c,t_2) + C(p_c,t_4) \geq 0$

Separation condition:

(1.*v*) $(y_5 \xrightarrow{t_2} y_8)$:

$M_0(p_c) + C(p_c,t_4) + C(p_c,t_2) < 0$

Note that the two last inequalities are contradictory. So, there is no pure control place to solve this problem. However, an impure control place exists as it will be shown in the next subsection.

3.3 Impure control places

Self-loops are introduced to increase the control power of control places. For any event separation instance $(M \xrightarrow{t} M')$ in Ω, only control places p_c with self-loop connecting p_c and t need to be considered. According to Lemma 1, cycle equation (2) and reachability condition (3) hold.

Since the control places are impure, the reachability conditions (3) are no longer enough to guarantee the reachability of markings M in $RG^{CO}(N, M_0)$. Additional conditions are needed to ensure firability of a transition t enabled at any markings M in $RG^{CO}(N, M_0)$:

$$M(p_c) = M_0(p_c) + C(p_c,.)\vec{\Gamma}_M \geq C^-(p_c,t), \quad \forall M[t\rangle \text{ in } RG^{CO}(N, M_0) \qquad (5)$$

The event separation condition related to $(M^* \xrightarrow{t} M')$ becomes:

$$M_0(p_c) + C(p_c,.)\vec{\Gamma}_{M^*} < C^-(p_c,t) \qquad (6)$$

Theorem 2: *A desired behavior $RG^{CO}(N, M_0)$, the* Controlled Observer Reachability Graph *of a bounded Petri net* (N, M_0)*, can be realized by adding impure control places to* (N, M_0) *if and only if there exists a solution* ($M_0(p_c)$, $C(p_c, .)$, $C^-(p_c, t)$) *satisfying conditions* (*2*)*,* (*3*)*,* (5) *and* (6) *for any event separation instance in* Ω*.*

Proof: The proof of this theorem is similar to that of theorem 1

Let consider again the forbidden state-transition problem of figure 5. It may be solved by an impure control place p_c *iff* p_c satisfies relations (1.*i*), (1.*ii*), (1.*iii*), (1.*iv*) and:

$$M_0(p_c) \geq C^-(p_c,t_2)$$

$$M_0(p_c) + C(p_c,t_4) < C^-(p_c,t_2)$$

A solution may be given by $M_0(p_c)=1$, $C(p_c, t_4) = -1$, $C(p_c, t_2) = 0$ and $C^-(t_2) = 1$ (see figure 5).

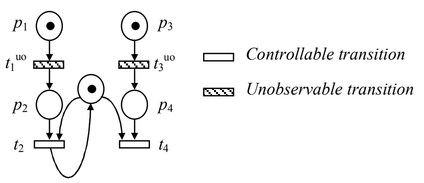

Fig. 5. The controlled net corresponding to problem of Figure 5

Note that combination of (5) and (6) leads to:

$$C(p_c,.)\left(\vec{\Gamma}_M - \vec{\Gamma}_{M^*}\right) > 0, \quad \forall M[t\rangle \text{ in } RG^{CO}(N, M_0) \qquad (7)$$

Corollary 1: There exists an impure control place solving an event separation instance $(M^* \xrightarrow{t} M')$ iff the linear system defined by relations (2) and (7) has a solution.

4. EXISTENCE OF CONTROL PLACES

In some cases, the unobservability constraint makes that optimal PN controller might not exist.

Consider the example of figure 6. Assume that the control specification requires that transition T should be disabled by the controller when p_3 and p_5 are marked i.e. T should not be fired from y_3, while it is firable. Let y' be such that $y_3[T>y'$. The event separation instance is then $(y_3 \xrightarrow{T} y')$. Let y'' be such that $y_8[T>y''$. Let us use the theory of regions to determine the controller.

Remark In the $RG^{CO}(N, M_0)$ given in figure 6, all reachable macro-states by firing transition T are not represented.

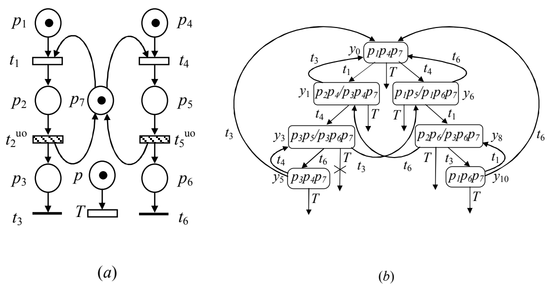

Fig. 6. A forbidden state-transition problem 2

Consider the reachability condition (3) of macro-state y'': (2.*i*) y'':

$$M_0(p_c) + C(p_c,t_4) + C(p_c,t_1) + C(p_c,T) \geq 0$$

Consider now the separation condition of macro-state y': (2.*ii*) $(y_3 \xrightarrow{T} y')$:

$$M_0(p_c) + C(p_c,t_1) + C(p_c,t_4) + C(p_c,T) < 0$$

Note that the inequalities are contradictory. So, there is no control place to solve optimally this problem. However, a more restrictive controller exist and cane be solved by a control places (figure 7).

Corollary 2: There exists a control place that forbids the firing of a transition t at some macro-state y' such that $y_0[\sigma_1>y'$ if it does not exist a macro-state y'' such that $y_0[\sigma_2>y''$ and $\vec{\sigma}_1^o = \vec{\sigma}_2^o$ from which t can fire.

Proof: Consider a Controlled Observer Reachability Graph $RG^{CO}(N, M_0)$ and a event separation instance $(y_1 \xrightarrow{t} y_1')$ where $y_0[\sigma_1>y_1$. Let y_2 a macro-state such that $y_0[\sigma_2>y_2$ and $y_2[t> y_2'$. From theorem 1, a pure control place exists iff there exists a solution ($M_0(p_c)$, $C(p_c, .)$) satisfying conditions (*2*), (*3*) and (*4*) for any event separation instance in Ω. However, (*3*)$\Rightarrow y_2'$: $M_0(p_c) + C(p_c,.)\vec{\sigma}_2^o + C(p_c,t) \geq 0$, and (*4*)$\Rightarrow (y_1 \xrightarrow{t} y_1')$: $M_0(p_c) + C(p_c,.)\vec{\sigma}_1^o + C(p_c,t) < 0$, two contradictory inequalities if $\vec{\sigma}_1^o = \vec{\sigma}_2^o$. So, there is no pure control place to solve optimally this problem.

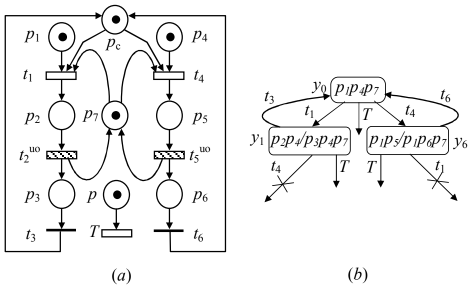

Fig. 7. Restrictive controller corresponding to problem of Figure 7

Similarly, From theorem 2, an impure control place exists iff there exists a solution ($M_0(p_c)$, $C(p_c, .)$, $C^-(p_c, t)$) satisfying conditions (*2*), (*3*), (5) and (6) for any event separation instance in Ω. However, (*5*) ⇒ $y_2^{'}$: $M_0(p_c)+C(p_c,.)\vec{\sigma}_2^o \geq C^-(p_c,t)$, and (*6*) ⇒ $(y_1 \xrightarrow{t} y_1^{'})$: $M_0(p_c)+C(p_c,.)\vec{\sigma}_1^o < C^-(p_c,t)$, two contradictory inequalities if $\vec{\sigma}_1^o = \vec{\sigma}_2^o$. So, there is no impure control place to solve optimally this problem.

In the same spirit, control places for event separation do not exist and the theory of regions fails to provide a solution if the set of cycle equations has full rank with zero as the unique solution for $C(p_c, .)$. Figure 8 is such example.

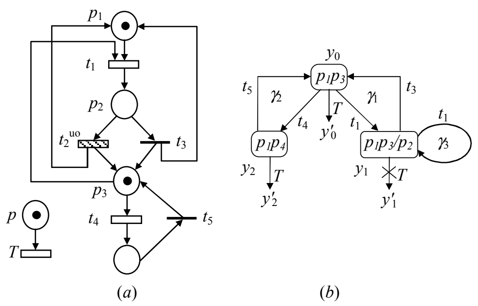

Fig. 8. A forbidden state-transition problem 3

Assume that the control specification requires that transition *T* be disabled by the controller when p_2 is marked i.e. *T* cannot fire from y_1, while it can fire. Let Use theory of regions to determine the control place to solve this forbidden state-transition problem: Consider first the cycle equations:

(3.*i*) $\gamma_1 : C(p_c,t_1)+C(p_c,t_3)=0$

(3.*ii*) $\gamma_2 : C(p_c,t_4)+C(p_c,t_5)=0$

(3.*iii*) $\gamma_3 : C(p_c,t_1)=0$

Consider the reachability conditions of the admissible macro-states:

(3.*iv*) y_0: $M_0(p_c) \geq 0$

(3.*v*) $y_0^{'} : M_0(p_c)+C(p_c,T) \geq 0$

(3.*vi*) $y_1 : M_0(p_c)+C(p_c,t_1) \geq 0$

(3.*vii*) $y_2 : M_0(p_c)+C(p_c,t_4) \geq 0$

(3.*viii*) $y_2^{'} : M_0(p_c)+C(p_c,t_4)+C(p_c,T) \geq 0$

Consider now the separation condition of the inadmissible macro-state:

(3.*ix*) $(y_1 \xrightarrow{T} y_1^{'})$:
$M_0(p_c)+C(p_c,t_1)+C(p_c,T) < 0$

Note that $C(p_c,t_1)=0$ and $C(p_c,t_3)=0$. Then it dose not exist a control place to solve this forbidden state-transition problem.

Assume now that the control specification requires that transition *T* be disabled by the controller when p_4 is marked i.e. *T* cannot fire from y_2, while it can fire. In this case, a solution exist and the control place is characterised as fallows:
$M_0(p_c)=1,\ C(p_c,t_4)=-1,\ C(p_c,t_5)=1,\ C(p_c,T)=-1.$

5. CONCLUSION

In this paper, we present a rigorous setting for the general PN controller design problem given desired behavior. Desired behaviors are defined according to general forbidden state-transition specifications which include the case of forbidden state specifications.

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