ROBUST LOAD-SHARING CONTROL OF SPACECRAFT FORMATIONS

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Abstract: This paper focuses on the design of autonomous and collaborative control strategies to govern the relative distances among multiple spacecraft in formation with no ground intervention. A coordinate load-sharing control structure for formation flying and a methodology to control their dynamic models with slow time-varying and uncertain parameters are the main objectives of this work. The method is applied to a deep space formation example, where the uncertainty in spacecraft fuel masses is also considered.

Keywords: Formation Flying, Multi-spacecraft, Robust Control, Co-operative control.

1. INTRODUCTION

Formation Flying (FF) of multiple spacecraft poses significant research issues for future NASA and ESA missions. Due to limitations of launch vehicle fairing sizes and of the ability to phase optical elements over long distances on flexible structures, separated spacecraft formation flying is the only viable means to enable imaging at micro-arc-second resolution. Several NASA and ESA missions, with high priority science objectives that exploit formation flying technology in the next two decades include Terrestrial Planet Finder, Stellar and Planet Imager and Life Finder missions.

Spacecraft in Formation is mainly a load-sharing control problem when the spacecraft try collaboratively to control the relative distances and angles among them. Typically FF approaches avoid the load-sharing problem by moving only one spacecraft at a time (e.g. leader/follower, cyclic architectures, etc.). Moving all the satellites at the same time has additional challenges as interactive loops and stability problems. In fact non-collaborative controllers at every spacecraft can only be applied with reduced bandwidth objectives to preserve stability.

This paper deals with the problem of control of multiple collaborative spacecraft in a formation -with no ground intervention. The paper focuses on the theory needed to design autonomous and collaborative control strategies to govern the relative distances among satellites, sharing the load according to frequency specifications. The problem is solved by combining Load-Sharing control theories (LS) and the Quantitative Feedback Theory (QFT), both in the frequency domain approach.

A coordinate load-sharing control structure for spacecraft in formation and a methodology to deal with their dynamic models with slow time-varying and uncertain parameters are the main objectives covered for this work.

The paper also applies the methodology to a 3 degrees-of-freedom (DOF) deep space problem, where uncertainty in spacecraft fuel masses is also considered.

The model of FF spacecraft in deep space is presented in Section 2. Section 3 provides an example to show the limitation of non-collaborative control in FF. Sections 4 and 5 introduce a coordinate load-sharing control structure for spacecraft in formation, its main equations and the related controller's synthesis methodology. Section 6 applies the methodology to a deep space formation example. Section 7 summarizes the conclusions.

2. MODEL OF FF SPACECRAFT

The non-linear equations of motion of a formation consisting of n spacecraft (s/c) were derived via Lagrange's equations by Ploen *et al* (2004). For low Earth Keplerian orbit, with a slow variation of the orbit radius of the center of mass of the formation R_0 (which means $\dot{R}_0 \approx 0;\ \ddot{R}_0 \approx 0$), the relative equations of motion of a spacecraft j relative to a spacecraft i are (see Fig. 1),

$$\left(\ddot{x}_j - \ddot{x}_i\right) - 2\ \omega_0\ \left(\dot{y}_j - \dot{y}_i\right) - 3\ \omega_0^2\ \left(x_j - x_i\right) = \frac{Q_{xj}}{m_j} - \frac{Q_{xi}}{m_i} \quad (1)$$

$$\left(\ddot{y}_j - \ddot{y}_i\right) + 2\ \omega_0\ \left(\dot{x}_j - \dot{x}_i\right) = \frac{Q_{yj}}{m_j} - \frac{Q_{yi}}{m_i} \quad (2)$$

$$\left(\ddot{z}_j - \ddot{z}_i\right) + \omega_0^2\ \left(z_j - z_i\right) = \frac{Q_{zj}}{m_j} - \frac{Q_{zi}}{m_i} \quad (3)$$

where m_i and m_j are the mass of the i and j s/c respectively, $\omega_0^2 = \mu / R_0^3$, μ = 3.9860 10^5 Km^3/s^2 denotes the gravitational parameter of the Earth, R_0 and ω_0 are slow time-varying functions, and

$$Q_i = \left[Q_{xi}, Q_{y_i}, Q_{z_i}\right]^T$$

$$Q_{xi} = \vec{F}_i \cdot \vec{O}_1,\quad Q_{yi} = \vec{F}_i \cdot \vec{O}_2,\quad Q_{zi} = \vec{F}_i \cdot \vec{O}_3$$

where, $\vec{F}_i$ is the resulting force acting on the s/c[i].

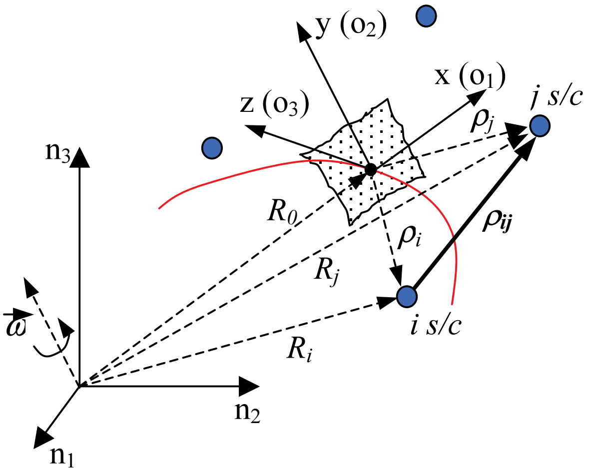

Fig. 1. Spacecraft FF. Orbital geometry.

Grouping the variables, the relative distances between two spacecraft are:

$dx_{ji} = x_j - x_i$; $dy_{ji} = y_j - y_i$; $dz_{ji} = z_j - z_i$;

and the control signals are:

$$U_{xji} = \frac{Q_{xj}}{m_j} - \frac{Q_{xi}}{m_i};\ U_{yji} = \frac{Q_{yj}}{m_j} - \frac{Q_{yi}}{m_i};\ U_{zji} = \frac{Q_{zj}}{m_j} - \frac{Q_{zi}}{m_i}$$

The origin of the load-sharing problem in the formation are the variables U_{xji}, U_{yji} and U_{zji}, where every one has two components: Q_j and Q_i, one from each spacecraft. Applying the Laplace Transform, equations (1) to (3) are written as,

$$\begin{bmatrix} dx_{ji} \\ dy_{ji} \\ dz_{ji} \end{bmatrix} = \begin{bmatrix} \frac{1}{s^2+\omega_0^2} & \frac{2\ \omega_0}{s\ (s^2+\omega_0^2)} & 0 \\ \frac{-2\ \omega_0}{s\ (s^2+\omega_0^2)} & \frac{s^2 - 3\ \omega_0^2}{s^2\ (s^2+\omega_0^2)} & 0 \\ 0 & 0 & \frac{1}{s^2+\omega_0^2} \end{bmatrix} \begin{bmatrix} U_{xji} \\ U_{yji} \\ U_{zji} \end{bmatrix} \quad (4)$$

which is a 2x2 MIMO system (axes X and Y) plus a SISO system (axis Z) –Low Earth Orbit-. If R_0 tends to infinity (i.e. deep space case) then $\omega_0 = \sqrt{\frac{\mu}{R_0^3}}$ tends to zero, and Eq. (4) becomes,

$$\begin{bmatrix} dx_{ji} \\ dy_{ji} \\ dz_{ji} \end{bmatrix} = \begin{bmatrix} \frac{1}{s^2} & 0 & 0 \\ 0 & \frac{1}{s^2} & 0 \\ 0 & 0 & \frac{1}{s^2} \end{bmatrix} \begin{bmatrix} U_{xji} \\ U_{yji} \\ U_{zji} \end{bmatrix} \quad (5)$$

which presents a three independent SISO systems.

3. NON-COLLABORATIVE CONTROL IN FF

Control of spacecraft in a formation is mainly a load-sharing control problem, where the relative distance $[dx_{ji},\ dy_{ji},\ dz_{ji}]$ between two spacecraft (j,i) is controlled by moving both satellites at the same time $[(Q_{xj}, Q_{yj}, Q_{zj}), (Q_{xi}, Q_{yi}, Q_{zi})]$. This section introduces an example of two satellites in deep space. It shows how two non-collaborative control systems, one at each spacecraft, do not work properly when controlling the relative distance between them. The example compares the case of only one spacecraft controlling the relative distance, with the case of two spacecraft controlling it at the same time.

Let us consider two spacecraft, i and j, flying in formation in deep space. Let us also consider the plant models with uncertainty due to fuel consumption, so that,

$$P_i = \frac{-1}{m_i},\quad m_i \in [360,\ 460]\ \text{kg} \quad (6)$$

$$P_j = \frac{1}{m_j},\quad m_j \in [1350,\ 1500]\ \text{kg} \quad (7)$$

$$A = \frac{1}{s^2} \quad (8)$$

where the 3 DOF system is represented by equation (5), and the control diagram of every axis is represented by Fig. 2.

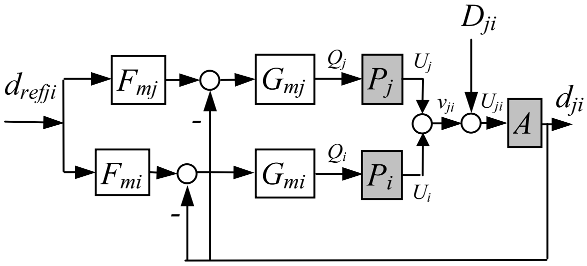

Fig. 2. Two Spacecraft. Non collaborative control diagram for every axis.

The compensators (loop compensator G_m and prefilter F_m) of every spacecraft are designed so that:

$$G_{mj}(s) = G_{mi}(s) = \frac{k_g \left(1+\dfrac{s}{0.05}\right)\left(1+\dfrac{s}{1.2}\right)}{s\left(1+\dfrac{s}{114}\right)\left(1+\dfrac{s}{114}\right)} \tag{9}$$

where $k_{gj} = 3290$, and $k_{gi} = -12600$.

$$F_{mj}(s) = F_{mi}(s) = \frac{\left(1+\dfrac{s}{30}\right)}{\left(1+\dfrac{s}{5}\right)\left(1+\dfrac{s}{7}\right)} \tag{10}$$

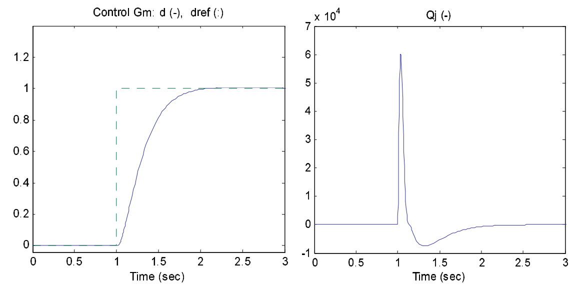

Fig. 3. One compensator. G_{mj} ON, G_{mi} OFF.
a) d_{ji} (solid line), d_{refji} (dashed line); b) Q_j

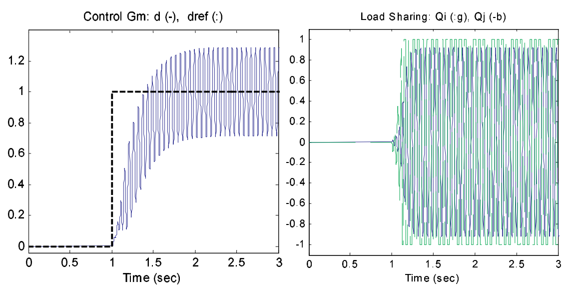

Fig. 4. Two compensators. G_{mj} ON, G_{mi} ON.
a) d_{ji} (solid line), d_{refji} (dashed line); b) Q_j and Q_i

Fig. 3 shows that if there is only one spacecraft controlling the distance with Eqs. (9) and (10) while the second one is Off, then the system is stable and works well. However, when the system tries to control the distance moving both spacecraft at the same time with the same compensators, then the result is undamped, and almost unstable (Fig. 4). Note that in both cases the bandwidth specifications are not very tight.

Because of that, and to reach more demanding control specifications, a coordinated load sharing control structure for Formation Flying is needed. It is introduced in next section.

4. LOAD SHARING CONTROL STRUCTURE

Many physical and engineering systems share the burden of controlling some signals (plant outputs) between two or more actuators. This is the case of the electrical network, where the frequency (50 or 60 Hz) is controlled by many different power stations distributed across the geography. It is also the case of the macro-economic system, where many different inputs can change the stock-prices, inflation, rates, etc. It is definitely the case of the formation-flying problem, where several spacecraft try to control the relative distances among them at the same time.

In these three cases, and also in many other systems, some special control measures must be adopted to share the load among different actuators. An excellent analysis of that problem can be found in the book *Load Sharing Control*, written by Eitelberg (1999).

4.1.- Independent Load-Sharing Control

An independent implementation of the load-sharing control problem is the simpler option, but in many cases not the best solution. Fig. 5 shows a typical independent load sharing control structure, with a common plant A, a common load D and output y; n individual plant inputs u_i, plant references r_i and noise measurements n_i.

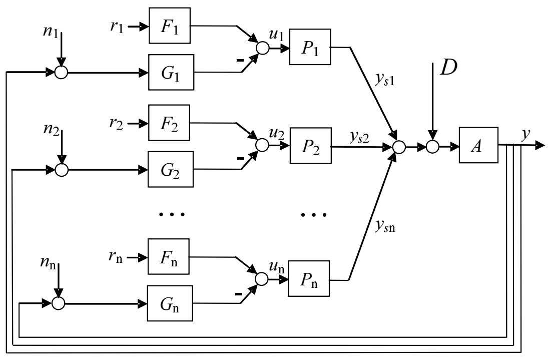

Fig. 5. Independent load-sharing control structure

As shown in Eitelberg book, such an independent structure presents significant control problems, because the pair-wise discrepancies between feedback signals errors ($N_i - N_j$) and mismatches between the filtered references $((F_i/G_i)\, R_i$ - $(F_j/G_j)\, R_j)$. Both are amplified by the supply distribution cross-sensitivity function $S_{ij} = L_i\, L_j\, /\, (P_c\, (1+L))$, where $L = L_1 + L_2$ …, and $L_i = A\, P_i\, G_i$, and finally affect the behavior of the plant output y.

4.2.- Coordinated Load-Sharing Control

To avoid the problems presented in sections 3 and 4.1, an alternative structure for coordinated load-sharing control in formation flying is presented in Fig. 6. The structure is based on a previous work introduced by Shinskey (1988), modified to take into account the special characteristic of the Formation Flying model [Eqs. (4) and (5)].

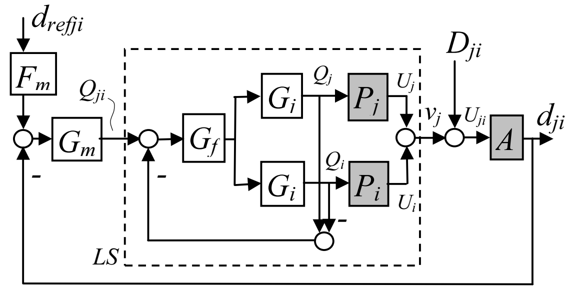

Fig. 6 Coordinate load-sharing control structure of Two S/C for every axis, where: $G_i < 0$, $P_i < 0$.

The proposed control diagram avoids the pair-wise discrepancies problems that present the independent load-sharing control structures (Figs. 2 and 5). In addition it can deal with model uncertainty and does not introduce RHP zeros, as the original Shinkey structure does, when it is applied to the Formation Flying problem. The main equations of the block diagram are,

$$d_{ji} = \frac{G_m \ LS \ A}{1+G_m \ LS \ A} F_m \ d_{ref \ ji} + \frac{A}{1+G_m \ LS \ A} D_{ji} \tag{11}$$

$$LS = \frac{v_{ji}}{Q_{ji}} = \frac{G_f \, (G_j \ P_j + G_i \ P_i)}{1+G_f \ (G_j - G_i)} \tag{12}$$

Remark

The relative distance [dx_{ji}, dy_{ji}, dz_{ji}] between every two spacecraft (j,i) is controlled tracking a reference [$drefx_{ji}$, $drefy_{ji}$, $drefz_{ji}$] and moving the two satellites at the same time [(Q_{xj},Q_{yj},Q_{zj}), (Q_{xi},Q_{yi},Q_{zi})] (Fig. 6).

The extension to multiple spacecraft needs a hierarchical structure, where a first level calculates the trajectory in terms of relative distances between two spacecraft at a time. These values are the set points for the second level (Fig. 6). From them, every pair of satellites (i, j) applies the signals [(Q_{xi}, Q_{yi}, Q_{zi}), (Q_{xj}, Q_{yj}, Q_{zj})] respectively.

5.- COMPENSATOR SYNTHESIS

The methodology to design the compensators of the proposed structure has three steps, starting from the inner compensators to the outer ones.

First the inner compensators G_j(s) and G_i(s), have to be defined. Their objective is to share the load in terms of frequency. The proposed structure gives the freedom to define the frequency response that the designer wants for every channel (i and j), regarding to fuel consumption, time requirements or other politics. For instance, if the system has two satellites with different masses, it is possible to assign to the biggest one an active response at low frequency and an attenuated response at high frequency. Similarly, for the smallest one, it is possible to assign an attenuated response at low frequency and an active response at high frequency. Thus the control structure is very general and allows defining the load sharing strategy in many ways, according to the requirements of the formation. Moreover, it is also possible to define different frequency responses for every axis (X,Y,Z) and channel if it is necessary.

The second step is the design of the G_f(s) compensator. The main objective of this compensator is to keep the sum of the two channels, Q_i(s) + Q_j(s), equal to the demand Q(s) of the outer compensator G_m(s). It is also possible to define different dynamics for every axis (X,Y,Z) if it is necessary. The Quantitative Feedback Theory (QFT) is applied in this work to design the G_f(s) compensator. The integral part of the compensators has to receive an special treatment. To avoid the channels fighting against each other, if the system needs an inner compensator with an integral part, then it is necessary to implement it in only one channel, G_j(s) or G_i(s), or better only in the inner compensator G_f(s).

The third and last step is the design of the outer compensators G_m(s) and F_m(s). The objective for these compensators is to govern the distance [dx_{ji}, dy_{ji}, dz_{ji}] between every two spacecraft (j,i) according to some previous performance specifications (tracking, disturbance rejection, stability, etc).

The design of the G_m(s) and F_m(s) compensators depends on the formation-flying scenario. If the formation is in deep space, then equation (5) governs the dynamics and there is an independent SISO problem for every axis. QFT is applied to design G_m(s) and F_m(s).

6. S/C FORMATION IN DEEP SPACE

Let us consider again the two spacecraft presented in Section 3 [Eqs. (6) and (7)], but now flying in formation in deep space Eq. (5).

6.1.- G_j, G_i Compensators

First, the inner compensators, G_j(s) and G_i(s), have to be defined. The objective is to share the load in terms of frequency. Here two transfer functions are chosen so that the biggest spacecraft P_j shows an active response at low frequency and an attenuated response at high frequency, and the smallest one P_i an attenuated response at low frequency and an active response at high frequency (see Fig. 7).

The selected G_j(s) and G_i(s) compensators are:

$$G_i(s) = \frac{-800 \ s}{\left(1+\dfrac{s}{0.628 \ 10^{0.3}}\right)\left(1+\dfrac{s}{0.628 \ 10^{2}}\right)} \tag{13}$$

$$G_j(s) = \frac{3000}{\left(1 + \frac{s}{0.628\,10^{-0.3}}\right)} \tag{14}$$

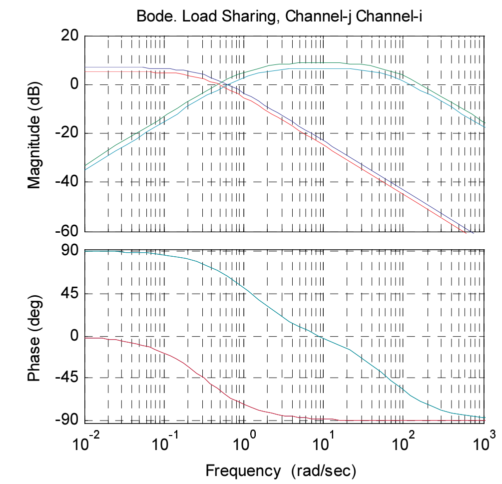

Fig. 7. [G_j(s) P_j(s)] and [G_i(s) P_i(s)] transfer functions, with fuel mass uncertainty.

6.2.- G_f Compensator

The second step is the design of the G_f(s) compensator. Its main objective is to keep the sum of the two channels, Q_i(s) + Q_j(s), equal to the output Q(s) of the outer compensator G_m(s).

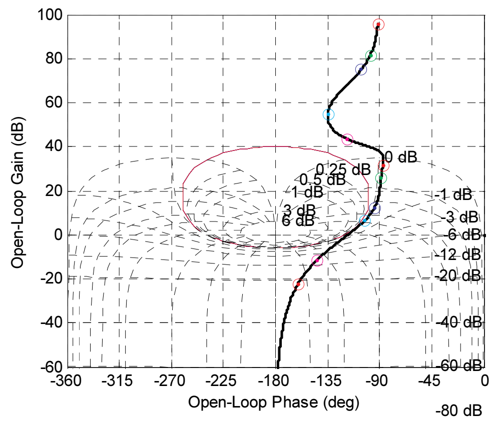

Fig. 8. Loop Shaping of G_f (G_j - G_i).

The transfer function between Q and x_1 is P_L, according to Eq. (12). The plant to be controlled by G_f is (G_j - G_i). The complexity of the design of G_f(s) is conditioned by the compensators G_j and G_i, selected previously. In this paper G_f(s) is designed using standard QFT -Eq. (15)-. The loop shaping of G_f (G_j - G_i) on the Nichols Chart (NC) is shown in Fig. 8.

$$G_f(s) = \frac{0.2\left(1 + \frac{s}{55}\right)}{s\left(1 + \frac{s}{340}\right)} \tag{15}$$

6.3.- G_m, F_m Compensators

The third and last step of the method is the design of the outer compensators G_m(s) and F_m(s). Their objective is to govern the distance d between the spacecraft according to some previous performance specifications. In this example the selected specifications are:

i. Robust Stability. $\left|\frac{G_m\, P_L\, A}{1 + G_m\, P_L\, A}\right| \le 1.1,\ \forall\omega,$

which involves a phase margin of at least 55º and a gain margin of at least 1.99 (5.9 dB).

ii. Disturbance rejection.

$$\left|\frac{1}{1 + G_m\, P_L\, A}\right| \le \left|\frac{2\,\omega_0\,(j\omega)\left(\frac{j\omega}{8\,\omega_0} + 1\right)}{\left(\frac{j\omega}{\omega_0} + 1\right)\left(\frac{j\omega}{4\,\omega_0} + 1\right)}\right|,$$

where ω_0 = 12 rad/sec, and for ω=[0.001,0.01,0.05,0.1,0.5,1,5,10,50,100,500,1000]rad/s.

iii. Tracking specifications.

$$T_{R_L}(\omega) \le \left|\frac{G_m\, P_L\, A}{1 + G_m\, P_L\, A}\right| \le T_{R_U}(\omega),$$

where,

$$T_{R_U}(\omega) = \left|\frac{\left(\frac{\omega_n^2}{a}\right)(j\omega + a)}{(j\omega)^2 + 2\,\zeta\,\omega_n\,(j\omega) + \omega_n^2}\right|$$

$$T_{R_L}(\omega) = \left|\frac{1}{\left(\frac{j\omega}{\sigma_1} + 1\right)\left(\frac{j\omega}{\sigma_2} + 1\right)\left(\frac{j\omega}{\sigma_3} + 1\right)}\right|$$

and where,
ω_n = 9 ; a = 30 ; σ_1 = 2 ω_n ; σ_2 = 0.3 ω_n ; σ_3 = ω_n ; for ω=[0.001,0.01,0.05,0.1,0.5,1,5,10,50,100,500,1000]rad/s.

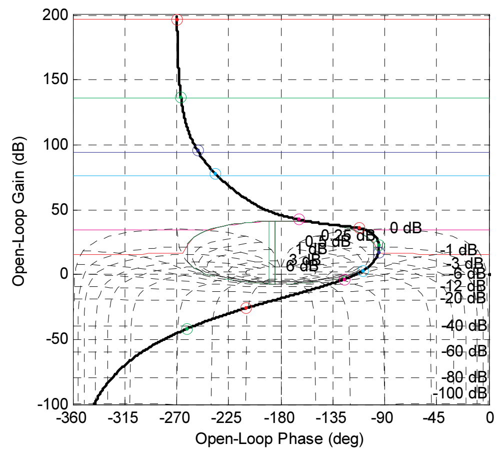

Fig. 9. Loop Shaping of G_m P_L A

Using standard QFT, the G_m(s) compensator and the prefilter F_m(s) -Eqs. (16),(17)- are designed. Fig. 9 shows the QFT loop shaping on the NC.

$$G_m(s) = \frac{10800\left(1+\frac{s}{0.15}\right)\left(1+\frac{2\ 0.6}{1.08}s+\frac{s^2}{1.08^2}\right)\left(1+\frac{2\ 0.6}{280}s+\frac{s^2}{280^2}\right)}{s\left(1+\frac{s}{0.5}\right)\left(1+\frac{s}{250}\right)\left(1+\frac{s}{900}\right)\left(1+\frac{s}{900}\right)} \tag{16}$$

$$F_m(s) = \frac{\left(1+\frac{s}{30}\right)}{\left(1+\frac{s}{5}\right)\left(1+\frac{s}{7}\right)} \tag{17}$$

6.4.- Simulation Results

This section applies the previous design to control the 3 DOF distance between two spacecraft in formation in deep space, moving both spacecraft at the same time. Three unit step inputs at the references of the three channels, X, Y, Z, are applied at different times: t = 5, t = 10 and t = 15 sec respectively. Fig. 10 shows a good tracking control with more demanding specifications than in case of Figs. 3, 4.

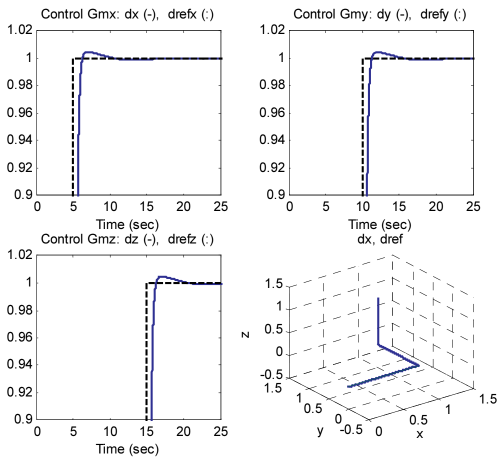

Fig. 10. Tracking in channels X, Y and Z, and 3D

Once the 3 DOF reference is reached, a sinusoidal disturbance of unit amplitude and f = 70 rad/sec is applied to axis X from t = 30 sec to t = 40 sec. Fig. 11 shows how the spacecraft i (the fastest one) carries with most of the load, and the spacecraft j (the slowest one) does not carry with load.

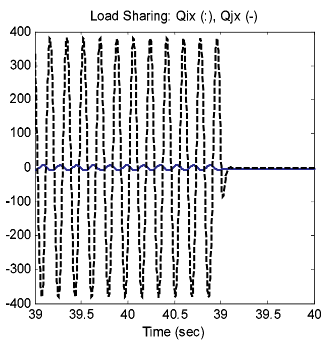

Fig. 11. Load sharing: Q_{xi} (dashed), Q_{xj} (solid)

Similarly, a sinusoidal disturbance of unit amplitude and f = 0.05 rad/sec is applied to axis Y from t = 60 sec to t = 180 sec. Fig. 12 shows how the spacecraft j (the slowest one) carries with most of the load, and the spacecraft i (the fastest one) does not. Both agree with the design.

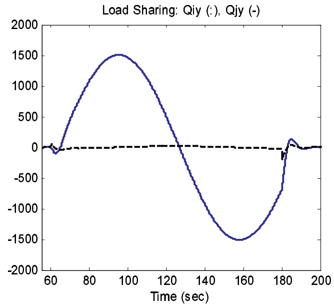

Fig. 12. Load sharing: Q_{yi} (dashed), Q_{yj} (solid)

7. CONCLUSIONS

This paper introduced the design of an autonomous and collaborative control strategy to govern the relative distances among spacecraft in formation. A coordinate load-sharing control structure for formation flying that shares the load between spacecraft according to frequency specifications and a methodology to control their uncertain dynamic models were presented. The method was applied to a deep space formation example with fuel masses uncertainty.

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