ACTIVE VIBRATION DAMPING OF A SMART FLEXIBLE STRUCTURE USING PIEZOELECTRIC TRANSDUCERS: $\mathcal{H}_\infty$ DESIGN AND EXPERIMENTAL RESULTS

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Abstract: This paper deals with active vibration control of a plate like smart flexible structure. This plate is equipped with several thin piezoelectric patches. Some of them are used as sensors and the others as actuators. They are optimally positioned and not collocated. The main goal of control is to reduce the most energetic vibrating modes. By using a state-space representation of a *MIMO* model of the equipped structure, derived from *Finite Elements Modeling* and modal analysis, a synthesis setup is derived to design an $\mathcal{H}_\infty$ controller. The resulting controller is reduced and tested experimentally.

Keywords: Active vibration damping, Robust control, Experiment results, Smart structure

1. INTRODUCTION

Smart structures have occupied a major place in the control research area during the last two decades. They find their utility for example in aerospace and civil engineering for the capability they bring to modify the structure geometry and/or physical properties (Kajiwara and Uehara, 2001). Their adaptative nature to external stimuli makes them the best candidates in vibration control applications. Structural control systems by using piezoelectric materials have shown to be effective in vibration suppression. Nevertheless, achieving an active vibration control is still a very complex problem, particulary for flexible structures which have lightly damped modes with closely spaced resonant frequencies. One of the reasons leading to consider the robust control tools is the difficulty in obtaining a model which correctly represents the real dynamical behaviour of the structure (Balas and Doyle, 1994). Although some sophisticated technics, such as finite element modeling, lead to good quality models, the real behaviour can be very different when this structure is connected with an other one due to dynamical coupling or ill realised boundary conditions. Moreover, when a control law has to be implemented in real time, several limitations such as saturation of voltage amplifiers or signal levels adaptations must be taken into account. Often the designed controller has to be retuned.

An application of a $\mathcal{H}_\infty$ robust control law will be presented for the active vibration damping of such a plate like smart structure.

The paper is organized as follows. The considered problem is described in section 2 and the main features of the experimental smart structure are given. Section 3 introduces the obtained numerical model. Section 4 details the control procedure design for robust active damping. In section 5, simulations results are examined through different figures. Section 6 shows the experimental results of the obtained $\mathcal{H}_\infty$ controller and the paper ends with some final observations.

2. PROBLEM DESCRIPTION

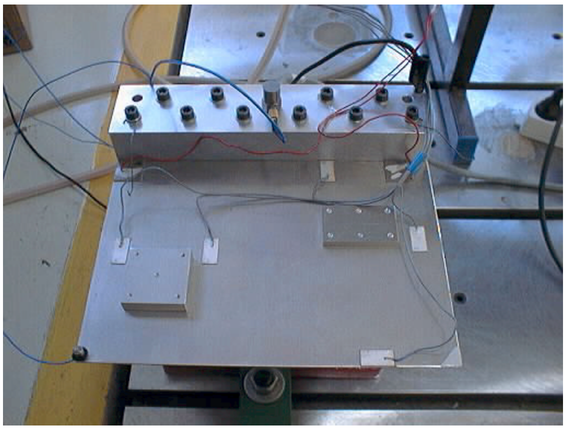

Fig. 1. experimental device

This paper is concerned with the active vibration damping of the plate like structure described by the photo of Figure 1. The frequency range of interest is $[0-2000]$ Hz. This structure is an aluminium based. The dimensions are $230 \times 160 \times 1.6$ millimeters. The plate is clamped into a stiff movable support at one edge. The movable support is linked to a mechanical vibrator which can move the structure as desired in the vertical direction. It is used to generate a perturbation. The perturbation is monitored by an accelerometer located at the center of the movable support. Two aluminium based components, thick and stiff, with different dimensions, are fixed on it, as represented on Figure 1, in order to brake the structure geometrical symmetry. For actuation and sensing purposes, small piezoelectric patches in piezoceramic lead zirconite titanate (PZT) are bonded on the plate. Their dimensions are $20 \times 20 \times 0.7$ millimeters for the actuators and $20 \times 10 \times 0.4$ millimeters for the sensors. They have been optimally positioned in order to increase both modal controllability and observability by using the criteria proposed in (Leleu *et al.*, 2000). The structure has been equipped with 2 piezoelectric patches used as actuators and 5 used as sensors. Sensors and actuators are voluntary not collocated and bounded in one face. The resulting equipped structure is very singular in comparison with other usual smart structures (Moheimani, 2001). The corresponding control problem is a hard challenge as explained in (Smith *et al.*, 1994). A small accelerometer is bonded at the plate corner on the left of the photo in figure 1. It is used to measure the efficiency of the active control.

In (Tliba and Abou-Kandil, 2003), it has been shown that the first bending mode is the most energetic vibrating one. It represents nearby 95 percent of the vibrating energy when the structure is excited by a white noise. We shall present a control strategy in order to damp this mode by combining the action of both actuators.

3. NUMERICAL MODEL

Finite element modeling associated with modal analysis permit to get the input-output frequency response between actuators and sensors as well as between the perturbation and the acceleration of any point of the structure. The model is obtained in the state-space form:

$$\begin{cases} \dot{x} = Ax + B_w w + Bu \\ z_p = C_p x \\ y = Cx \end{cases} \tag{1}$$

where x is the state-vector containing the modal coordinates, y is the five dimension output vector measuring the voltage of each sensor, and z is the position of the point coinciding with the small accelerometer relatively to its undeformed position frame. The two dimensional vector u is the input corresponding to the voltage applied to each actuator and w is a scalar input corresponding to the clamped edge's acceleration. The units are those of the international system. The matrices A,B,B_w,C_p and C depend on geometrical and physical properties of the mechanical structure and the piezoelectric material.

The obtained model of our equiped structure is of order 44. It represents the 20 first vibrating modes for each input-output transfer. All of them are statically corrected with a 2^{nd} order dynamic. The mode shape and resonant frequency of the 4 first modes are presented in figure 2.

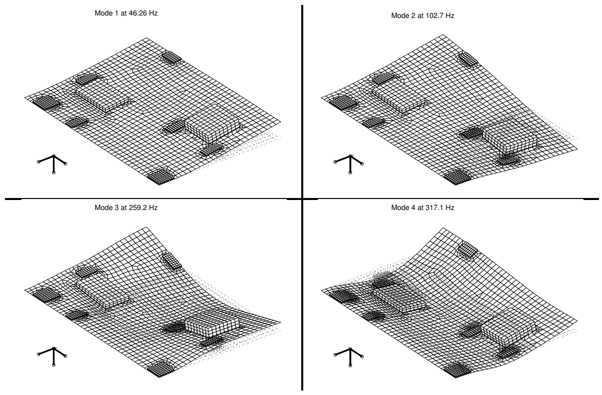

Fig. 2. 4 first modes shapes

In the sequel, some results are presented when using only one sensor and two coupled actuators as a single input. It means that the two actuators are controlled with one signal. The controller is so a SISO system. This is made possible thanks to the actuators positionning, optimally placed to control the first modes (Formosa, 2002). The concerned sensor is the $20 \times 10 \times 0.4mm$ piezoelectric patch at the clamped edge.

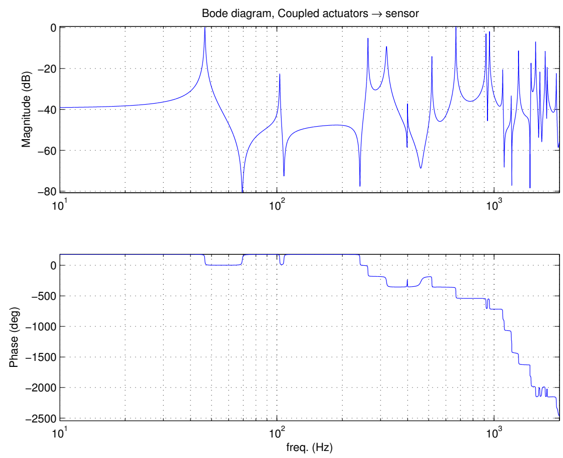

Fig. 3. Actuators to sensor voltage frequency response

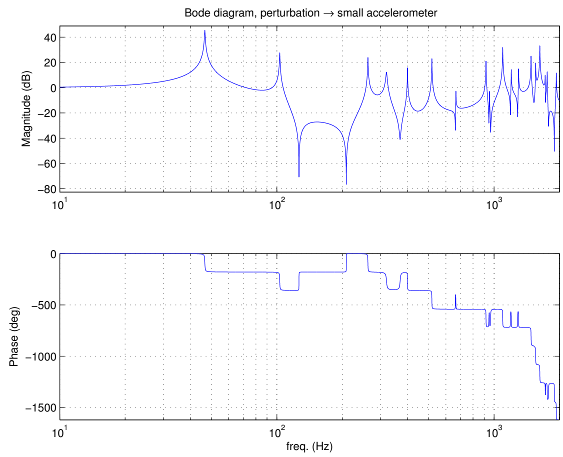

Fig. 4. Accelerometric frequency response

The numerical Bode frequency response between the coupled actuators voltage and the sensor voltage is given in figure 3 and those between the perturbation acceleration and the small accelerometer measure is given in figure 4.

4. CONTROLLER DESIGN PROCEDURE

The particularity of the active vibration damping control problem is its multiobjective feature. The main control objectives are summarized thereafter and regrouped into a standard setup for the robust performance problem described in Figure 5.

The 3 first modes have been choosen to be concerned by the active damping and the control

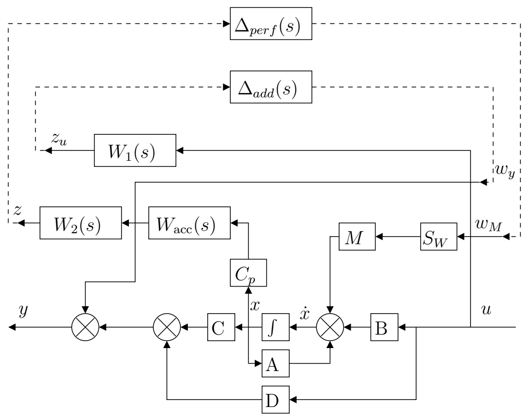

Fig. 5. standard setup for robust active damping

design procedure must permit to decrease mainly the first resonance peak in the *accelerometric frequency response*. To this end, the model is reduced via a residualization method to take into account the 3 first modes dynamic. This synthesis model is then of order 6. This reduced model allows to alleviate the optimisation process and the final controller dimension. This is an appreciable property for experimental purposes.

The z_p output is filtered by a double derivative filter $W_{\rm acc}(s)$ leading to the acceleration signal of the plate corner. The resulting output is weighted by a two order low pass filter $W_2(s)$ cutting at 280 Hz so as to specify the level of damping to obtain.

A perturbation input w_M is introduced to act as a particular state noise perturbing the state dynamic. This state noise has an input direction modeled by a matrix M. This is the modal damping factor uncertainties input direction into the A matrix as presented in (Alazard, 2002) and applied in (Tliba and Abou-Kandil, 2003). This technique has the advantage to consider the parametric uncertainties as some state noises affecting the A matrix. A diagonal scaling matrix S_W allows to weight each modal damping factor relatively to its importance.

The robust performance is specified by the minimization of the $\mathcal{H}_\infty$ norm on the transfer function between the perturbation w_M and the output z.

The controller also have to ensure robustness against the neglected dynamics in order to prevent the high frequencies modes spill-over phenomenon. Due to the closeness of the resonant mode frequencies, poor performances are sought for the closed loop damping factor of the mode closest to the crossover frequency region, say the third flexible mode. This can be improved by adjusting the scalar of matrix S_W weighting the third flexible mode.

The neglected dynamics between the full order and the reduced model have been modeled by an additive uncertainty $\Delta_{\rm add}(s)$. They are taken

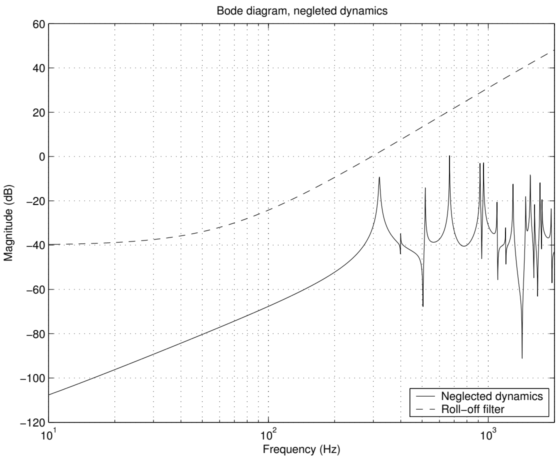

Fig. 6. Neglected dynamics shaping

into account by adding in the setup of Figure 5 the input w_y and the output z_u with $w_y = \Delta_{\text{add}}(s)\, z_u$. The corresponding weight $W_1(s)$ is chosen such that $\|\Delta_{\text{add}}(s)\, W_1(s)\|_\infty \leq 1$. After several tests, a high pass filter has been tuned with a cutting frequency at 295 Hz and of order 3, weighting the control signal as shown in figure 6. This filter penalizes high frequency efforts on the actuators by imposing a 60 dB per decade roll-off specification on the control signals u.

One should also be careful that the control signal applied to the piezoelectric actuators must not exceed ± 500 Volts, for all admissible disturbances affecting the clamped edge. This constraint impose to tune the low frequency magnitude of the $W_1(s)$ roll-off filter so that the low frequency magnitude of the resulting controller remains acceptable. The perturbation is characterized by its frequency spectrum. Indeed, its Power Spectral Density is nearby $0.001 g^2/Hz$ in the frequency range $[0 - 2000Hz]$. This perturbation is modeled by a low-pass filter of order 1, represented in Figure 7.

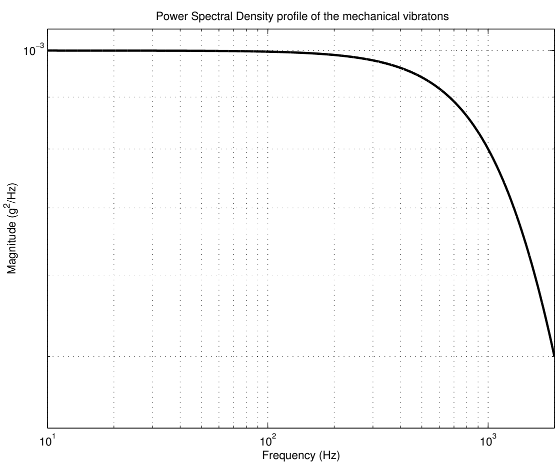

Fig. 7. Power Spectral Density of the mechanical vibrations

The augmented plant of Figure 5 is of order 13.

The formulation of the $\mathcal{H}_\infty$-control proposed considers all the additive uncertainties as some perturbations and the robustness objectives as performance objectives. The controller is obtained by solving the optimal $\mathcal{H}_\infty$ control problem :

$$\begin{aligned} \gamma &= \|\mathcal{F}_l\{P(s), K(s)\}\|_\infty \\ K(s) &= \arg \min_{K(s) \in \mathcal{RH}_\infty} \gamma \end{aligned} \qquad (2)$$

where, $\mathcal{F}_l\{P(s), K(s)\}$ denotes the closed loop transfer matrix between the two vectors $[w_y, w_M]^T$ and $[z_u, z]^T$.

To solve problem (2), a $\gamma-$iteration algorithm (Doyle *et al.*, 1989) is performed [1].

5. NUMERICAL RESULTS AND ANALYSIS

The optimized controller is of order 13. It has been reduced to order 8 by using a balanced reduction method (Enns, 1984). The figure 8 shows

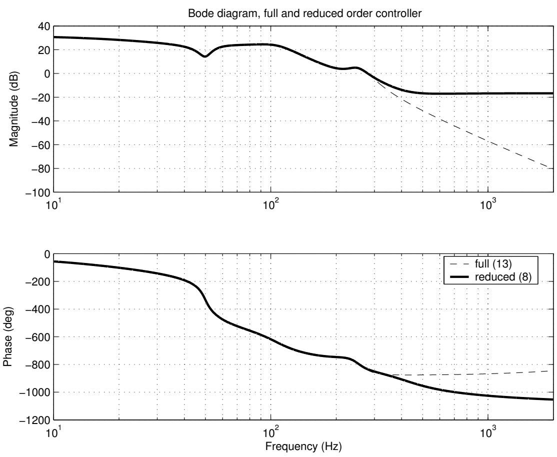

Fig. 8. Full and reduced order controller Bode diagram

the corresponding Bode diagram. The best value obtained for γ is $\gamma_{\text{opt}} = 81.4$. This value proves the difficulty to manage the trade-off between the robustness against neglected dynamics and the requested damping level. The reduced controller has been applied to the full order model. Figure 9 shows the eigenvalues locus between open and closed loop. The crosses $\times$ and the plus $+$ correspond respectively to the open and the closed loop system eigenvalues. The circles $\circ$ are the open loop system transmission zeros and the squares $\square$ are the controller open loop eigenvalues. The closed loop stability is achieved since the neglected dynamics did not move.
Figure 10 shows that the first vibrating mode has been damped by 18.6 dB. The second and the third modes have been damped by 3 to 4 dB.

[1] *hinfsyn* command of the *Matlab*, *$\mu-$analysis and synthesis toolbox* (Balas *et al.*, 1991)

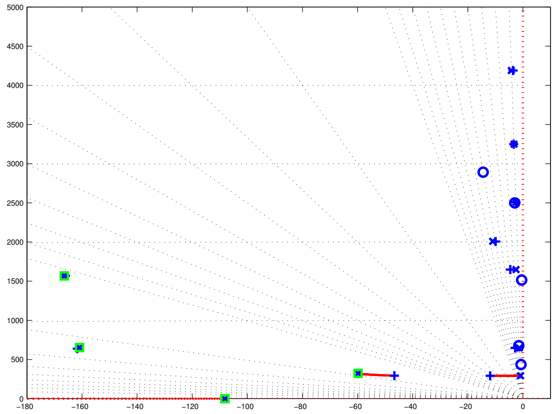

Fig. 9. Eigenvalues locus for the full open loop model

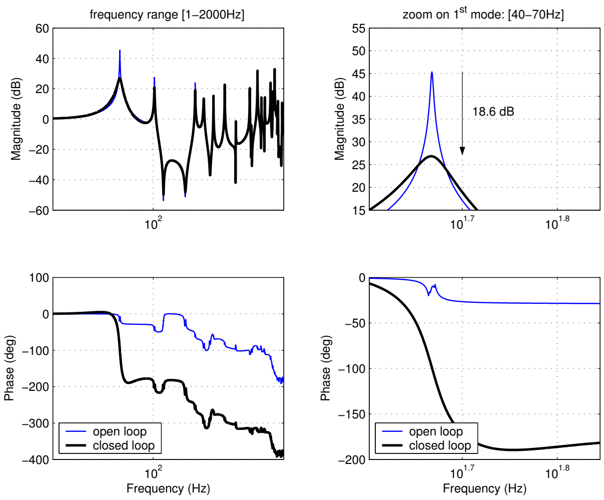

Fig. 10. closed loop performance vs. open loop

6. EXPERIMENTAL TEST

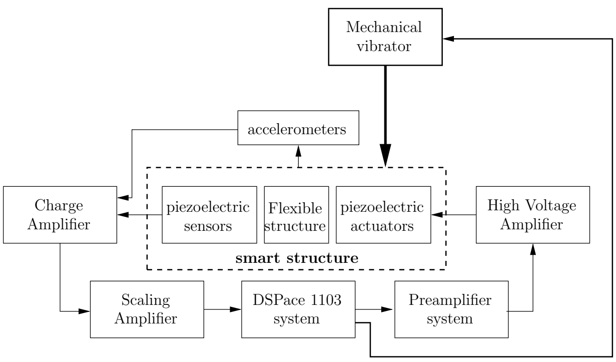

Fig. 11. Experimental setup

The experimental device is described in Figure 11. This makes possible the fast prototyping controller. The 8 order controller has been implemented into a DSP board DS1103 from dSPACE via a SIMULINK diagram. The sampling frequency has been set to 20000 Hz. We used two charge amplifiers to get the piezoelectric sensor voltage and the accelerometer signals. Both piezoelectric and accelerometric signals are scaled using a multichannel preamplifier. A high voltage amplifier of gain 100 has been used to drive the two coupled actuators. The vibrator has been fixed onto a vibration plateform and is driven by the DSP board. Figure 13 shows the open loop

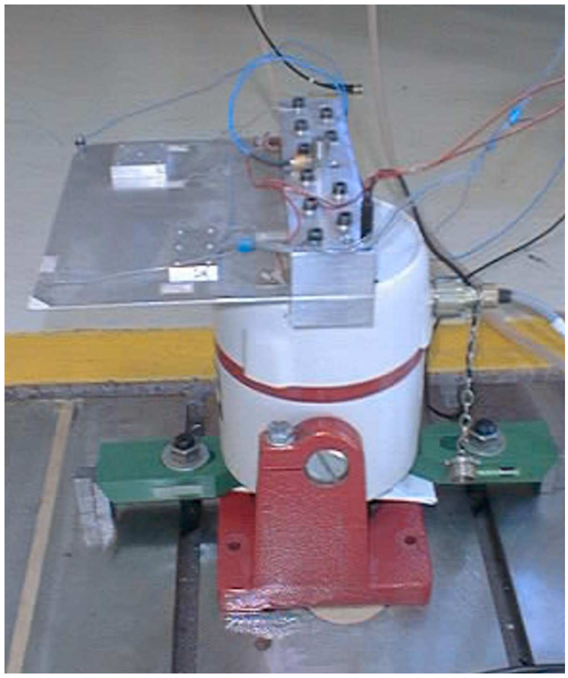

Fig. 12. Smart structure on vibrator

frequency response from the coupled piezoelectric actuators to the piezoelectric sensor, when the structure is fixed on the vibrateur (red plot) and when it is fixed alone on the vibration plateforme (blue plot). The blue plot is very similar to the one obtained by finite element method. The comparison of the red and the blue plots underlines the dynamical coupling phenomenon. Indeed, the resonant frequencies are slightly different. In the red plot, there is a resonant frequency at about 33 Hz associated with the mechanical vibrator structure which is neither present in the blue plot

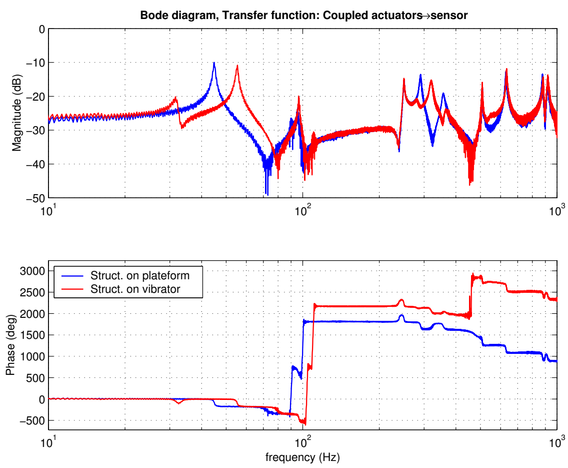

Fig. 13. Frequency response: coupled actuators to sensor

or in the numerical model. This dynamic was not taken into account in the numerical model even though it is apparently controlable by the coupled piezoelectric actuators and observable by the sensor. Moreover, the resonant frequency associated with the plate structure first bending mode is at 54 Hz instead of 46 in the numerical model presented before, say a difference of 17%. The second and third modes are about 6.5% different from the numerical model. The consequences of these differences is obvious in the closed loop performances. The experimental open and closed loop accelerometric frequency responses are shown in figure 14. The experimental gain on the first mode resonant frequency peak is about 3 dB instead of 18.6 in the simulation results.

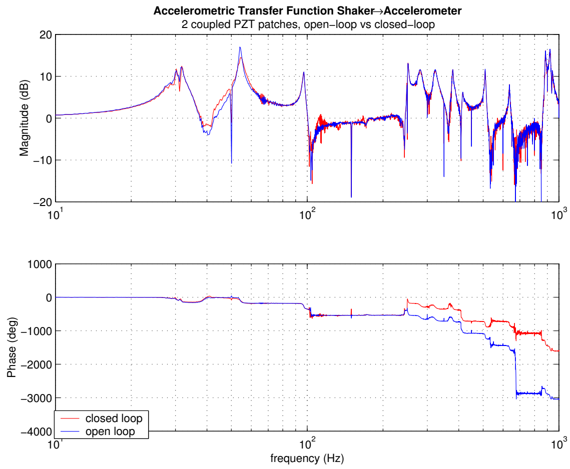

Fig. 14. Experimental accelerometric frequency response

7. CONCLUSION

The results presented in this paper confirm several interesting points. First, the active vibration damping of plate like flexible structures can be efficiently experimented, although the experimental results do not match with the simulations. The use of several, but electrically coupled, piezoelectric patches as a single actuator has been both simulated and experimented successfully. Then, the robust control method we proposed have given good simulated results. The study of the parametric robustness of the performances in relation with frequency and damping factor uncertainties was promising, it was not presented in this paper by lack of space. But in presence of structure dynamical coupling, the performances become degraded. The unmodeled dynamic of the mechanical vibrator perturbed significantly the closed loop properties. This emphasizes the inevitability of the controller retuning or the model update. The robustness of performances theme for such problems is still a topic of research.

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