IDENTIFICATION OF AN LPV VEHICLE MODEL BASED ON EXPERIMENTAL DATA FOR BRAKE-STEERING CONTROL

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Abstract: A physically parameterized continuous-time velocity-scheduled LPV state-space model of a heavy-truck is identified from measurement data. The aim is to develop a model for controller which steers the vehicle by braking either the one or the other front wheel. It can be applied in many vehicles, where the sole possibility to automate the steering in emergency situations, like e.g. unintended lane departure, is the application of the electronic brake system. Such steering controllers usually require the prediction of the yaw rate and the steering angle on every possible velocity. This problem defines the requirements for the model. Four different order model structures are derived from a certain physical description. Assuming state and output noise, all of them are identified in parameter-varying observer form using prediction error method. The quadratic criterion function is composed from measurement data of several different experiments. Each experiments are carried out on constant velocities but the cost is constituted from different velocity experiments. That structure is selected for controller design which has the best cost on test data out of those the poles of which are in the control bandwidth. The poles are defined on constant velocity. The resulted nominal model consists of the feedback connection of the yaw dynamics with one state-variable and the steering system dynamics with two states and of a first order actuator dynamics with time-delay. The predicted outputs show a good fit to the measurements.

Keywords: LPV identification, steering dynamics, differential braking, unintended lane departure

1. INTRODUCTION

In the most common vehicles, where no electronic steering system is available but the braking is controlled by onboard computers, the only way to automate or assist steering is the use of the electronic brake system, the application of individual or unilateral wheel brakes. There are many papers concerning different approaches that develop steering by the braking systems. In (Ackermann *et al.* 1998) active steering and individual wheel braking is compared in yaw and roll control point of view. A method for unilateral braking for rollover avoidance can be found in (Gáspár *et al.* 2003*a*, Gáspár *et al.* 2003*b*), for avoiding unintended lane departure in e.g. (Kovács *et al.* 1998, Pilutti *et al.* 1995). The model presented in this paper is designed for this latter application. In the latter two works local linear models valid on a single constant velocity were applied for control design. In both works the steering effect was caused by the moment due to the brake forces and half width of the vehicle as force arm. But in case of some trucks (and load distribution) this has little effect and in general requires heavy braking.

In the present paper only the first wheels are braked by small ($\leq 1.2bar$) pressure in order to turn aside the steering system. The goal of a steering controller and its performance can be expressed for example in terms of the yaw rate and the steering angle. Thus the requirements for the model are given: it has to estimate the yaw rate and the steering angle on the whole range of velocities when applying asymmetric front wheel braking.

The dynamics of a vehicle is nonlinear in general, but steering dynamics is well captured in normal driving situations by linear models that depend on the velocity (Mitschke 1990, Kiencke and Nielsen 2000). For these models linear parameter-varying (LPV) controllers can be designed by solving linear matrix inequalities guaranteing stability and performance for the closed-loop on the whole operating region of the scheduling parameters. Our aim in this paper is to identify an LPV model, where the scheduling parameter is the velocity.

The paper is organized as follows. After explaining the experimental conditions the derivation of the physical model of the vehicle is presented in section 3. Then the steps of identification are detailed in separate sections: choice of model set and parameterization, identification method and model validation.

2. EXPERIMENTAL CONDITIONS

The experimental vehicle is an MAN truck (F2000 26.403 Silent), the third axle is lifted up. The front axle is equipped with disk breaks. The electronic brake system is EBS 2.3. The steering gear of the vehicle is hydraulic power assisted ball and nut type of ZF-Servocom 8098. During the data acquisition the vehicle is driven on a 10m wide dry asphalt road with constant velocity. The front wheels are braked alternately by an operator at a computer connected to the EBS system through Controller Area Network (CAN) bus. The operator tries to produce a pseudo random binary (PRBS) signal, Δp_c (for commanded brake pressure difference), in order to have a large energy excitation on wide bandwidth. The experiments are repeated on different velocities. The brake pressures (p_l for front left, p_r for front right and $\Delta p := p_r - p_l$) in the brake cylinders are measured by sensors. The threshold pressure required the brake pad to be in contact with the disk is about 0.08 bar. The driver releases the handwheel so the handwheel is turning according to the turning of the steered wheels. The handwheel angle (δ_m) is measured by a sensor mounted onto the upper steering column. The handwheel angle is scaled with the steer-transmission factor i_s: $\delta_m := \frac{\delta_m}{i_s}$ and is redefined in this way in the following. The yaw rate r is also measured. The forward velocity v is computed as the average of the rotational equivalent velocities of the driven rear wheels. The difference of the rotational equivalent velocities of the braked front wheels is denoted by Δv_R. The sampling-time is given: $T_s = 10ms$.

3. MODELLING

The system from the commanded pressure difference Δp_c to the measurable outputs $y = \left[\, r \;\, \delta_m \;\, \Delta v_R \,\right]^T$ is described as a serial connection of two systems

$$\Delta p = G_1 \Delta p_c + \nu_1$$
$$y = G_2 \Delta p + \nu_2,$$

where ν_1 and ν_2 are some disturbances. The first system G_1, can be considered as actuator dynamics, consist of some delay due to the software timings in the computer, CAN and EBS and an unknown and complex software-controlled mechanic-pneumatic dynamics of the brake system.

The dynamic equations of the second system G_2 can be derived as follows. First nonlinear equations are written from force and torque balance equations for the chassis, the steering system and the wheels. The following modelling paradigm is applied. The center of gravity of the vehicle is sunk down onto the surface of the road. Thus the variation of tire load, the suspension dynamics, the coupled roll, pitch and heave motions are not modelled. The plain of wheels are vertical. The wheel casters $n_{RK} = n_R + n_K$ are negligible as compared to the distance between the wheels and the center of gravity (CG), but significant as the force arm of the aligning torque $n_{RK} F_y$ (F_y denotes the lateral wheel force). The tire-road adhesion model is static and empirical. The rolling resistance and drag are neglected.

The brake-steering controllers are designed for normal driving situations with lateral acceleration less then 0.4g, so linearized models are appropriate (Mitschke 1990). The next step is the linearization of equations around the straight driving state using first order Taylor-series approximation. During the brake-steering just small brake pressure of about 1 bar is applied. Therefore the direct turning effort of the brake force (through the half vehicle width as force arm) to the chassis can also be neglected and the adhesion function $\mu(s)$ can be approximated in the region of small slip values s as $\mu(s) \approx \bar{\mu}s$. For the driver releases the handwheel (driver torque T_d is zero), the fast dynamics of the upper steering column and handwheel is replaced with its DC component.

The block scheme of the linearized vehicle model can be seen on Figure 1. The final equations can be written as

$$\begin{aligned}
\dot{x} &= A_0x + B_0u + w, \\
y &= C_0x + \nu_2 \qquad (1)\\
x &= \begin{bmatrix} \beta & r & \delta & \dot{\delta} & \Delta v_R \end{bmatrix}^T, \\
u &= \Delta p,
\end{aligned}$$

$$A_0 = \begin{bmatrix} \frac{a_{11}}{v} & -1 + \frac{a_{12}}{v^2} & \frac{a_{13}}{v} & 0 & 0 \\ a_{21} & \frac{a_{22}}{v} & a_{23} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ a_{41} & \frac{a_{42}}{v} & a_{43} & a_{44} & \frac{a_{45}}{v} \\ 0 & \frac{a_{52}}{v} & 0 & 0 & \frac{a_{55}}{v} \end{bmatrix},$$

$$B_0 = \begin{bmatrix} 0 & 0 & 0 & 0 & b_1 \end{bmatrix}^T, C_0 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

where w is the effect of exogenous disturbances and neglected dynamics and

$$\begin{aligned}
a_{11} &= -\frac{c_f + c_r}{m}, & a_{12} &= \frac{c_r l_r - c_f l_f}{m}, \\
a_{13} &= \frac{c_f}{m}, & a_{21} &= \frac{c_r l_r - c_f l_f}{J_z}, \\
a_{22} &= -\frac{c_r l_r^2 + c_f l_f^2}{J_z}, & a_{23} &= \frac{c_f l_f}{J_z}, \\
a_{41} &= \frac{n_{RK} c_f}{J_s}, & a_{42} &= \frac{n_{RK} c_f l_f - 0.5 r_s c_f l_w}{J_s}, \\
a_{43} &= -\frac{n_{RK} c_f}{J_s}, & a_{44} &= -\frac{k_s}{J_s}, \\
a_{45} &= \frac{0.5 r_s c_f}{J_s}, & a_{52} &= \frac{0.5 c_f r_{eff}^2 l_w}{J_w}, \\
a_{55} &= -\frac{0.5 c_f r_{eff}^2}{J_w} & b_1 &= -\frac{C_{pT} r_{eff}}{J_w}.
\end{aligned}$$

The physical parameters are l_f, l_r distances between CG and axles; c_f, c_r front and rear cornering stiffness; m total mass; J_z mass moment of inertia of the vehicle around the vertical axis; $n_{RK} = n_R + n_K$ longitudinal caster + caster; r_s force arm of the brake forces; J_s, C_s, k_s resultant mass moment of inertia, spring and damping coefficients, resp. of the steering system below the torsion bar; r_{eff} effective wheel radius; J_w mass moment of inertia of a front wheel; C_{pT} brake transmission factor;

4. CHOICE OF MODEL SET

The systems G_1 and G_2 are independently identified. For G_1 linear discrete-time black-box models are chosen in elementary subsystem (ESS) representation. The structure estimation method

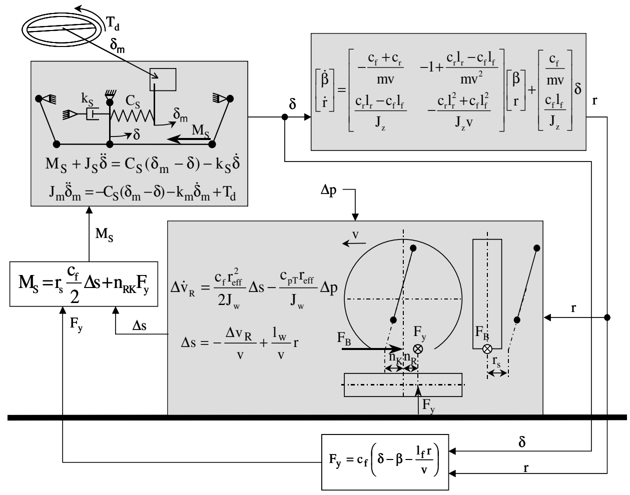

Fig. 1. The three components of the physical model of G_2. Chassis described by single-track model (top right gray box), differential wheel model (bottom) and steering system (top left).

developed by (Keviczky *et al.* 1979) and the recursive algorithm by (Bokor and Keviczky 1985) starts from an overestimated structure and using prediction error method a maximum likelihood estimator finds the smallest required structure and its parameters, and converges with numerically advantageous properties. In case of output-error estimation we have the following discrete-time model set

$$\begin{aligned}
y(t) = &\sum_{i=1}^{n_r} \frac{b_i z^{-1}}{1 + a_i z^{-1}} u(t-d) + \\
&+ \sum_{i=1}^{n_c} \frac{q_{1i} z^{-1} + q_{2i} z^{-2}}{1 + p_{1i} z^{-1} + p_{2i} z^{-2}} u(t-d) + e(t),
\end{aligned}$$

where the wanted parameters are n_r, n_c for the number of real poles and complex pairs, resp., d for time delay, furthermore a_i, b_i, q_{ij} and p_{ij}.

Concerning G_2, physically parameterized continuous-time LPV state-space family is chosen in predictor form

$$\begin{aligned}
\dot{\hat{x}}(t) &= A(\theta,\rho)\hat{x}(t) + B(\theta,\rho)u(t) + L(\theta)e(t), \\
\hat{y}(t) &= C(\theta,\rho)\hat{x}(t) + D(\theta,\rho)u(t), \qquad (2) \\
e(t) &= y(t) - \hat{y}(t).
\end{aligned}$$

Advantages of physical parameterization:

- Known dependence on the scheduling variable avoids searching in an infinite function space.
- Less parameters then in case of black-box models.
- Physical insight into the relevance of the model components.
- Initialization for higher fidelity nonlinear models.

- Physical insight into the role of parameters helps to estimate parametric uncertainty to the nominal model. (E.g. the change of load, load distribution and cornering stiffness.)

A disadvantage is that the parameters in canonical forms tend to be ill-conditioned, criterion function may be quasi-convex. Let the system (1) be denoted by $\mathcal{M}_0$. This is in LPV form with scheduling parameter v. From $\mathcal{M}_0$ four different LPV model structures can be derived by applying the following simplification assumptions.

- A1. In simulations the sideslip angle and yaw rate appeared to be approximately proportional. Therefore state β is omitted and $\frac{\beta(t)}{r(t)}$ approximated as
$$\left.\frac{\beta(s)}{r(s)}\right|_{s=0} = \frac{l_r}{v} - \frac{ml_f}{c_r l} v$$
is inserted in the expression for $\dot{r}$.
- A2. The wheel model is assumed to be fast for the excitation bandwidth and $\Delta v_R(t)$ is replaced by the DC component of its Laplace-transform:
$$\Delta v_R(s)|_{s=0} = l_w r - 2\frac{C_{pT} v}{c_f r_{eff}} \Delta p.$$
- A3. The steering system is assumed to be fast for the excitation bandwidth,
$$\delta(t) \approx \delta(s)|_{s=0} = (\frac{l}{v} - \frac{r_s l_w}{2n_{RK} v} - \frac{ml_f}{c_r l} v) r + \\ + \frac{r_s}{2n_{RK} v} \Delta v_R.$$

Applying the simplifications from A1 to A3, the model graph on figure 2 is generated. Here the model $\mathcal{M}_2$ is stated in details.

$$\hat{x}(t) = \begin{bmatrix} \hat{r} & \hat{\delta} & \dot{\hat{\delta}} \end{bmatrix}^T,$$
$$A(\theta, v) = \begin{bmatrix} \frac{-p_1 l}{v} + p_2 v & p_1 & 0 \\ 0 & 0 & 1 \\ \frac{p_3 l}{v} + p_4 v & -p_3 & p_5 \end{bmatrix},$$
$$B(\theta, v) = \begin{bmatrix} 0 \\ 0 \\ p_6 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, D = 0,$$

where

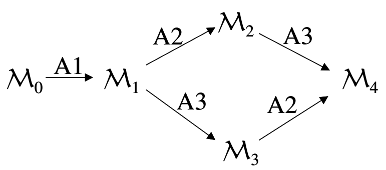

Fig. 2. Structure graph generated from $\mathcal{M}_0$ by assumptions A1-A3

$$p_1 = \frac{c_f l_f}{J_z}, \quad p_2 = -\frac{ml_f(c_r l_r - c_f l_f)}{J_z c_r l},$$
$$p_3 = \frac{n_{RK} c_f}{J_s}, \quad p_4 = -\frac{ml_f n_{RK} c_f}{J_s c_r l},$$
$$p_5 = -\frac{k_s}{J_s}, \quad p_6 = -\frac{C_{pT} r_s}{J_s r_{eff}}.$$

The noise model due to the filter gain L was directly parameterized. Because of the ill-conditioned parameterization of the canonical structure, in case of full-parameterized 3x2 L matrix the criterion function became almost quasi-convex and the parameter vector θ did not converge. A sufficient parameterization was found as $L = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & p_7 \end{bmatrix}^T$, so $\theta = \begin{bmatrix} p_1 & \cdots & p_7 \end{bmatrix}$, i.e. feeding back the steering angle error.

5. IDENTIFICATION METHOD

The ESS structure and parameter estimation method for G_1 starts from an overestimated structure. The prediction error optimization problem is solved with constraints on the poles and numerator coefficients. The absolute value of a pole is bounded between the bandwidth of the input (or the Nyquist-frequency) and the frequency according to the observation time. The angle of a complex pole pair is constrained in order to avoid rare sampling and to make distinction between a large time constant and a low frequency sine wave during the observation time. The numerator coefficients should be positive real numbers. When the procedure idles over local minima, the values of the parameters should be considered. If all b_i and q_{1i} are nonsignificant, the delay is increased. Vanishing numerator coefficients or too small poles indicate the presence of too much poles. If a complex pair achieves the real axis, it is changed to two real poles. Then the search is continued.

The system G_2 is identified with prediction error method with the criterion

$$V(\theta) = \sum_{i=1}^{n_v} \frac{1}{N_i} \sum_{k=1}^{N_i} \varepsilon_i(kT_s, \theta)^T \Lambda_i^{-1} \varepsilon_i(kT_s, \theta),$$
$$\Lambda_i^{-1} = diag \left[\frac{w_1}{a_{1,i}}, \ldots, \frac{w_{n_y}}{a_{n_y,i}} \right], \tag{3}$$
$$a_{p,i} = \frac{1}{N_i} \sum_{k=1}^{N_i} y_{p,i}^2(kT_s), \quad p = 1, 2, \ldots, n_y,$$

where $\varepsilon_i(kT_s, \theta) = y_i(kT_s) - \hat{y}_i(kT_s, \theta)$ is the prediction error vector at time $t = kT_s$, N_i is the number of data-points in the ith experiment. The velocity v_i is constant over each experiments. The n_y is the number of outputs, n_v is the number of experiments composing the criterion function. The model outputs $\hat{y}_i(kT_s, \theta)$ are computed by transforming the continuous-time linear models

(2) with $\rho = v_i$ into the discrete time-domain assuming zero-order-holder as follows

$$\begin{aligned}
\xi(k+1) &= F(\theta, v_i)\xi(k) + G(\theta, v_i)u(k) + H(\theta)\nu_2(k) \\
z(k) &= C(\theta, v_i)\xi(k) + D(\theta, v_i)u(k), \\
\xi(0) &= \hat{x}(0) = 0, \\
F(\theta, v_i) &= e^{A(\theta, v_i)T_s}, \\
G(\theta, v_i) &= \int_0^{T_s} e^{A(\theta, v_i)(T_s - \tau)} B(\theta, v_i) d\tau, \\
H(\theta) &= \int_0^{T_s} e^{A(\theta, v_i)(T_s - \tau)} L(\theta) d\tau,
\end{aligned}$$

then $\hat{y}(kT_s) := z(k)$, $k = 1, \ldots, N_i$. This transformation is required because the inputs of G_2 are sampled, measured and therefore noisy. Using integration methods - e.g. Runge-Kutta - the computation error would be larger. This is the reason of fixing the scheduling parameter during the experiments.
The optimization was first run with Λ being the covariance matrix of the innovations. This assumes Gaussian distribution of the innovation e. The results were very similar when L was zero (OE model). But with parameter p_7 the criterion become quasi-convex, therefore the constant weighting (3) was applied where the w_is set the relative importance of the outputs in the matching.

6. RESULTS AND MODEL VALIDATION

Considering the actuator model G_1 the algorithm was started from $(n_r = 3, n_c = 2)$ with delay $d = 0$. It was find that the most significant parameter was the time-delay. With the best choice of the delay $d = 4T_s$ the worst - and also the simplest - model structure $(n_r = 1, n_c = 0)$ differs in criterion value from the best $(n_r = 1, n_c = 2)$ only by 10%. Therefore the following model is selected for G_1:

$$\hat{\Delta p}(t) = G_1(q^{-1})\Delta p_c(t) = \frac{0.1581q^{-1}}{1 - 0.8362q^{-1}} \Delta p_c(t - 4T_s),$$

where q^{-1} is the backward-shift operator. On figure 3 the input $\Delta p_c(t)$ generated on the standing vehicle by a PRBS generator is plotted together with the measured $\Delta p(t)$ and model output $\hat{\Delta p}(t)$. The static errors at the plateaus are due to the about 0.025bar error tolerance of the pressure controller in the cylinders.

The parameters of the G_2 model structures are initialized by least-square estimation (LSE): the unmeasurable state variables and its first derivatives are computed by numeric derivation of the measured outputs by fitting a 3-order polynomial to a moving window of length 15 points. Thus

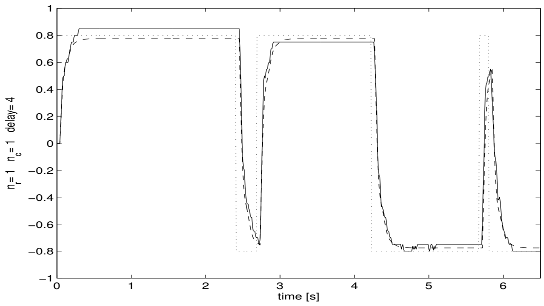

Fig. 3. Validation of model G_1. Δp_c: dotted, Δp measurement: solid, $\hat{\Delta p}$ model output: dashed.

(2) became a linear regression model in the parameters. The parameter in L was random generated. After this initialization the prediction error method was used as presented in the previous section. An additional time-delay of $2T_s$ for G_2 was estimated.

First the resulted structures are validated. On figure 4 the pole-location of the four LPV models are pictured with fixed scheduling variable from 8m/s to 20m/s. The input bandwidth is also plotted as a circle with radius of 8rad/s. The bandwidth of the control will be below this value. Those structures can be accepted of which poles are placed inside the circle. Poles outside the circle imply the presence of not excited, fast dynamics. According to figure 4 it can be concluded that the wheel dynamics is too fast. Since $\mathcal{M}_2$ is better predictor then $\mathcal{M}_4$, $\mathcal{M}_2$ is selected as a candidate for controller design.

On figure 5 $\mathcal{M}_2$ is validated on the time domain. On the left side the velocity was 8.05 m/s during the experiment, on the right it was 17.49m/s. The measured input $\Delta p(t)$, the yaw rate r and the steering angle δ are plotted. The outputs of the predictor $\mathcal{M}_2$ (solid lines) fit the measurements well-markedly. The achieved peak-to-peak prediction error per measured output ratio is below 18% for the yaw rate and below 32% for the steering angle. In many control synthesis problems the role of the filter $L(\theta)$ is redesigned, only the $A(\theta)$, $B(\theta)$ and $C(\theta)$ matrices of the predictor are kept. The outputs of the predictor without the filter $(L(\theta) = 0)$ are plotted with dashed lines.

Connecting the models G_1 and G_2 into a single model, the outputs are almost the same as on figure 5, where the G_2 was driven by the measured Δp. Therefore the outputs are not plotted again. This proves the sufficiency of the actuator model G_1. The resulted model structure is summarized on figure 6, where G_1 is rewritten as

$$G_1(q^{-1}) = \frac{0.1581q^{-1}}{1 - 0.8362q^{-1}}, \qquad (4)$$

and $G_2 = \mathcal{M}_2$ with parameters given in table 1.

Table 1. Parameters of the resulted model G_2.

p_1	p_2	p_3	p_4	p_5	p_6	p_7
14.54	0.06	20.60	-0.25	-4.96	-0.32	38.87

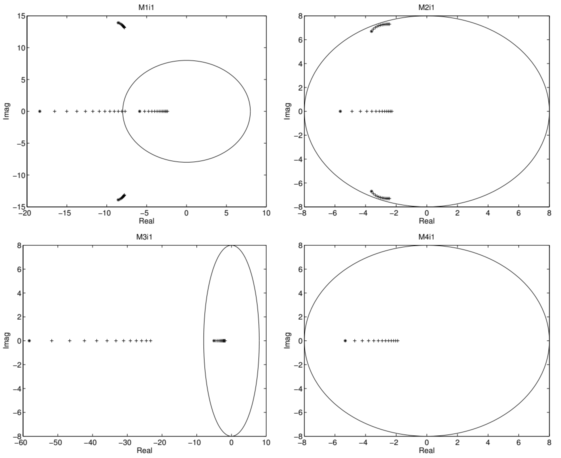

Fig. 4. Structure validation: the excitation bandwidth and pole locations of $A - LC$ from $v = 8m/s$ to $20m/s$

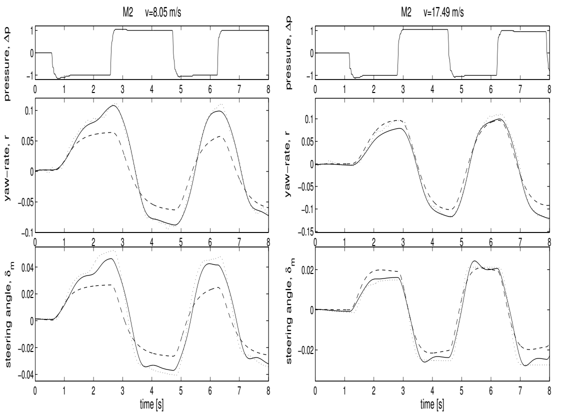

Fig. 5. Validation of the nominal model in time domain. Measurements - dotted, predictor - solid, with filter gain $L = 0$ - dashed.

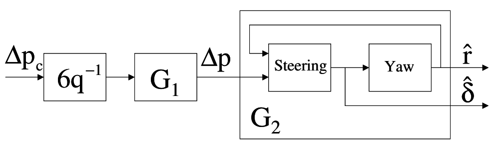

Fig. 6. The resulted model structure.

7. CONCLUSIONS

A linear velocity-varying model was identified from measurement data for steering controller which brakes the front wheels. The unknown constant parameters of the LPV model and the structure selection by looking at the pole locations were performed with the help of experiments with fixed scheduling variable, i.e. keeping the velocity constant. The parameters being functions of physical parameters allow the insight into the role of them, so it will be possible to estimate parametric uncertainty to the nominal model.

The resulted model is the feedback connection of the yaw dynamics with one state-variable and the steering system dynamics with to states. The achieved noise-to-signal ratio promises sufficient model performance for controller design.

Future work will include uncertainty modelling from experimental data.

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