VIRTUAL REFERENCE FEEDBACK TUNING IN RESTRICTED COMPLEXITY CONTROLLER DESIGN OF NON-MINIMUM PHASE SYSTEMS

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Abstract: This paper discusses the applicability of the identification-based Virtual Reference Feedback Tuning scheme in (Campi *et al.*, 2002; Campi *et al.*, 2003; Sala and Esparza, 2005) to reduced-order controller design. As the presence of zeros outside the unit circle is quite usual in sampled-data systems, a particular discussion on the topic is carried out. Also, reduced-order controllers identified with the VRFT scheme may not meet the specifications (or even be unstable) so some invalidation tests are needed prior to experiment.

Keywords: system identification, reduced order controller, non-minimum phase systems, stability tests

1. INTRODUCTION

When addressing a controller design task, there are many issues to take into account. In many industrial applications it is essential to achieve a low order design (Landau *et al.*, 2003) preserving the desired specifications of the plant.

Some approaches have been recently developed to design a (reduced order) controller based on experimental data collected from the real plant. They can be summarized as “indirect” and “direct” techniques. The first ones (see, for example, (Anderson, 2002; Gevers, 1993; Partanen and Bitmead, 1993; Van den Hof and Schrama, 1995; Schrama, 1992)) identify a model of the plant which will be used to design a controller. The “direct” methods (as (Campi *et al.*, 2002; Hjalmarsson *et al.*, 1998; Hjalmarsson, 2002; Spall and Criston, 1998)) use the data to directly identify a controller without the plant model. In both cases the resulting controller can either have a reduced complexity or be of high order (in this case, a model reduction technique must be used (Anderson and Liu, 1989; Obinata and Anderson, 2001; Cordons *et al.*, 1999)).

The Virtual Reference Feedback Tuning (VRFT) (Campi *et al.*, 2002) has appeared as a useful and straightforward method to design low order controllers (Campi *et al.*, 2003). This is one of the model-free direct techniques above mentioned. It uses available open loop data to identify a controller with the prespecified order, *i.e.*, it can be used to directly identify low order controllers.

When designing discrete controllers some additional issues must be taken into account.

- The plant to be controlled could be a non-minimum phase (NMP) one. In fact, the poles outside the unit circle may be originated both from right half plane (RHP) zeros of the continuous plant and as a discretisation artifact at high sampling rates.
- The reduced-order fit to a set of experimental data may be not close enough to achieve the required performance (even compromising closed-loop stability). So, a controller invalidation test, which targets predicting at least loop stability before implementation, appears to be necessary.

In (Sala and Esparza, 2005) some extensions to the VRFT algorithm have been outlined, addressing, among others, the above mentioned issues and applying the methodology to open-loop unstable systems. The present contribution discusses the use of all-pass elements in the filtering sections to generate, if needed, some of the virtual signals in the controller ID and invalidation steps. Furthermore, as shown by simulation examples, the inclusion of approximate information about the non-minimum-phase behaviour greatly improves the chances of obtaining a useful reduced-order controller; this information may be obtained with the plant input-output data.

The paper is organised as follows: section 2 introduces some basic concepts concerning the VRFT scheme. An extension of the method to NMP systems is addressed in section 3. In section 4 an invalidation test is proposed based on the data available and the controller previously designed via the VRFT technique. Section 5 discusses some practical issues concerning the proposed methodology. The proposed extension of the VRFT algorithm to address NMP systems is illustrated with two examples in section 6. Finally, the paper is closed with a conclusion section.

2. PRELIMINARIES

This section will summarise the main ideas in the VRFT methodology described in (Campi *et al.*, 2002). The procedure identifies a controller from input-output data (u, y) gathered from a process P, given a target closed-loop behaviour M. The steps followed are:

(1) From the recorded signals u, y a virtual reference r_v is built such that $y = M \cdot r_v$.
(2) Then the tracking error $e = r_v - y$ is computed. In fact, it can be obtained directly from the output y of the plant as $e = (M^{-1} - 1)y$.
(3) The controller design reduces to an identification problem between the signals e and u, obtaining a parameterised controller $u = C(\theta)e$, $\theta \in \mathbb{R}^n$.

The “ideal” controller achieving a tracking behaviour M is the one with transfer function $C_0 = \frac{M}{(1-M)P}$. Depending on the parameterisation of $C(\theta)$, C_0 may not belong to the controller set $\mathscr{C} = \{C(\theta) | \theta \in \mathbb{R}^n\}$, such as, likely, the case of reduced-order controllers.

The cost index of the identification carried out is

$$J_{VR}^N = \frac{1}{N}\sum_{i=1}^{N}(u_L - C(\theta)e_L)^2 \qquad (1)$$

where u_L and e_L denote sequences obtained by filtering by a particular filter L the input and virtual error sequences. “Perfect” control would be achieved if a parameter value θ^* could be found so that $C(\theta^*)e = u$, *i.e.*, $C(\theta^*) = C_0$.

In (Campi *et al.*, 2002) a prefilter $L = M(1-M)T_u^{-1}W$ is proposed, where T_u is a filter such that $|T_u|^2 = \Phi_u$ (Φ_u is the power spectral density of $u(t)$), being L derived to approximately minimise the frequency integral of $W^2|M - \hat{M}|^2$, where $\hat{M}$ represents the achieved closed loop function.

The controller inverse can also be identified minimising the index:

$$J_{VR}^N = \frac{1}{N}\sum_{i=1}^{N}(C^{-1}(\theta)u_L - e_L)^2 \qquad (2)$$

with advantages if additive noise corrupts e regarding the need of instrumental variables in (Campi *et al.*, 2002). In this case, following a similar methodology, the prefilter L would be $L = M^2 T_y^{-1} W$, with $|T_y|^2 = \Phi_y$.

3. VRFT APPLIED TO NON-MINIMUM PHASE SYSTEMS

Pole-zero cancellation issues may arise when applying the technique to unstable or NMP systems. This section will study the tuning knobs of the methodology when applied to NMP systems. As stated in the introduction, discrete zeros outside the unit circle appear naturally in fast-sampled systems and also arise from continuous-time RHP zeros.

Let G be a NMP system, which can be decomposed as:

$$G = G^* \cdot G_{nm} \qquad (3)$$

where G^* denotes the minimum phase and G_{nm} the NMP part of G (expressed as an all-pass factor). Delays, if any, should also be included in G_{nm}.

Identification. Given any input signal for the VRFT experiment (u), the output signal (y) contains the effect of a NMP factor and delays, in the sense that, for any stable filters Q, L, an ID setup for a transfer function H minimising $||Ly - QH(\theta)u||$ will estimate an NMP factor in the numerator of H if enough flexibility is available and the referred factor is not present in Q. This is the case if the controller inverse were identified in a setup like equation 2. Conversely, if the ID is to minimise $||Qu - LH(\theta)y||$, the NMP factor may be identified in the denominator of H, if enough freedom is available.

Given the above circumstances, the recommended ID setting when dealing with NMP systems in VRFT is to identify the controller minimising expression 1 avoiding the possibility of unstable denominators in $C(\theta)$. In (Campi *et al.*, 2002), the denominator was fixed to avoid these issues. However, using any ID method guaranteeing an stable result, such as OE is also an option, with full parameterisations.

Reference model. A second issue is how to set up the target reference model M. As G_{nm} always appears in the closed-loop response, if an approximation of G_{nm} is known, then the closed loop model reference should include the NMP term in order to improve the chances of achieving a stable closed loop, particularly under reduced-order controller ID. In this way, denoting as M the reference model, its proposed definition will be:

$$M = M^* \cdot M_{nm} \approx M^* \cdot G_{nm} \tag{4}$$

where M^* is the minimum phase part of M and M_{nm} is a NMP term, expressed as an all-pass plus delay function. As an example, let us consider a plant with one NMP zero β outside the unit circle. The model reference will be $M = M^* \cdot M_{nm}$, with M_{nm} defined, for instance, as:

$$M_{nm} = \frac{z - \beta'}{1 - \beta' z} \tag{5}$$

where β' is an approximation of the NMP zero, if available.

Ideally, M_{nm} should contain an approximation of G_{nm}. Indeed, if it were exactly G_{nm} then the virtual error would not contain the NMP behaviour, so the problems associated with ID of an unstable regulator would dissapear. A good approximation mitigates these issues and allows for a better resulting performance.

This may seem unnatural, in the sense that the requirement of the information of the NMP zeros appears to spoil the advantage of the original VRFT method. However, although ideally the VRFT methodology does not require any process model information, for practical success it is convenient to have such information in order to design a sensible M, as the examples in Section 6 illustrate.

Virtual error generation. An open loop experiment, applying u_{ex} to the plant input, gives y_{ex} as the output. Then, next expression relates y_{ex} and the virtual reference r_v:

$$y_{ex} = M^* \cdot M_{nm} \cdot r_v \Rightarrow r_v = (M^* M_{nm})^{-1} y_{ex} \tag{6}$$

r_v cannot be evaluated from expression 6, as the inversion of M_{nm} is unstable. Then, auxiliary signals, r^* and y^*, are needed for the VRFT setup. They are defined as:

$$\begin{aligned} r^* &= r_v \cdot M_{nm} = {M^*}^{-1} \cdot y_{ex} \\ y^* &= y_{ex} \cdot M_{nm} \end{aligned} \tag{7}$$

The virtual loop error, e, defined as: $e = r_v - y_{ex}$ cannot be evaluated either. So, an auxiliary e^* is defined as follows:

$$e^* = e \cdot M_{nm} = r^* - y^* \tag{8}$$

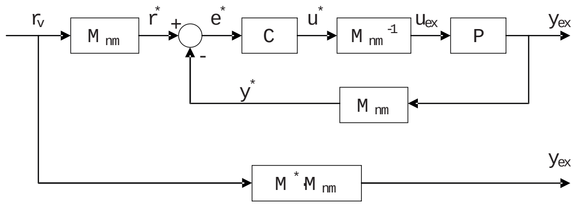

Figure 1. Relationship among closed loop signals

The design method involves identifying a controller which satisfies the following condition:

$$u_{ex} = C \cdot e = C \cdot M_{nm}^{-1} \cdot e^* \tag{9}$$

The auxiliary signal u^* is defined as:

$$u^* = u_{ex} \cdot M_{nm} \Rightarrow u^* = C \cdot e^* \tag{10}$$

In this way, all the signals needed in the VRFT procedure can be obtained from input-output data filtered by stable transfer functions, if needed. The above can be shown to be equivalent to setting the data filter L as a multiple of M_{nm}.

Figure 1 shows the relationship among the signals defined above. The controller has to be identified between e^* and u^*. The relationship of these signals with the actual VRFT ones e and u is multiplication by an all-pass factor so the frequency weighting is not modified.

4. CONTROLLER INVALIDATION

Once a controller C has been identified, assessing the closed loop stability before implementing it, if possible, would be a convenient step. Of course, identifying a model of the plant with the available data records and then checking closed-loop stability is a viable approach, and it is the path followed in (Campi *et al.*, 2000), including estimated modelling error bounds.

However, a closed loop function can be directly identified in order to check its stability, with the same data, as discussed below. The chosen function is the achieved input sensitivity function defined as:

$$CS_a = \frac{C}{1 + CP} \tag{11}$$

obtained experimentally with the relation

$$u_{ex} = CS_a \cdot r_a \tag{12}$$

where r_a is the "achieved" virtual reference to be defined below. Let us now discuss how ID of CS_a may be carried out.

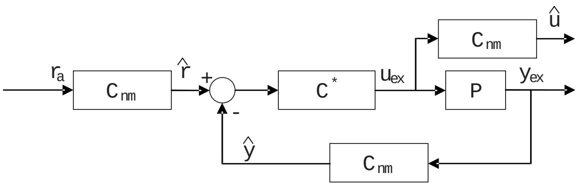

Figure 2. Relationship among signals used to identify KS_a when C is NMP.

Minimum phase controller. As the measured signals are u_{ex} and y_{ex}, the reference r_a must be recalculated from them using the current controller C:

$$r_a = C^{-1} \cdot u_{ex} + y_{ex} \tag{13}$$

In this way, using 12, $r_a = (CS_a)^{-1} \cdot u_{ex}$, and the inverse input sensitivity can be identified from data. Note that additive output disturbances appear also as such in r_a. Different model structures (output error, ARMAX, etc.) and instrumental variables may be used. As a result of the procedure, if CS_a were an unstable transfer function, the controller would be invalidated.

Non-minimum phase controller. If the identified controller happens to be NMP (in the form C^*C_{nm} as discussed before), its inverse is unstable and r_a cannot be computed. In this case, the controller inverse can be split into two parts: the stable ($C^{*^{-1}}$) and the unstable (C_{nm}^{-1}) ones: $C^{-1} = C^{*^{-1}} \cdot C_{nm}^{-1}$. Hence,

$$\underbrace{C_{nm} \cdot r_a}_{\hat{r}} = C^{*^{-1}} \cdot u_{ex} + \underbrace{C_{nm} \cdot y_{ex}}_{\hat{y}}$$
$$\Rightarrow \hat{r} = C^{*^{-1}} \cdot u_{ex} + \hat{y} \tag{14}$$
$$\hat{u} = C_{nm} \cdot u_{ex}$$

Figure 2 shows the relationship among these signals. The ID of CS_a is carried out between $\hat{u}$ and $\hat{r}$ defined in expression 14. The magnitude of the spectrum of u_{ex} and r_a is not modified by the multiplication by C_{nm}, because it is an all-pass filter.

5. METHODOLOGY

Given a target performance, from which M is built, the above discussed procedures can be used to identify a controller from experimental data. Some fixed terms (integrators, high-frequency notch) may also be added to the controller, being the procedure quite straightforward, omitted for brevity.

As in every ID experiment, however, issues such as model structures, disturbance models, etc. must be sorted out. The above setups may need instrumental variables, as suggested in (Campi *et al.*, 2002).

In many practical cases, reduced-order controllers are sought. In this case, the desired excitation profile (frequency content in u) to achieve the correct fit in M at the most relevant frequencies may be also an issue. Invalidation of a reduced-order controller can be carried out with the procedure in section 4, noting that ID of CS_a can be done with a parameterised model with a higher-order than that used for controller ID.

Furthermore, when designing a controller for an unknown plant, the first thing to decide is the desired performance. If there is no information available about the plant behaviour it may be difficult to know its performance limitations. So, it would be advisable to progressively increment the loop performance in an iterative way, comparing the target performance with the experiment results after each run (Anderson, 2002; Sala and Esparza, 2003) in a practical setup, even if it seems to be no theoretical need for it.

6. EXAMPLES

In this section two examples will be presented. The first one is the same used in (Sala and Esparza, 2003), in order to compare both approaches to reduced-order controller design. The second one is a sampled continuous NMP system. In both cases it has been assumed that the designer knows an approximation of the unstable zero.

First example The continuous real plant P_c to be controlled is:

$$P_c = \frac{1.83 \cdot 10^9 (s+70)(s+15)}{(s+1600)(s+800)(s+200)(s+45)(s+4)} \tag{15}$$

It is discretised using a sampling rate of 1 KHz. This causes an NMP zero to appear:

$$P = \frac{0.166(z+2.06)(z-0.98)(z-0.93)(z+0.13)}{(z-0.996)(z-0.956)(z-0.819)(z-0.45)(z-0.202)} \tag{16}$$

To the data collected in open loop operation (u_{ex}, y_{ex}), a 0.01 variance noise signal has been added.

The model reference is a first order one with one step delay and the NMP part (as stated in expression 5). Its transfer function is:

$$M = M^* \cdot M_{nm} = \frac{(1-\alpha)z^{-1}}{1-\alpha z^{-1}} M_{nm}$$
$$\alpha = \exp^{-T_s \lambda} \tag{17}$$

where λ is the bandwidth expressed in rad/sec, T_s is the sampling period (0.001 sec) and β' is the approximation of the NMP zero (in the example, $\beta' = 2$).

Let us consider a target bandwidth of $\lambda = 500$ rad/sec. With an integrator ($C_{11} = \frac{0.01372z}{z-1}$) the system step response is very poor. In this controller, the coefficient 0.01372 was identified and $\frac{z}{z-1}$ was a fixed factor. Increasing the order ($C_{12} = \frac{0.63654z(z-0.9696)}{(z-1)(z+0.8145)}$) the closed loop behaviour is less oscillatory and faster. With a third order controller ($C_{13} = \frac{0.77454z(z-0.873)(z-0.4303)}{(z-1)(z^2+0.4674z+0.1097)}$)

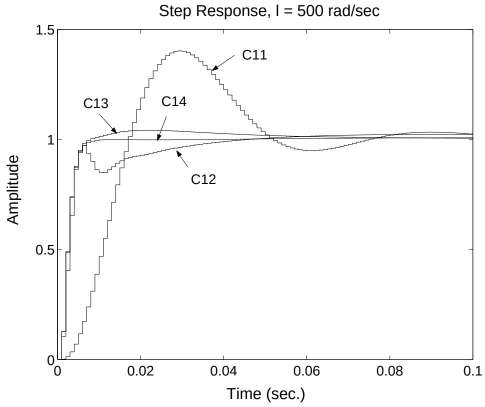

Figure 3. Step response for first, second, third and fourth order controllers with $\lambda = 500$ rad/sec

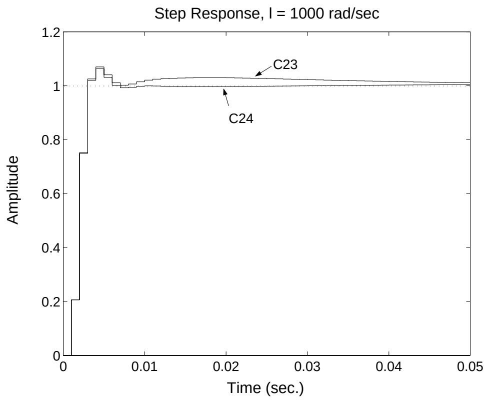

Figure 4. Step response for $\lambda = 1000$ rad/sec with a third and a fourth order controller

the response is much better. If the order of the controller is four the behaviour is almost the same as the model reference. In this case, the choice of the order of the controller will depend on the tolerance over the specifications. It must be remarked that the stability test has been successfully applied to every controller before implementing it.

Figure 3 shows the system step response with C_{11}, C_{12}, C_{13} and C_{14}. With a third order controller (C_{13}) the overshoot is about 4%, so the system behaviour could be acceptable.

If the desired bandwidth is $\lambda = 1000$ rad/sec, the first and second order controller designed provide an unstable loop (do not pass the invalidation test). If the controller is a third order one ($C_{23} = \frac{1.24z(z-0.866)(z-0.466)}{(z-1)(z^2+0.519z+0.108)}$) the system is stable and the response has a 7% overshoot. If a fourth order controller is identified ($C_{24} = \frac{1.23z(z-0.978)(z-0.799)(z-0.52)}{(z-1)(z-0.937)(z^2+0.497z+0.09)}$) the overshoot does not decrease, but the transient is much shorter, as can be seen in figure 4. Higher orders do not appear to substantially improve the loop behaviour.

Remark: Sampling-induced NMP zeros may be difficult to estimate in order to set up M_{nm}. Notwithstanding, if no factor M_{nm} were used, the VRFT procedure works reasonably well in a wide bandwidth range. However, at high target bandwidths, problems do occur. For instance, for $\lambda = 500$ rad/sec, the procedure with $M_{nm} = 1$ and a third order controller ($C_{1,500} = \frac{0.773z(z-0.99)(z-0.38)}{(z-1)(z^2+0.82z+0.21)}$) provided a worse response than the controller designed with the suitable M_{nm} ($C_{2,500} = \frac{0.774z(z-0.87)(z-0.43)}{(z-1)(z^2+0.47z+0.11)}$). This results can be seen in figure 5.

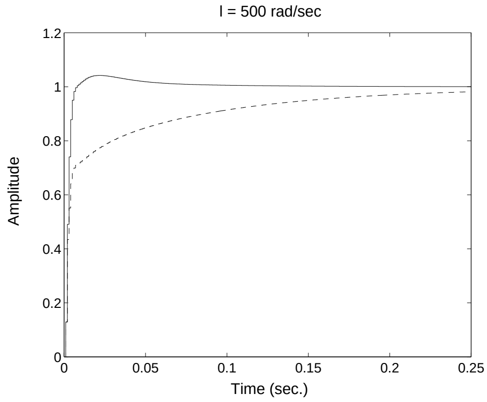

Figure 5. Comparison of the system response with a controller designed with $M_{nm} \approx G_{nm}$ (solid line) or $M_{nm} = 1$ (dash line)

In addition, if the desired bandwidth is $\lambda = 1000$ rad/sec, all controllers up to fifth order designed with $M_{nm} = 1$ yielded an unstable closed loop, however, if M_{nm} is defined as in expression 17 the resulting third order controller stabilized the plant (figure 4).

Non-minimum phase system. In order to show the behaviour of the proposed approach with plants with continuous-time RHP zeros, let us discuss the case where the continuous plant P_c to be controlled is:

$$P_c = \frac{-1.83 \cdot 10^9 (s-70)(s+15)}{(s+1600)(s+800)(s+200)(s+45)(s+4)} \quad (18)$$

The discrete transfer function, using a sampling rate $f_s = 400$ Hz, is:

$$P = \frac{-1.07(z+0.88)(z-0.96)(z-1.19)(z+0.04)}{(z-0.99)(z-0.89)(z-0.61)(z-0.13)(z-0.018)} \quad (19)$$

A 0.01 variance noise signal has also been added to the open loop data. In this case, the continuous time plant is NMP, so the unstable zero is not due to the discretisation.

The model reference is a first order plant with one step delay and including the NMP term. Its definition is the same as in expression 17. In this example, λ is the bandwidth expressed in rad/sec, $T_s = 0.0025$ sec and $\beta' = 1.2$.

As the NMP zero imposes a natural limitation to the real plant performance, the model reference bandwidth chosen is $\lambda = 30$ rad/sec. Figure 6 depicts the step response of the system using controllers of first, second, third and fourth order (C_{11}, C_{12}, C_{13} and C_{14}, respectively). The system behaviour does not improve

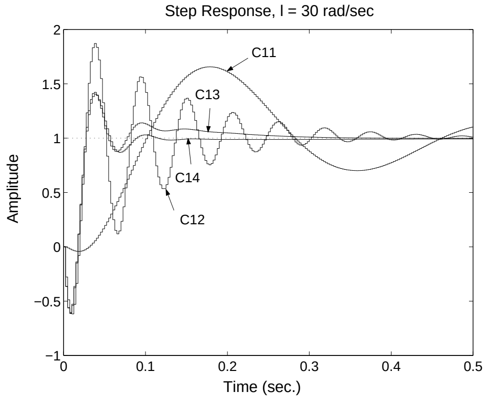

Figure 6. Step response of the system using controllers of different order

when designing controllers with an order higher than four.

In figure 6 can be seen that the system response using C_{13} and C_{14} is quite similar. The steady state is reached faster with the fourth order controller. So in this case, the choice of the controller complexity depends on the tolerance over the specifications that have to be met.

7. CONCLUSION

In this paper the use of the Virtual Reference Feedback Tuning controller design method for reduced-order controller design is investigated. As NMP zeros appear quite frequently in fast sampled systems, suitable modifications to the basic algorithm have been discussed in order to deal with them. Importantly, a controller invalidation step has been proposed prior to testing the controller on the real plant, also based on ID experiments on the available data. This step is crucial when operating with low-complexity controllers that might not stabilise the actual plant.

Some examples show the suitability of the approach, in the sense that the scheme using VRFT methods to design reduced order controllers gives good results with low computational cost and a reduced number of experiments.

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