OBSERVER DESIGN WITH GUARANTEED BOUND FOR LPV SYSTEMS

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Abstract: This paper deals with state observation of discrete time LPV systems in the special situation when the parameters are not exactly known (estimated with a finite accuracy or affected by noise or bounded disturbances during their measurement). In ((Millerioux *et al.*, 2004)), it was shown that despite of the resulting mismatch between the true parameters and the available ones, the state reconstruction error boundedness can be guaranteed and an explicit but possibly conservative bound can be derived. The objective here is to propose a procedure in order to improve the gap between this bound and the actual one. The main result consists in a convex optimization problem which allows to select among all possible polytopic observers the one which minimizes the observation error bound in the steady state.

Keywords: LPV systems, polytopic observers, bounded estimation.

1. INTRODUCTION

Linear Parameter Varying (LPV) systems have received considerable attention by the automatic control researchers (Shamma and Athans, 1991), (Becker and Packard, 1994), (Apkarian and Gahinet, 1995), (Scherer, 1996)... Usually, in the context of LPV systems and gain scheduling strategies, the parameters are assumed to be available online and exactly known. For practical reasons, this can not be the case and one may have to use a parameter estimator (determined for example by standard nonlinear identification techniques) and causing a bounded estimation error. Also, even if measuring the parameters is possible, one may be faced with bounded disturbances on the dynamics and/or the measurements. That's why investigating the situation where the time-varying parameters are estimated with a given accuracy or affected by noise and bounded disturbances during the measurement is a challenging problem.

As far as uncertain systems are considered, estimation problems have been considered in (Geromel *et al.*, 2002) (de Oliveira and Geromel, 2003), (Barbosa *et al.*, 2002)... To our knowledge, among the published papers related to LPV problems where the controller or the observer is scheduled without assuming real-time availability of all the parameters, only (Kose and Jabbari, 1997) addressed a problem similar to the one presented here. In (Kose and Jabbari, 1997), partly measured parameters have been considered. The LPV system is assumed to depend on time-varying real uncertain parameters and only some of the parameters are exactly known and available for feedback. A rank minimization algorithm with LMI constraints has been proposed for dynamic output feedback design. Here, we assume that all the parameters are not exactly known. We consider a parameter dependent observer where instead of using the exact values of the parameters, only estimated (or uncertain but bounded) values can be used. It was proved in (Millerioux *et al.*, 2004) that an explicit bound on the state reconstruction error can be derived by using the concept of Input-to-State Stability (ISS) (Sontag, 1989). However, the main drawback states in the fact that the gap between the provided bound and the real one can be large. Here, we propose a modification to the state observer design procedure which allows to minimize the provided bound and hence to reduce the gap between the obtained bound and the actual one.

We mention that the problem under study differs from the one involving adaptive approaches where the goal is to simultaneously estimate the state and the parameters. The design often requires the use of a global state space diffeomorphism such that, in the new coordinates, the nonlinearities are restricted to be functions of available signals and the system becomes linear with respect to both state and parameters (Bastin and Gevers, 1988). It can be shown that the effect of a bounded estimation error is similar to the effect of a bounded unknown exogenous input acting on the system. And yet, it is well known that a bounded disturbance may drive to infinity a nonlinear system (Marino *et al.*, 2001). That's why analyzing the impact of the parameter estimation error (or the parameter uncertainty) on the state reconstruction error is an interesting problem.

This paper is organized as follows. In section 2, we give the problem statement and recall the main contribution of (Millerioux *et al.*, 2004) which consists in giving an explicit bounded on the state reconstruction error despite of the parameter estimation error using the concept of Input-to-State Stability. Section 3 gives the main result that is a convex optimization problem where the minimization of the bound is included in the observer design procedure. Finally, the improvement provided by this solution is illustrated through the same numerical example used in (Millerioux *et al.*, 2004).

Notation : $\mathbb{R}^n$, the real n-vectors; M^T, the transpose of the matrix M; $\lambda_{min}(M)$, $\lambda_{max}(M)$, the minimum and maximum eigenvalue of the real matrix $M = M^T$, $\|x_k\|$, the usual Euclidean norm $\sqrt{x_k^T x_k}$ of the vector x_k; $\|x\|_\infty$, the supremum norm $\sup_{k\geq 0} \|x_k\|$ of a discrete sequence x; $\|M\|$, the spectral norm $\sqrt{\lambda_{max}(M^T M)}$ of the matrix M.

2. PROBLEM STATEMENT

We consider LPV discrete-time systems given by :

$$\begin{cases} x_{k+1} = A(\rho_k)x_k + Bu_k \\ y_k \quad\;\; = Cx_k \quad\quad\; + Du_k \end{cases} \tag{1}$$

where $x_k \in \mathbb{R}^n$, $y_k \in \mathbb{R}^m$, $u_k \in \mathbb{R}^r$, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times r}$, $C \in \mathbb{R}^{m\times n}$, $D \in \mathbb{R}^{m\times r}$. It is assumed that the discrete trajectory $x(., x_0)$ with initial state x_0 is bounded, that is $\|x\|_\infty < \infty$. Some usual assumptions and considerations for LPV systems are recalled. In particular, A is of class C^1 with respect to the entries of a L-dimensional time-varying parameter vector $\rho_k = (\rho_k^1, \ldots, \rho_k^L)^T$ bounded in a hypercube Θ. For a general parameter dependence of the system and a general parameter dependent Lyapunov function, it is known that controllers or observers design may lead to a convex but infinitely constrained problem (ElGhaoui and Niculescu, 2000). Thus, one usually must resort to "gridding" the range of all admissible values of the parameter in order to obtain a finite set of constraints. To overpass it, a solution consists in carrying out a polytopic decomposition. Indeed, since ρ_k is valued in the hypercube Θ, A lies in a compact set which can always be embedded in a polytope, that is :

$$A(\rho_k) = \sum_{i=1}^{N} \xi_k^i(\rho_k) A_i \tag{2}$$

where the A_i's correspond to the vertices of the convex hull $\mathbf{Co}\{A_1, \ldots, A_N\}$. The ξ_k's belong to the compact set $\mathcal{S} = \{\mu_k \in \mathbb{R}^N, \mu_k = (\mu_k^1, \ldots, \mu_k^N)^T,\ \mu_k^i \geq 0\ \forall i \text{ and } \sum_{i=1}^N \mu_k^i = 1\}$ and they can always be expressed as functions of class C^1 with respect to the ρ_k's. Such a decomposition turns the design problem into the resolution of a finite set of constraints involving only the vertices of the convex hull.

Here, we focus on the situation where the true parameter ρ_k is actually not available but it is assumed that an estimator provides at each discrete-time k an estimated $\hat{\rho}_k$ fulfilling $\|\rho_k - \hat{\rho}_k\|_\infty < \Delta$. It is a typical situation when the estimator results from standard nonlinear identification techniques based upon learning machines. Obviously, it includes the case where $\rho_k = \rho^*$, a constant value.

For the reconstruction of the state x_k, the following so-called polytopic observer is proposed.

$$\begin{cases} \hat{x}_{k+1} = A(\hat{\rho}_k)\hat{x}_k + Bu_k + L(\hat{\rho}_k)(y_k - \hat{y}_k) \\ \hat{y}_k = C\hat{x}_k + Du_k \end{cases} \tag{3}$$

with $A(\hat{\rho}_k) = \sum_{i=1}^{N} \hat{\xi}_k^i(\hat{\rho}_k)\hat{A}_i$, $\hat{\xi}_k^i \in \mathcal{S}$, $\hat{A}_i \in \mathbf{Co}\{\hat{A}_1, \ldots, \hat{A}_N\}$, L being a time-varying gain defined by $L(\hat{\rho}_k) = \sum_{i=1}^{N} \hat{\xi}_k^i(\hat{\rho}_k)L_i$. The L_i's are some constant gains to be computed.
The motivation of such an observer stems from the fact that, for the polytopic decomposition (2) and a perfect estimation corresponding to $\hat{\rho}_k = \rho_k$ and so $\mathbf{Co}\{\hat{A}_1, \ldots, \hat{A}_N\} = \mathbf{Co}\{A_1, \ldots, A_N\}$, a global convergence of the state reconstruction error is obtained. On one hand, from (1) and (3), it is easy to see that for $\rho_k = \hat{\rho}_k$, the state reconstruction error $\epsilon_k \stackrel{\triangle}{=} x_k - \hat{x}_k$ is governed by the dynamics :

$$\epsilon_{k+1} = \mathcal{A}(\rho_k)\epsilon_k \tag{4}$$

with $\mathcal{A}(\rho_k) = \sum_{i=1}^{N} \xi_k^i(\rho_k)\tilde{A}_i$ and

$$\tilde{A}_i = A_i - L_iC$$

Following similar details as in (Daafouz and Bernussou, 2001), one can build the observer gain matrices and ensure the global stability of the observation error using a parameter dependent Lyapunov function.

Now, the situation when $\rho_k \neq \hat{\rho}_k$ is considered. In this case, (4) does no longer hold and turns into :

$$\epsilon_{k+1} = \mathcal{A}(\hat{\rho}_k)\epsilon_k + v_k \tag{5}$$

where it can easily be seen that $v_k = (A(\rho_k) - A(\hat{\rho}_k))x_k$.

In (Millerioux *et al.*, 2004), the observer gains were computed using a poly-quadratic stability based approach (Daafouz and Bernussou, 2001) that is solving the LMIs

$$\begin{bmatrix} P_i & A_i^T G_i^T - C^T F_i^T \\ G_i A_i - F_i C & G_i^T + G_i - P_j \end{bmatrix} > 0 \tag{6}$$

$\forall (i,j) \in \{1, \ldots, N\} \times \{1, \ldots, N\}$ where the positive definite matrices P_i, the matrices F_i and G_i are the unknowns and $L_i = G_i^{-1}F_i$. It was shown that with such an observer the boundedness (in the sense of the supremum norm) of the resulting state reconstruction error is guaranteed. Input-to-State Stability (ISS) concept was used and an explicit bound in terms of the estimation error bound Δ was proposed. We recall here the main result of (Millerioux *et al.*, 2004).

Definition 1. (Jiang *et al.*, 1999) System (5) is said to be Input-to-State Stable if there exist a $\mathcal{KL}$ [1] function $\beta : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ and a $\mathcal{K}$ function γ such that, for each input sequence v fulfilling $\|v\|_\infty < \infty$ and each $\epsilon_0 \in \mathbb{R}^n$, the discrete trajectory associated with the initial condition ϵ_0 and the input v fulfills :

$$\|\epsilon_k\| \leq \beta(\|\epsilon_0\|, k) + \gamma(\|v\|_\infty) \quad \forall k \tag{7}$$

Theorem 1. (Millerioux *et al.*, 2004) (5) is Input-to-State Stable, that is there exist a $\mathcal{KL}$ function $\beta : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ and a strictly positive quantity α_3 such that

$$\|\epsilon_k\| \leq \beta(\|\epsilon_0\|, k) + \alpha_3\|v\|_\infty \quad \forall k \tag{8}$$

The proposed explicit bound α_3, in the steady state ($\beta(\|\epsilon_0\|, k) \rightarrow 0$ as $k \rightarrow \infty$), is given by

$$\alpha_3 = \sqrt{\frac{c_2 + \delta^{-1}c_4^2}{c_1} \cdot \frac{c_2}{c_3 - \delta}} \tag{9}$$

with $c_1,\ c_2,\ c_3,\ c_4$ constant scalars given by

$$c_1 = \min_{1 \leq i \leq N} \lambda_{min}(P_i), \qquad c_2 = \max_{1 \leq i \leq N} \lambda_{max}(P_i)$$

$$c_3 = \min_{1 \leq i \leq N, 1 \leq j \leq N} \lambda_{min}(P_i - (\hat{A}_i - L_iC)^T P_j(\hat{A}_i - L_iC))$$

$$c_4 = (\max_{1 \leq i \leq N} \|\hat{A}_i - L_iC\|) \cdot (\max_{1 \leq i \leq N} \|P_i\|)$$

and

$$\delta \in]0, c_3[$$

From the bound expression (9), the best result is obtained by selecting δ in $]0, c_3[$ which gives the smallest bound α_3. However, selecting the best δ is constrained by the conservative inequalities used in the proof of Theorem 1 to provide the bound (9) and hence will not reduce significantly the gap between the proposed bound and the actual one.

3. MAIN RESULT

The purpose of this section is to improve the gap reduction by including the minimization of the bound α_3 in the observer design procedure that is to select among all possible polytopic observers (3) the one which minimizes the steady state bound. The main contribution is stated in the following Theorem.

Theorem 2. Assume that the following convex optimization problem

[1] A function $\gamma : \mathbb{R} \rightarrow \mathbb{R}$ is a $\mathcal{K}$ function if it is continuous, strictly increasing and $\gamma(0) = 0$.
A function $\beta : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ is a $\mathcal{KL}$ function if, for each $t \geq 0$, the function $\beta(., t)$ is a $\mathcal{K}$ function, and for each fixed $s \geq 0$, the function $\beta(s, .)$ is decreasing and $\beta(s, t) \rightarrow 0$ as $t \rightarrow \infty$.

$$\min_{\substack{P_i = P_i' \\ G_i = G_i' \\ F_i, \alpha}} \quad \alpha$$

under

$$\begin{bmatrix} \mathbf{1} - P_i & A_i'G_i - C'F_i' & A_i'G_i - C'F_i' \\ G_iA_i - F_iC & P_j - 2G_i & \mathbf{0} \\ G_iA_i - F_iC & \mathbf{0} & 2G_i - \alpha\mathbf{1} \end{bmatrix} < \mathbf{0} \tag{10}$$

has a solution $P_i^* \in \mathbb{R}^{n\times n}$, $G_i^* \in \mathbb{R}^{n\times n}$, $F_i^* \in \mathbb{R}^{n\times m}$ and α^*, then the error dynamic (5) with $L_i = G_i^{*-1}F_i^*$ is Input-to-State Stable, that is

$$\|\epsilon_k\| \le \beta(\|\epsilon_0\|, k) + \alpha^*\|v\|_\infty \quad \forall k \tag{11}$$

with $\beta : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ a $\mathcal{KL}$ function.

Proof : Assume that the LMIs in (10) are feasible. Recall that $\tilde{A}_i = A_i - L_iC$. Multiply these inequalities on the left by

$$\mathcal{T} = \begin{bmatrix} \epsilon' & (\tilde{A}_i\epsilon + v)' & v' \end{bmatrix}$$

and on the right by $\mathcal{T}'$ one gets

$$\epsilon'(\mathbf{1} - P_i)\epsilon + (\tilde{A}_i\epsilon + v)'P_j(\tilde{A}_i\epsilon + v) - \alpha v'v < 0$$

which is equivalent by Schur formula to

$$\begin{bmatrix} \epsilon'(\mathbf{1} - P_i) - \alpha v'v & (\tilde{A}_i\epsilon + v)'P_j \\ P_j(\tilde{A}_i\epsilon + v) & P_j \end{bmatrix} < \mathbf{0}$$

Multiply by $\hat{\xi}^i$ and sum, multiply by $\hat{\xi}^j$ and sum, and using Schur formula we obtain

$$\epsilon_k^T(\mathcal{A}^T\mathcal{P}_{k+1}\mathcal{A} - \mathcal{P}_k)\epsilon_k < -\epsilon'\epsilon + \alpha v'v$$

with $\mathcal{P}_k = \sum_{i=1}^N \hat{\xi}_k^i P_i$ which is nothing than

$$V(\epsilon_{k+1}, \hat{\xi}_{k+1}) - V(\epsilon_k, \hat{\xi}_k) < -\epsilon'\epsilon + \alpha v'v \tag{12}$$

with

$$V = \epsilon_k^T\mathcal{P}_k\epsilon_k$$

Moreover, the feasibility of the LMIs (10) implies that

$$P_i > \mathbf{1} \quad \text{and } P_i < 2G_i < \alpha\mathbf{1} \quad \text{and hence} \quad \alpha > 1$$

then

$$\|\epsilon_k\|^2 \le V(\epsilon_k, \hat{\xi}_k) \le \alpha\|\epsilon_k\|^2 \;\; \forall\epsilon_k \in \mathbb{R}^n, \;\; \forall\hat{\xi}_k \in \mathcal{S}, \;\; \forall k \tag{13}$$

Consequently, from (12) and (13), one has :

$$V(\epsilon_{k+1}, \hat{\xi}_{k+1}) \le (1 - \frac{1}{\alpha})V(\epsilon_k, \hat{\xi}_k) + \alpha\|v_k\|^2 \tag{14}$$

Applying the Gronwall-lemma in the discrete time case, we obtain :

$$\begin{aligned} V(\epsilon_k, \hat{\xi}_k) \le\; & (1 - \frac{1}{\alpha})^k V(\epsilon_0, \hat{\xi}_0) + \\ & \alpha\sum_{l=0}^{k-1}(1 - \frac{1}{\alpha})^{k-l-1}\|v_l\|^2 \\ \le\; & (1 - \frac{1}{\alpha})^k V(\epsilon_0, \hat{\xi}_0) + \alpha^2\|v\|_\infty^2 \end{aligned}$$

Finally, by using again (13), taking the square root, the main inequality is obtained :

$$\|\epsilon_k\| \le \sqrt{\alpha}(1 - \frac{1}{\alpha})^{k/2}\|\epsilon_0\| + \alpha\|v\|_\infty \tag{15}$$

This inequality completes the proof according to the definition of ISS. Moreover, as $\alpha > 1$, $\sqrt{\alpha}(1 - \frac{1}{\alpha})^{k/2}\|\epsilon_0\| \to 0$ when $k \to \infty$. □

The convex optimization problem given in the previous Theorem states that the best quantity α which explicitly bounds the state reconstruction error in the steady state is obtained by selecting among all possible solutions α, P_i, G_i, and F_i for the LMIs (10) the ones leading to the smallest value for α.

4. ILLUSTRATIVE EXAMPLE

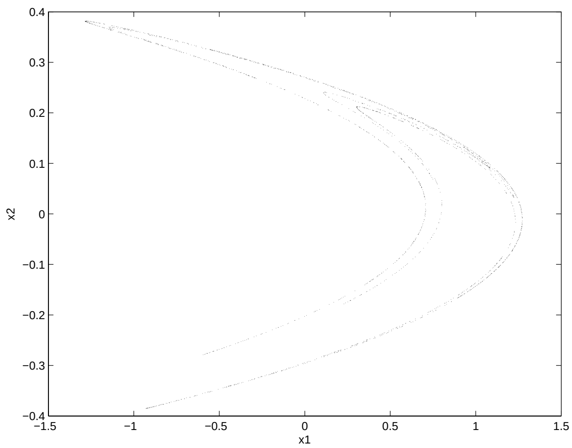

Fig. 1. Chaotic behavior

To illustrate the improvement provided by Theorem 2, we consider the same example as in (Millerioux *et al.*, 2004). The system is given by

$$A(\rho_k) = \begin{bmatrix} \rho_k & 1 \\ 0.3 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad D = \mathbf{0}$$

with

$$\rho_k(x_k) = -1.4x_k^1, \qquad \text{and} \qquad x_k = \begin{bmatrix} x_k^1 \\ x_k^2 \end{bmatrix}$$

This non-linear system has a chaotic behavior (Figure 1) which implies that the state trajectory is bounded. The goal is to assess the impact of a bounded disturbance acting on the signal $y_k = Cx_k = x_k^1$ and consequently on the parameter ρ_k. It is assumed that a uniformly distributed disturbance w_k in the range -0.0025 and 0.0025 acts on the system output. As the parameter ρ_k is directly related to the first state component, the used scheduling parameter $\hat{\rho}$, given by the noisy output, in the observer structure is different from the actual parameter and the difference $\|\rho_k - \hat{\rho}_k\|$ is bounded. From a simple numerical study, we find that $\|x_k\|_\infty = 1.34$ and hence $\|\rho_k - \hat{\rho}_k\|_\infty < \Delta$ with $\Delta = 0.007$. For this system, a polytopic description with 2 vertices

$$A_1 = \begin{bmatrix} -1.7850 & 1 \\ 0.3 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1.7995 & 1 \\ 0.3 & 0 \end{bmatrix}$$

is used and an observer of the form (3) was designed in (Millerioux *et al.*, 2004). This observer is characterized by the gain matrices

$$L_1 = \begin{bmatrix} -1.7878 \\ 0.3 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 1.7982 \\ 0.3 \end{bmatrix} \qquad (16)$$

and leads to a bound $\alpha_3 = 18.21$. The numerical computation of $\|\epsilon_k\|$ in the steady state (figure 2) shows that the norm is always less than 0.01 and so, less than $\alpha_3\|v\|_\infty = 0.16$.

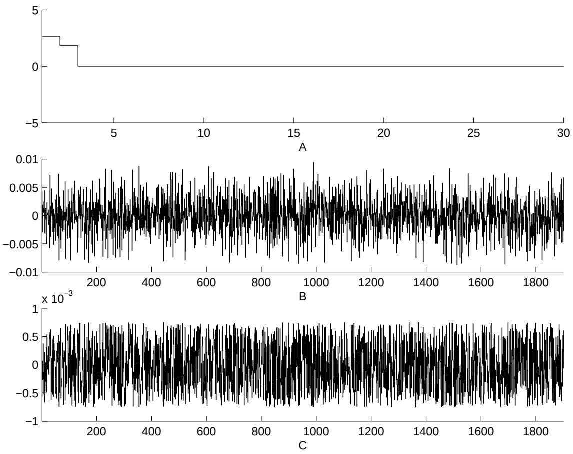

Fig. 2. A: $\|\epsilon_k\|$ with respect to k. B,C: each component of ϵ_k in the steady state.

Notice the large gap between the proposed bound and the actual observer behavior. As the observer designed in (Millerioux *et al.*, 2004) is not unique, one may obtain an observer with a very large gap. As an example, the observer characterized by the gain matrices

$$L_1 = \begin{bmatrix} -2.9448 \\ 0.6000 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 2.3759 \\ 0.2999 \end{bmatrix}$$

satisfies the LMIs (6) with

$$P_1 = \begin{bmatrix} 0.2773 & 0.2329 \\ 0.2329 & 0.5597 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 0.1836 & 0.1268 \\ 0.1268 & 1.1971 \end{bmatrix},$$

$$G_1 = \begin{bmatrix} 0.3065 & 0.4854 \\ 0.4867 & 2.2146 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 0.2806 & 0.2338 \\ -0.0079 & 1.0011 \end{bmatrix}$$

and leads to a bound $\alpha_3 = 5812$. The numerical computation of $\|\epsilon_k\|$ in the steady state (figure 3) shows that the norm is always less than 0.02 which is quite far from $\alpha_3\|v\|_\infty = 67.89$. Figure 4 shows the effect of δ, involved in the bound expression (9), on α_3. One can notice that the improvement provided by an optimization using δ as a degree of freedom is not sufficient.

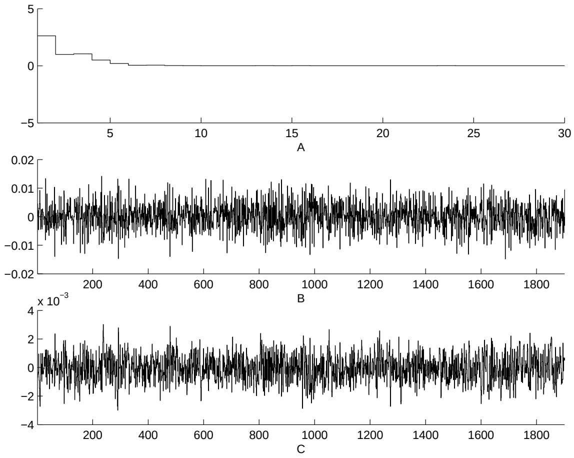

Fig. 3. A: $\|\epsilon_k\|$ with respect to k. B,C: each component of ϵ_k in the steady state.

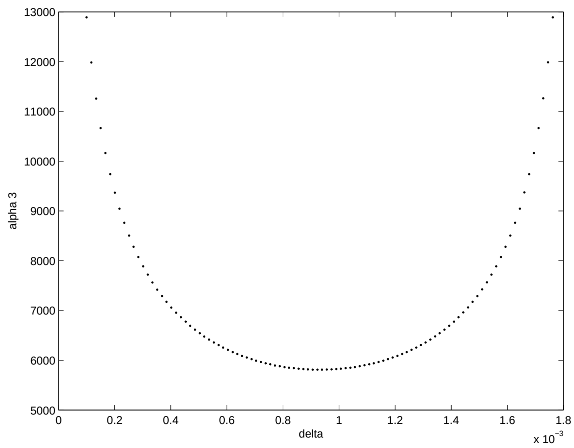

Fig. 4. α_3 versus δ

Now using Theorem 2, the proposed optimization problem leads to a solution with

$$L_1 = \begin{bmatrix} -1.7850 \\ 0.3000 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 1.7995 \\ 0.3000 \end{bmatrix}$$

and a bond $\alpha = 2.6180$. The numerical computation of $\|\epsilon_k\|$ in the steady state (figure 5) shows that the norm is always less than 0.01 and so, less than $\alpha\|v\|_\infty = 0.0220$. The gap between the proposed bound and the actual behavior has been significantly improved. Notice that the observer gains are close to the ones obtained "randomly" in (Millerioux *et al.*, 2004) and recalled in (16). This illustrates the fact that one may get a "good" observer using the approach presented in

(Millerioux *et al.*, 2004) but with very conservative bound. This conservatism is also illustrated by the computation of the bound expression α_3 using the results obtained by the optimization problem (10), that is $\alpha_3 \|v\|_\infty = 0.0808$ instead of $\alpha\|v\|_\infty = 0.0220$ guaranteed by Theorem 2.

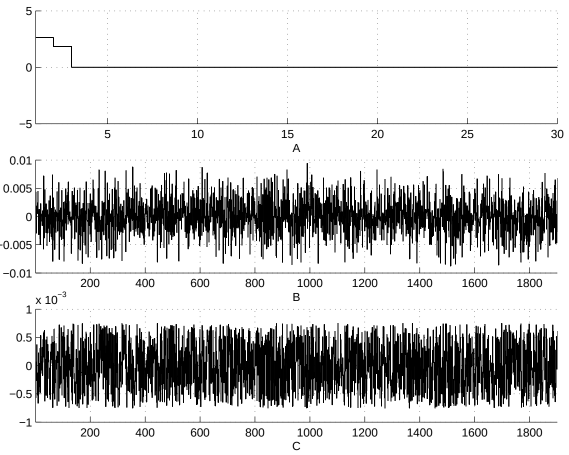

Fig. 5. A: $\|\epsilon_k\|$ with respect to k. B,C: each component of ϵ_k in the steady state.

5. CONCLUSION

In this paper, a convex optimization problem has been proposed to improve the state reconstruction bound proposed in (Millerioux *et al.*, 2004) for LPV systems with uncertain parameters (estimated parameters with finite accuracy or noisy parameters). The result is obtained by including the bound minimization in the observer design procedure. When compared to methods based on H_∞ disturbance rejection or peak-to-peak optimization which focus on the worst case disturbance effect including during the transient, here the proposed solution is based on the minimization of the reconstruction error bound in the steady state which is a more realistic. Such a bound has a practical interest in engineering when considering for instance the truncation error arising in digital systems. And yet, assessing the impact of the resulting state reconstruction error, for fault detection purposes or non destructive measurement, is of first importance.

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