ON CONTROL BY INTERCONNECTION OF PORT HAMILTONIAN SYSTEMS

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Abstract: In the standard approach of control by interconnection the plant and the controller are assumed to be passive and coupled via a power–preserving interconnection—generating an overall passive system with storage function the sum of the plant and controller storage functions. To achieve stabilization of a desired equilibrium one must make this point a minimum of the new storage function. Towards this end, dynamic invariants—called Casimirs—are first computed. Restricting the dynamics to the level sets of the Casimirs, the overall storage function becomes a *bona fide* function of the plant states and the storage function can be shaped. Unfortunately, this procedure is applicable only if one fixes the initial conditions of the controller to some specific values. To remove this drawback we propose in this paper to carry out the stability analysis in the full plant and controller state spaces. The new storage function is then the sum of the plant and the controller Hamiltonians and an arbitrary functions of the corresponding Casimir functions. We also provide some examples which illustrate the possibilities and limitations of the new method.

Keywords: Interconnection, Lyapunov stability.

1. INTRODUCTION

Port Controlled Hamiltonian (PCH) models are natural candidates to describe many physical systems (van der Schaft, 2000). Basically, PCH systems are systems defined with respect to a power–conserving geometric structure capturing the basic interconnection laws, and a *Hamiltonian* function given by the total stored energy of the system. A very important property of PCH systems, which may be directly inferred from the structure matrix, is the existence of dynamical invariants *independent* of the Hamiltonian called *Casimir functions*, the existence of which has an immediate consequence on stability analysis.

In this paper we are interested in stabilization of equilibria via *Control by Interconnection* (van der Schaft, 2000; Ortega *et al.*, 2001), whose central component is the generation of Casimir functions. In the standard approach the plant and the controller are PCH systems coupled via a power–preserving interconnection—generating an overall PCH system with storage function the sum of the plant and controller storage functions. To achieve stabilization one

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must make the desired equilibrium a minimum of the new storage function. Restricting the dynamics to the level sets of the Casimirs, the overall storage function becomes a *bona fide* function of the plant states and the storage function can be shaped. Unfortunately, this procedure is applicable only if one fixes the initial conditions of the controller to some specific values.

To remove this drawback in this paper we propose not to restrict the dynamics to the Casimir level sets, but to carry out the stability analysis in the full plant and controller state spaces. The new storage function, defined in the full state space, is then the sum of the plant and the controller Hamiltonians and an arbitrary function of the corresponding Casimir functions. Although we get nice results for some electrical systems and "fully actuated" mechanical systems, in the case of electromechanical systems (an also for some electrical systems) we see that due to the dissipation obstacle we cannot assign the desired equilibrium point.

2. CONTROL BY INTERCONNECTION: GENERAL THEORY

Control by Interconnection is a controller design procedure to stabilize the equilibria of passive systems via passive controllers (van der Schaft, 2000; Ortega *et al.*, 2001). [1] In this paper we are interested in PCH systems of the form

$$\begin{cases} \dot{x} \;=\; [J(x)-R(x)]\dfrac{\partial H}{\partial x}+g(x)u \\ y \;=\; g^{\top}(x)\dfrac{\partial H}{\partial x} \end{cases} \tag{1}$$

where $x \in \mathcal{X}$ is the state vector, $u \in \mathbb{R}^m$, $m < n$ is the control action, $H : \mathcal{X} \to \mathbb{R}$ is the total stored energy, and $J(x) = -J^{\top}(x)$, $R(x) = R^{\top}(x) \geq 0$ are the natural interconnection and damping matrices, respectively.

The controller is also a PCH system of the form

$$\begin{cases} \dot{\xi} \;=\; [J_c(\xi)-R_c(\xi)]\dfrac{\partial H_c}{\partial \xi}+g_c(\xi)u_c \\ y_c \;=\; g_c^{\top}(\xi)\dfrac{\partial H_c}{\partial \xi} \end{cases} \tag{2}$$

with state $\xi \in \mathcal{X}_c$, input $u_c \in \mathbb{R}^p$, $J_c(\xi) = -J_c^{\top}(\xi)$, $R_c(\xi) = R^{\top}(\xi) \geq 0$, and $H_c : \mathcal{X}_c \to \mathbb{R}$ the energy of the controller.

Interconnecting plant (1) and controller (2) via the standard (power preserving) feedback interconnection [2]

$$u = -y_c, \quad u_c = y \tag{3}$$

[1] As indicated in (Ortega *et al.*, 2001) the procedure is actually applicable for a larger class of plants and controllers, namely, those satisfying the energy balance equation—that may be even unstable.
[2] We present here the simplest case of unitary feedback, but the results carry through for other more general power preserving interconnections.

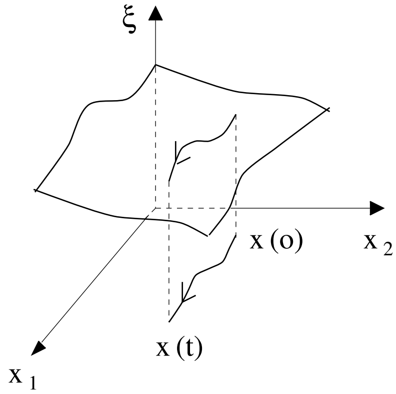

Fig. 1. Invariant subspaces.

we get that the composed system is still PCH and can be written as

$$\begin{bmatrix}\dot{x}\\ \dot{\xi}\end{bmatrix} = [J_{cl}(x,\xi)-R_{cl}(x,\xi)]\begin{bmatrix}\dfrac{\partial H}{\partial x}(x)\\ \dfrac{\partial H_c}{\partial \xi}(\xi)\end{bmatrix} \tag{4}$$

$$\begin{bmatrix}y\\ y_c\end{bmatrix} = \begin{bmatrix}g(x) & 0\\ 0 & g_c(\xi)\end{bmatrix}\begin{bmatrix}\dfrac{\partial H}{\partial x}(x)\\ \dfrac{\partial H_c}{\partial \xi}(\xi)\end{bmatrix}$$

with state space the product space $\mathcal{X} \times \mathcal{X}_c$,, the total Hamiltonian $H_{cl}(x,\xi) = H(x) + H_c(\xi)$ and the matrices $J_{cl}(x,\xi)$, $R_{cl}(x,\xi)$ defined as

$$J_{cl}(x,\xi) = \begin{bmatrix} J(x) & -g(x)g_c^{\top}(\xi)\\ g_c(\xi)g^{\top}(x) & J_c(\xi)\end{bmatrix} \tag{5}$$

$$R_{cl}(x,\xi) = \begin{bmatrix} R(x) & 0\\ 0 & R_c(\xi)\end{bmatrix} \tag{6}$$

To achieve the stabilization objective we shape the energy function $H_{cl}(x,\xi)$ to assign a minimum at the desired equilibrium point $x_\star$. For, we restrict the motion of the closed–loop system to a certain subspace of the extended state space (x,ξ) by rendering it invariant (see Fig. 1). [3] Consider, for instance, the subspace

$$\Omega = \{(x,\xi) | \mathcal{C}(x,\xi) = 0\}$$

where $\mathcal{C}(x,\xi) = \xi - F(x)$. The closed–loop total energy restricted to the level sets of Ω becomes $H(x) + H_c(F(x)+c)$, $c \in \mathbb{R}$. Now, given $F(x)$, we can shape this total energy function with a suitable selection of the controller energy $H_c(\xi)$.

It is clear that Ω is invariant if and only if

$$\left.\frac{d}{dt}\mathcal{C}(x,\xi)\right|_{\Omega} = 0, \tag{7}$$

along the dynamics of the closed–loop system (4). In the Control by Interconnection method we look for *Casimir functions*, which are dynamic invariants independent of the Hamiltonian function. That is, we

[3] A set $\Omega \subset \mathcal{X} \times \mathcal{X}_c$ is invariant if $(x(0),\xi(0)) \in \Omega \Rightarrow (x(t),\xi(t)) \in \Omega \; \forall t \geq 0$

look for solutions of the partial differential equations (PDEs)

$$\left[-\frac{\partial^\top F}{\partial x} \,\vdots\, I_m \right] \begin{bmatrix} J(x) - R(x) & -g(x)g_c^\top(\xi) \\ g_c(\xi)g^\top(x) & J_c(\xi) - R_c(\xi) \end{bmatrix} = 0 \tag{8}$$

As shown in (van der Schaft, 2000), the admissible functions $F(x)$ are characterized by the following proposition.

Proposition 1. (van der Schaft, 2000) $F(x)$ is a solution of the PDEs (8) if and only if

$$\frac{\partial^\top F}{\partial x}(x)J(x)\frac{\partial F}{\partial x}(x) = J_c(\xi) \tag{9}$$

$$R(x)\frac{\partial F}{\partial x}(x) = R_c(\xi) = 0 \tag{10}$$

$$\frac{\partial^\top F}{\partial x}(x)J(x) = g_c(\xi)g^\top(x) \tag{11}$$

Notice that the function $H(x)+H_c(F(x)+c)$ depends on a constant vector c determined by the controller initial conditions. Although in analog controller design this is natural, for more practical discrete–time implementations it is desirable to remove this restriction. Removing this restriction is one of the main motivations of the present work.

Example 1. (van der Schaft, 2000) Consider the equations of a normalized pendulum

$$\ddot{q} + \sin\, q + d\dot{q} = u, \tag{12}$$

with d a positive damping constant. The total energy is given by $H(q,p) = \frac{1}{2}p^2 + (1 - \cos\, q)$. The solution of (8) are functions of the form $F(q) = q$. Let $q_\star$ be a desired position of the pendulum. The objective is to shape the potential energy $P(q) = 1 - \cos\, q$ in such a way that it has a minimum at $q = q_\star$. Choosing

$$P_c(\xi) = \cos\, \xi + \frac{1}{2}(\xi - q_\star)^2$$

and substituting $\xi = G(q) + c = q + c$ we get the shaped potential energy as

$$\begin{aligned} P_d(q) &= P(q) + P_c(G(q) + c) \\ &= \cos(q + c) + (1 - \cos\, q) + \frac{1}{2}(q + c - q_\star) \end{aligned}$$

However, in order to obtain a minimum at $q = q_\star$ the controller needs to be initialized in such a way that $c = 0$.

Remark 1. Equation (10) encodes the so–called *"dissipation obstacle"* of this methodology (Ortega *et al.*, 2001), and it represents a necessary condition for the existence of the Casimir functions, roughly speaking, equation (10) says that the Casimirs cannot depend on the coordinates where there is dissipation, i.e. dissipation is admissible only on the coordinates of the energy function that do not require shaping.

3. CASIMIRS IN THE EXTENDED STATE SPACE.

In this section we propose a modification of the Control by Interconnection method to overcome the problem of controller initialization mentioned above. The key idea is to analyze the closed-loop system (4) in the extended state space $\mathcal{X}\times\mathcal{X}_c$, with the control objective of stabilization of a desired equilibrium $(x_\star, \xi_\star)$, for some $\xi_\star$ satisfying the equilibrium equations of (4). To this end, we consider general Casimir functions $C : \mathcal{X} \times \mathcal{X}_c \to \mathcal{X}_c$, which means that we are looking for solutions of the PDEs

$$\left[\frac{\partial^\top C}{\partial x}(x,\xi) \quad \frac{\partial^\top C}{\partial \xi}(x,\xi) \right] [J_{cl}(x,\xi) - R_{cl}(x,\xi)] = 0 \tag{13}$$

where $J_{cl}(x,\xi)$ and $R_{cl}(x,\xi)$ are given by (5) and (6), respectively.

Equivalently, (13) can be written as

$$\begin{aligned} &\frac{\partial^\top C}{\partial x}(x,\xi)[J(x) - R(x)] + \frac{\partial^\top C}{\partial \xi}(x,\xi)g_c(\xi)g^\top(x) = 0 \\ &\frac{\partial^\top C}{\partial x}(x,\xi)g(x)g_c^\top(\xi) - \frac{\partial^\top C}{\partial \xi}(x,\xi)[J_c(\xi) - R_c(\xi)] = 0 \end{aligned} \tag{14}$$

Post-multiplying first equation of (14) by $\frac{\partial C}{\partial x}(x,\xi)$ and second equation by $\frac{\partial C}{\partial \xi}(x,\xi)$ yields

$$\frac{\partial^\top C}{\partial x}(x,\xi)J(x)\frac{\partial C}{\partial x}(x,\xi) = \frac{\partial^\top C}{\partial \xi}(x,\xi)J_c(\xi)\frac{\partial C}{\partial \xi}(x,\xi) \tag{15}$$

$$-\frac{\partial^\top C}{\partial x}(x,\xi)R(x)\frac{\partial C}{\partial x}(x,\xi) = \frac{\partial^\top C}{\partial \xi}(x,\xi)R_c(\xi)\frac{\partial C}{\partial \xi}(x,\xi) \tag{16}$$

Since by assumption $R(x) \geq 0$, $R_c(\xi) \geq 0$, then (16) implies

$$\begin{aligned} &\frac{\partial^\top C}{\partial x}(x,\xi)R(x)\frac{\partial C}{\partial x}(x,\xi) = 0, \\ &\frac{\partial^\top C}{\partial \xi}(x,\xi)R_c(\xi)\frac{\partial C}{\partial \xi}(x,\xi) = 0 \end{aligned}$$

The above equations are equivalent to

$$R(x)\frac{\partial C}{\partial x}(x,\xi) = R_c(\xi)\frac{\partial C}{\partial \xi}(x,\xi) = 0 \tag{17}$$

Summarizing we have obtained:

Proposition 2. $C(x,\xi)$ is solution of the PDEs (13) (and thus are Casimir functions for the closed-loop PCH system (2)) if and only if

$$\begin{aligned} \frac{\partial^\top C}{\partial x}(x,\xi)J(x)\frac{\partial C}{\partial x}(x,\xi) &= \frac{\partial^\top C}{\partial \xi}(x,\xi)J_c(\xi)\frac{\partial C}{\partial \xi}(x,\xi) \\ R(x)\frac{\partial C}{\partial x}(x,\xi) &= R_c(\xi)\frac{\partial C}{\partial \xi}(x,\xi) = 0 \\ \frac{\partial^\top C}{\partial x}(x,\xi)J(x) &= -\frac{\partial^\top C}{\partial \xi}g_c(\xi)g^\top(x) \\ \frac{\partial^\top C}{\partial \xi}(x,\xi)J_c(\xi) &= \frac{\partial^\top C}{\partial x}(x,\xi)g(x)g_c^\top(\xi) \end{aligned} \tag{18}$$

Proof. Only the last two equations need to be shown, which are easily obtained by substituting (17) into (14). ■

Remark 2. We see here again that the second equation of (18) represents the "dissipation obstacle", but to overcome this is not in the scope of the theory studied here.

Remark 3. We have considered the case in which we wish to relate all controller state variables to the plant state via Casimir functions $C(x,\xi)$. As discussed in (van der Schaft, 2000) other options are possible and should be explored.

4. STABILITY ANALYSIS

Consider the plant (1) and the the PCH controller (2) with power preserving interconnection (3). Suppose that there exist Casimirs for the plant controller interconnection satisfying (18). A Lyapunov function candidate is built as the sum of the plant and controller Hamiltonians and compositions of the Casimir functions as

$$V(x,\xi) = H(x) + H_c(\xi) + \Psi(C(x,\xi)) \qquad (19)$$

where $\Psi : \mathcal{X}_c \to \mathbb{R}$ is an arbitrary $\mathcal{C}_1$ function. We have

$$\begin{aligned}\frac{d}{dt}V(x,\xi) = &-\frac{\partial^\top H}{\partial x}(x)R(x)\frac{\partial^\top H}{\partial x}(x)\\ &-\frac{\partial^\top H_c}{\partial \xi}(\xi)R_c(\xi)\frac{\partial^\top H_c}{\partial \xi}(\xi) \le 0\end{aligned}$$

where we have used (13).

The next step is to shape the closed-loop energy in the extended state space (x,ξ) in such a way that it has a minimum at $(x_\star,\xi_\star)$. Therefore $V(x,\xi)$ should satisfy

$$\begin{bmatrix}\frac{\partial}{\partial x}[H(x)+\Psi(C(x,\xi))]\mid_{(x_\star,\xi_\star)}\\ \frac{\partial}{\partial \xi}[H_c(\xi)+\Psi(C(x,\xi))]\mid_{(x_\star,\xi_\star)}\end{bmatrix} = 0 \qquad (20)$$

and

$$\left.\begin{bmatrix}\frac{\partial^2}{\partial x^2}[H(x)+\Psi(C(x,\xi))] & \frac{\partial^2}{\partial \xi\partial x}\Psi(C(x,\xi))]\\ \frac{\partial^2}{\partial x\partial \xi}\Psi(C(x,\xi))] & \frac{\partial^2}{\partial \xi^2}[H_c(\xi)+\Psi(C(x,\xi))]\end{bmatrix}\right|_{(x_\star,\xi_\star)} \ge 0 \qquad (21)$$

Suppose that $V(x,\xi)$ has a strict local minimum at $(x_\star,\xi_\star)$. Furthermore assume that the largest invariant set under the dynamics (4) contained in

$$\begin{aligned}\{(x,\xi)\in\mathcal{X}\times\mathcal{X}_c \mid &\frac{\partial^\top H}{\partial x}(x)R(x)\frac{\partial^\top H}{\partial x}(x)=0,\\ &\frac{\partial^\top H_c}{\partial \xi}(\xi)R_c(\xi)\frac{\partial^\top H_c}{\partial \xi}(\xi)=0\}\end{aligned}$$

equals $(x_\star,\xi_\star)$. Then $(x_\star,\xi_\star)$ is a locally asymptotically stable equilibrium of (1)

5. ILLUSTRATIVE EXAMPLES

In this section we illustrate with some examples the application of Casimir functions in the extended state space plant–controller to the Control by Interconnection methodology.

Example 2. Consider a mechanical system with damping and actuated by external forces u described as a PCH system

$$\begin{aligned}\begin{bmatrix}\dot q\\ \dot p\end{bmatrix} &= (\begin{bmatrix}0 & I_k\\ -I_k & 0\end{bmatrix} - \begin{bmatrix}0 & 0\\ 0 & D(q)\end{bmatrix})\begin{bmatrix}\frac{\partial H}{\partial q}\\ \frac{\partial H}{\partial p}\end{bmatrix} + \begin{bmatrix}0\\ B(q)\end{bmatrix}u\\ y &= B^\top(q)\frac{\partial H}{\partial p}\end{aligned} \qquad (22)$$

with $x = \begin{bmatrix}q\\ p\end{bmatrix}$, where $q\in\mathbb{R}^k$ are the generalized configuration coordinates, $p\in\mathbb{R}^k$ the generalized momenta, and $D(q) = D^\top(q) \ge 0$ is the damping matrix. If $D(q) > 0$, then it is said that the system is fully damped. The outputs $y\in\mathbb{R}^m$ are the generalized velocities corresponding to the generalized external forces $u\in\mathbb{R}^m$. We consider the case where the Hamiltonian $H(q,p)$ takes the form

$$H(q,p) = \frac{1}{2}p^\top M^{-1}(q)p + P(q) \qquad (23)$$

where $M(q) = M^\top(q) > 0$ is the generalized inertia matrix, $\frac{1}{2}p^\top M^{-1}(q)p = \frac{1}{2}\dot q^\top M(q)\dot q$ is the kinetic energy, and $P(q)$ is the potential energy of the system.

Consider now the PCH controller (2), then the equations (18) for $C = (C_1(x,\xi),...,C_m(x,\xi))^\top$ take the form

$$\begin{gathered}\frac{\partial^\top C}{\partial p}\frac{\partial C}{\partial q} - \frac{\partial^\top C}{\partial q}\frac{\partial C}{\partial p} = \frac{\partial^\top C}{\partial \xi}J_c(\xi)\frac{\partial C}{\partial \xi}\\ D(q)\frac{\partial C}{\partial p} = 0 = R_c(\xi)\frac{\partial C}{\partial \xi}\\ \frac{\partial^\top C}{\partial p} = 0,\ \text{and}\ \frac{\partial^\top C}{\partial q} = -\frac{\partial^\top C}{\partial \xi}g_c(\xi)B^\top(q)\end{gathered}$$

or equivalently

$$\frac{\partial^\top C}{\partial \xi}J_c = 0,\quad \frac{\partial C}{\partial p} = 0,\quad \frac{\partial^\top C}{\partial q} + \frac{\partial^\top C}{\partial \xi}g_c(\xi)B(q) = 0 \qquad (24)$$

Hence if we can solve the PDE in the above equation, then the closed-loop port-Hamiltonian system with $J_c = 0$ admits Casimirs $C_i(x,\xi)$, $i = 1,...,m$. leading to a closed loop system

$$\begin{aligned}\begin{bmatrix}\dot q\\ \dot p\\ \dot \xi\end{bmatrix} &= \begin{bmatrix}0 & I_k & 0\\ -I_k & 0 & -B(q)g_c^\top(\xi)\\ 0 & g_c(\xi)B^\top(q) & 0\end{bmatrix}\begin{bmatrix}\frac{\partial H}{\partial q}\\ \frac{\partial H}{\partial p}\\ \frac{\partial H_c}{\partial \xi}\end{bmatrix}\\ y &= B^\top(q)\frac{\partial H}{\partial x}\\ y_c &= g_c^\top(\xi)\frac{\partial H_c}{\partial \xi}\end{aligned} \qquad (25)$$

If $H(q,p)$ is given as in (23), then the candidate Lyapunov function is built as

$$V(q,p,\xi) = \frac{1}{2}p^\top M(q)p + P(q) + H_c(\xi) + \Psi(C(x,\xi))$$

where $H_c(\xi)$ and $\Psi(C(x,\xi))$ are chosen to satisfy (20) and (21).

Example 3. Consider again the case of a normalized pendulum

$$\ddot{q} + \sin q + d\dot{q} = u$$

with d a positive damping constant, and the total energy function given as $H(q,p) = \frac{1}{2}p^2 + (1 - \cos q)$. The solution to (24) should be a function of the form $C(q,\xi) = q - \xi$

Let $(q_\star, \xi_\star)$ be the desired equilibrium. We shape the potential energy $P(q)$ in such a way that it has a minimum at $q = q_\star, \xi = \xi_\star$. This can be achieved by choosing a controller Hamiltonian of the form

$$H_c(\xi) = \frac{1}{2}\beta(\xi - \xi_\star - \frac{1}{\beta}\sin q_\star)^2$$

and the function $\Psi(C(q,\xi)) = \Psi(q - \xi)$ as

$$\Psi(q-\xi) = \frac{1}{2}k(q - q_\star - (\xi - \xi_\star) - \frac{1}{k}\sin q_\star)^2$$

where β and k are chosen to satisfy (20) and (21). Simple computations show that β and k should be such that

$$\cos q_\star + k > 0, \ \beta\cos q_\star + k\cos q_\star + k\beta > 0$$

The resulting input u, according to (2) and (3), is then given by

$$u = -\frac{\partial H_c}{\partial \xi}(\xi) = -\beta(\xi - \xi_\star - \frac{1}{\beta}\sin q_\star)$$

Remark 4. In the same way we can also stabilize a system of n "fully actuated" pendulums, in which case we have to solve n p.d.e.'s of the form (24), in order to find the corresponding Casimir functions.

Example 4. The model of a permanent magnet synchronous machine (Petrovic *et al.*, 2001), in the case of an isotropic rotor, in the dq frame can be written in PCH form (1), with the state vector $x = [x_1, x_2, x_3]^\top$ and

$$J(x) = \begin{bmatrix} 0 & \frac{LP}{J}x_3 & 0 \\ -\frac{LP}{J}x_3 & 0 & -\Phi \\ 0 & \Phi & 0 \end{bmatrix},$$

$$R(x) = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad g = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix},$$

where x_1, x_2 are the stator currents, x_3 is the angular velocity, P is the number of pole pairs, L is the stator inductance, R_s is the stator winding resistance, and Φ and J are the dq back emf constant and the moment of inertia both normalized with P. The inputs are the stator voltages $[v_d, \ v_q]^\top$. The energy function of the system is given by

$$H(x) = \frac{1}{2}\left(Lx_1^2 + Lx_2^2 + \frac{J}{P}x_3^2\right)$$

The desired equilibrium to be stabilized is usually selected based on the so–called "maximum torque per ampere" principle as $x_\star = [0, \ \frac{L\tau_l}{P\Phi}, \ \frac{J}{P}x_{3\star}]^\top$ where τ_l is the constant load torque. [4]

Interconnecting the plant system with a PCH control

$$\begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \end{bmatrix} = \begin{bmatrix} u_{c1} \\ u_{c2} \end{bmatrix}, \quad \begin{bmatrix} y_{c1} \\ y_{c2} \end{bmatrix} = \begin{bmatrix} \frac{\partial H_c}{\partial \xi_1}(\xi_1,\xi_2) \\ \frac{\partial H_c}{\partial \xi_2}(\xi_1,\xi_2) \end{bmatrix}$$

via the power preserving interconnection

$$v_d = -y_{c1}, \quad v_q = -y_{c2},$$
$$u_{c1} = \frac{\partial H}{\partial x_1}(x), \quad u_{c2} = \frac{\partial H}{\partial x_2}(x)$$

yields the closed–loop system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{\xi}_1 \\ \dot{\xi}_2 \end{bmatrix} = \begin{bmatrix} -R_s & \frac{LP}{J}x_3 & 0 & -1 & 0 \\ -\frac{LP}{J}x_3 & -R_s & -\Phi & 0 & -1 \\ 0 & \Phi & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial x_1} \\ \frac{\partial H}{\partial x_2} \\ \frac{\partial H}{\partial x_3} \\ \frac{\partial H_c}{\partial \xi_1} \\ \frac{\partial H_c}{\partial \xi_2} \end{bmatrix} \tag{26}$$

Using Proposition 2, we get that the Casimir function is given by $C = \frac{1}{\Phi}x_3 - \xi_2$. Thus, the resulting Lyapunov function would be of the form (19)

$$V(x,\xi) = \frac{1}{2}\left(Lx_1^2 + Lx_2^2 + \frac{J}{P}x_3^2\right) + H_c(\xi) + \Psi(\frac{1}{\Phi}x_3 - \xi_2).$$

However, we can see that the equilibrium assignment condition (20) cannot be satisfied, because we need to shape both x_2 and x_3 to assign $x_\star$, and the Casimir depends only on x_3. To overcome this problem, the interconnection matrix $J(x)$ should be modified, but this is not possible with the *Control by Interconnection* technique.

In general, it is not possible to apply the Control by Interconnection to the family of electromechanical systems described in (Rodriguez and Ortega, 2003). Firstly, in most cases, the closed–loop matrix $J_{cl}(x,\xi) - R_{cl}(x,\xi)$ is full–rank. Secondly, even if we can determine the Casimirs—as in the case of the permanent magnet synchronous machine—, these functions do not depend on the coordinates we need to shape. The source of the problem is the lack of interconnection between the electrical and mechanical

[4] In the PCH modeling of the permanent magnet synchronous machine, τ_l acts as a perturbation to the system.

subsystems, which can be solved modifying the interconnection matrix $J(x)$ (Ortega *et al.*, 2001; Ortega *et al.*, 2002).

In the case of electromechanical systems, using a control input $u = -\frac{\partial H_c}{\partial \xi} + \bar{v}$, with $\bar{v}$ a constant input, leads to a forced Hamiltonian system with dissipation. The analysis of (Maschke *et al.*, 2000) also allows to modify the interconnection structure to generate Lyapunov function for nonzero equilibria. However, even if Casimirs can be obtained (namely, microelectronics actuators, magnetic levitation system, etc), the stability analysis reveals that the minimum cannot be assigned. Hence, this issue remains open.

6. CONCLUSIONS AND FUTURE WORK

In this paper we have shown an extension of the Control by Interconnection methodology, to stabilize a system in the extended plant-controller state, and a construction of a Lyapunov function based on the plant and controller Hamiltonians and the corresponding Casimir functions.

Many problems and questions remain open, among then we might cite:

• In its general formulation Control by Interconnection consists of the power–preserving interconnection of two passive systems that admit the existence of dynamic invariants. In this paper we have followed (van der Schaft, 2000; Ortega *et al.*, 2001) and considered PCH systems—which are a particular class of passive systems—and taken the "natural" port variables to define the passive map. Some recent research has established the existence of alternative passive maps, even for PCH systems (Jeltsema *et al.*, 2004; Pérez *et al.*, 2004). With these new port variables the interconnected system might admit new dynamic invariants that we can use for energy shaping.

• As indicated in the paper, in some practical applications there are no Casimirs or even when they exist they do not depend on the coordinates that need to be "shaped", hence we cannot assign the minimum to the total energy functions. An alternative to Casimirs, already indicated in (Ortega *et al.*, 2001), is the generation of first integrals, that is, of solutions of (7). This is a set of PDEs whose solutions clearly contain all the Casimirs, but might include other functions useful for energy shaping.

Current research is under way in both of these directions.

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