NONLINEAR ROBUST PERFORMANCE ANALYSIS OF AN AEROELASTIC SYSTEM

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Abstract: The problem of analysing the worst-case performance of a nonlinear aeroelastic system is formulated as an optimisation problem and solved using gradient-based local optimisation methods. Two different approaches are considered. The first formulates the optimisation problem in the classical Euler-Lagrange setting and computes the gradient by backward integration of the resulting adjoint system. The second uses a Sequential Quadratic Programming (SQP) method which solves a Quadratic Programming (QP) subproblem at each iteration. The performance of both approaches is evaluated in terms of computational complexity and numerical accuracy, and compared with a standard industrial approach based on gridding the uncertain parameter space.

Keywords: Robust Performance, Optimization, Aerospace Control

1. INTRODUCTION

The robustness analysis of linear time invariant (LTI) systems subject to parametric and/or non-parametric uncertainty is now a relatively mature subject, and many powerful tools are available which can readily be applied to practical problems - see for example (Ferreres, 1999; Bates and Postlethwaite, 2002) for an overview of recent aerospace applications. The robustness analysis of nonlinear systems is, of course, a much more difficult problem, and, so far, most developments in this area have been at a theoretical level. Methods for computing upper bounds on robust stability or performance generally rely on generalisations of Lyapunov or Small-Gain theories, and are often computationally intractable and prone to conservatism. Due to the inherent non-convexity of the uncertain parameter space in nonlinear problems, very few methods are available with which to compute lower bounds on either robust stability or performance - the current industry standard is exhaustive nonlinear simulation using either stochastic (Monte-Carlo) or deterministic (gridding the uncertain parameter space) approaches, (Fielding *et al.*, 2002).

In (Tierno *et al.*, 1995), a promising new approach to nonlinear robust performance analysis was presented, which formulated the problem in the classical Euler Lagrange optimisation setting, (Bryson *et al.*, 1975). In this approach, gradient information is calculated via backward integration of an adjoint system, and a numerical algorithm for computing local solutions, i.e. lower bounds on worst-case performance, was described. This algorithm was applied successfully to the robust performance analysis of a ducted fan experimental test rig in (Tierno, 1996) and an F-16 autopilot simulation in (Gregory and Tierno, 1996). In (Ledegang, 1999), however, disappointing results (in particular, very poor convergence properties) were reported with the use of this approach for the robust performance analysis of a control law for a Cessna Citation aircraft.

In this paper, we apply a modified version of the approach proposed in (Tierno *et al.*, 1995) to the robust performance analysis of a nonlinear aeroelastic system, (Strganac *et al.*, 2000). Instead of updating the current estimate of the worst-case uncertain parameters using a simple steepest ascent method, as proposed in (Tierno, 1996), we use the *Minimisation Rule* (Bertsekas, 1999).[1] While this modification results in significantly improved convergence properties for the algorithm, some other practical difficulties with calculating gradient information via integration of the adjoint system are revealed in our study. As an alternative, an approach based on Sequential Quadratic Programming (SQP) is also proposed and evaluated.

2. A NONLINEAR AEROELASTIC SYSTEM

In this section, we briefly describe the aeroelastic system analysed in this study.

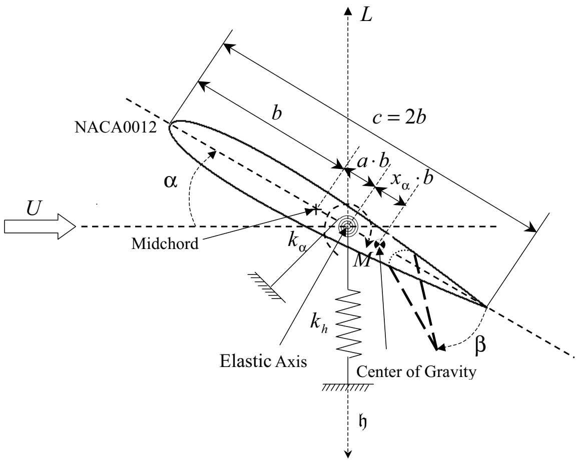

Fig. 1. The Aeroelastic system.

The aeroelastic system is a nonlinear model of a NACA 0012 airfoil with two degrees of freedom, i.e., angle of attack, α, and plunge displacement, $\mathfrak{h}$, which are shown in Figure 1. The equations of motion for the system are given by

$$\begin{bmatrix} m_T & m_W\, x_\alpha\, b \\ m_W\, x_\alpha\, b & I_\alpha \end{bmatrix} \begin{Bmatrix} \ddot{\mathfrak{h}} \\ \ddot{\alpha} \end{Bmatrix} + \begin{bmatrix} c_h & 0 \\ 0 & c_\alpha \end{bmatrix} \begin{Bmatrix} \dot{\mathfrak{h}} \\ \dot{\alpha} \end{Bmatrix} + \begin{bmatrix} k_h & 0 \\ 0 & k_\alpha\,(\alpha) \end{bmatrix} \begin{Bmatrix} \mathfrak{h} \\ \alpha \end{Bmatrix} = \begin{Bmatrix} -L \\ M \end{Bmatrix} \tag{1}$$

where m_T is the total mass, m_W is the mass of the wing only, and I_α is the moment of inertia about the elastic axis. The terms a and x_α represent the non-dimensionalised elastic axis and center of mass locations by the length of midchord, b, respectively. The location of the elastic axis, a, has a significant role in determining the overall stability of the system, however, its exact location is very difficult to determine accurately. To reflect this fact, it is represented in the model as

$$a = \tilde{a} + \Delta a \tag{2}$$

where $\tilde{a}$ is a nominal value and Δa is the predicted level of uncertainty. The terms c_h and c_α are the plunge and pitch structural damping coefficients, and the structural stiffness for the plunge and pitch motions is given by k_h and k_α, respectively.

The term $k_\alpha(\alpha)$ is a nonlinear function of α, given by (Strganac *et al.*, 2000):

$$k_\alpha(\alpha) = \sum_{i=0}^{\infty} k_{\alpha_i}\, \alpha^i \ [\text{N}\cdot\text{m/rad}] \tag{3}$$

where the k_{α_i}'s are constants. For numerical simulation purposes, the following 4^{th}-order approximation is used for $k_\alpha(\alpha)$ (Strganac *et al.*, 2000):

$$k_\alpha(\alpha) = k_{\alpha_0} + k_{\alpha_1}\alpha + k_{\alpha_2}\alpha^2 + k_{\alpha_3}\alpha^3 + k_{\alpha_4}\alpha^4 \tag{4}$$

where each of the coefficients is given by

$$k_{\alpha_i} = \tilde{k}_{\alpha_i} + \Delta k_{\alpha_i} \tag{5}$$

for $i = 1, 2, \ldots, 4$ and Δk_{α_i} represents a bounded level of uncertainty for each coefficient. As shown in (Strganac *et al.*, 2000), the above approximation closely matches experimental results for deviations in α up to $\pm 11.49°$. In addition, the following quasi-steady aerodynamic model for the lift, L, and the moment, M are used (Strganac *et al.*, 2000):

$$L = \rho\, U^2\, b\, c_{l\alpha} \left[\alpha + \frac{\dot{\mathfrak{h}}}{U} + \left(\frac{1}{2} - a \right) \frac{b\,\dot{\alpha}}{U} \right] + \rho\, U^2\, b\, c_{l\beta}\, \beta \tag{6a}$$

$$M = \rho\, U^2\, b^2\, c_{m\alpha} \left[\alpha + \frac{\dot{\mathfrak{h}}}{U} + \left(\frac{1}{2} - a \right) \frac{b\,\dot{\alpha}}{U} \right] + \rho\, U^2\, b^2\, c_{m\beta}\, \beta \tag{6b}$$

where ρ is air density, U is the freestream velocity, $c_{l\alpha}$ and $c_{m\alpha}$ are the aerodynamic lift and moment coefficients, respectively, and β is the flap defection. The freestream velocity, U, is another significant source of uncertainty in the model and is given by

$$U = \tilde{U} + \Delta U \ \ [\text{m/sec}] \tag{7}$$

where again $\tilde{U}$ denotes the nominal value and ΔU a bounded level of uncertainty.

The state-space form of (1) is given by

$$\dot{\boldsymbol{\phi}}(t) = \mathbf{f}(U,\, a,\, k_\alpha) + B(U,\, a)\, \beta \tag{8}$$

where

$$\boldsymbol{\phi} = \left[\phi_1,\, \phi_2,\, \phi_3,\, \phi_4\right]^T = \left[\alpha \ \dot{\alpha} \ \mathfrak{h} \ \dot{\mathfrak{h}}\right]^T \tag{9a}$$

$$\mathbf{f}(U, a, k_\alpha) = \begin{bmatrix} \phi_2(t) \\ f_1[\boldsymbol{\phi}(t)] \\ A_{32}\,\phi_2(t) + A_{34}\,\phi_4(t) \\ f_2[\phi_1(t)]\,\phi_1(t) + \sum_{i=2}^{4} A_{4i}\,\phi_i \end{bmatrix} \tag{9b}$$

$$B(U,\, a) = \left[0,\ g_4\, U^2,\ 0,\ 0\right]^T \tag{9c}$$

[1] In this paper we consider parametric uncertainty only - the approach of (Tierno, 1996) also considers signal uncertainty.

Table 1 Fixed parameters

Parameter	Value
m_T	12.3870 [kg]
m_W	2.0490 [kg]
b	0.1064 [m]
ρ	1.225 [kg/m^3]
c_α	0.036 [kg·m^2/sec]
$c_{l\alpha}$	6.28
$c_{l\beta}$	3.358
$c_{m\beta}$	-1.94
c_h	27.43 [kg/sec]
k_h	2844.4 [N/m]

Table 2 Uncertain parameters

Nominal Value	Uncertainty
$\tilde{U} = 16.0$	$-2.00 \leq \Delta U \leq 2.00$
$\tilde{a}$ = -0.6	$-0.15 \leq \Delta a \leq 0.15$
$\tilde{k}_{\alpha_0} = 6.833$	$-0.68 \leq \Delta k_{\alpha_0} \leq 0.68$
$\tilde{k}_{\alpha_1} = 0.0$	$9.00 \leq \Delta k_{\alpha_1} \leq 11.00$
$\tilde{k}_{\alpha_2} = 0.0$	$600.92 \leq \Delta k_{\alpha_2} \leq 734.45$
$\tilde{k}_{\alpha_3} = 0.0$	$23.91 \leq \Delta k_{\alpha_3} \leq 29.23$
$\tilde{k}_{\alpha_4} = 0.0$	$-4579.14 \leq \Delta k_{\alpha_4} \leq -5596.72$

Table 3 Dependent parameters

Parameter	Value
$c_{m\alpha}$	$(0.5 + a)\, c_{l\alpha}$
x_α	$[0.0873 - (b + a \cdot b)]\,/b$ [m]
I_α	$m_W\, x_\alpha^2\, b^2 + 0.0517$ [kg·m^2]

See (Strganac *et al.*, 2000) for the definitions of the other terms.

The value of each fixed parameter in the model is given in Table 1. There are seven uncertain parameters in the model, and the nominal value and the uncertainty bound for each of these parameters is given in Table 2. Three other parameters in the model, which are functions of the uncertain parameters, are given in Table 3.

In (Strganac *et al.*, 2000), a control law to regulate angle of attack by adjusting the flap deflection angle β was designed for the nominal system

$$\dot{\boldsymbol{\phi}} = \mathbf{f}(\tilde{U},\, \tilde{a},\, \tilde{k}_\alpha) + B(\tilde{U},\, \tilde{a})\, \beta_{\text{desired}} \tag{10}$$

Using feedback linearisation, the desired control input for the flap deflection angle is computed as:

$$\beta_{\text{desired}} = \frac{-f_1[\tilde{\boldsymbol{\phi}}] - p_1\phi_1 - p_2\phi_2}{\tilde{g}_4\, \tilde{U}^2} \tag{11}$$

where p_1 and p_2 are the control gains and are given by 4 and 1.2, respectively. [2] For this control law, the resulting zero dynamics are Hurwitz stable in the range of $-1 \leq a \leq 1$ and $0 < U \leq 30$ [m/sec], (Strganac *et al.*, 2000). By physical limitation, the actual flap deflection is restricted as follows:

$$\beta = \text{sign}(\beta_{\text{desired}})\, \min(|\beta_{\text{desired}}|, 12^\circ) \tag{12}$$

[2] In (Strganac *et al.*, 2000) an adaptive control law is also designed to estimate the nonlinear torsional spring constants, k_{α_i}. For simplicity, only the fixed gain part of the controller is used here.

3. ROBUST PERFORMANCE ANALYSIS

In (Tierno *et al.*, 1995) the robust performance analysis problem was formulated using the following finite L^2 gain cost function:

$$\max_{x_\delta \in \Delta_\delta} J = \int_{t_0}^{t_f} L(t)dt = \frac{1}{2}\|g(x)\|_2^2 \tag{13}$$

where t_0 and t_f are fixed, $g(x)$ is a piecewise continuous function, the cost function is subject to

$$\dot{x} = f(x, x_\delta) \tag{14a}$$
$$\dot{x}_\delta = 0 \tag{14b}$$

and the initial conditions are given by

$$x_0 = x(t_0) \tag{15}$$

Δ_δ is a hyperbox in $\Re^p$, where p is the number of uncertain parameters. Bounds for the values of the uncertain parameters x_{δ_i} are given by

$$\underline{x}_{\delta_i} \leq x_{\delta_i} \leq \bar{x}_{\delta_i} \tag{16}$$

for $i = 1, 2, \ldots, p$, where $\underline{x}_{\delta_i}$ and $\bar{x}_{\delta_i}$ are constants. Hence, the problem is to find the optimal initial condition x_δ to maximize the cost function.

For the aeroelastic system given in (1), the vector of uncertain parameters is given by

$$x_\delta = \begin{bmatrix} \Delta U & \Delta a & \Delta k_{\alpha_0} & \Delta k_{\alpha_1} & \Delta k_{\alpha_2} & \Delta k_{\alpha_3} & \Delta k_{\alpha_4} \end{bmatrix}^T \tag{17}$$

where the bound for each parameter is given in Table 2. To avoid numerical problems, each uncertain parameter is normalised so that it is bounded as follows:

$$-1 \leq x_{\delta_i} \leq 1 \tag{18}$$

for $i = 1, 2, \ldots, 7$.

Since the main control objective in the aeroelastic problem is regulation of angle of attack, an appropriate cost function for robust performance analysis is

$$\max_{x_\delta \in \Delta_\delta} J = \frac{1}{2}\int_{t_0}^{t_f} Q_1\, \alpha^2(t) + Q_2\, \dot{\alpha}^2(t)dt \tag{19}$$

where Q_1 and Q_2 are positive scaling factors, given by 10 and 1, respectively, which are used to (approximately) equally penalise large values of α and $\dot{\alpha}$, and t_0 and t_f are set equal to 0 and 5 sec, respectively. The initial condition for (1) is given by

$$\alpha(t_0) = 0.0483 \text{ [rad]} \tag{20a}$$
$$\dot{\alpha}(t_0) = 3.1819 \text{ [rad/sec]} \tag{20b}$$
$$\mathfrak{h}(t_0) = 0.0135 \text{ [m]} \tag{20c}$$
$$\dot{\mathfrak{h}}(t_0) = 0.2485 \text{ [m/sec]} \tag{20d}$$

3.1 Gridding the Uncertain Parameter Space

For the purposes of comparison, the worst case value of the cost function in (19) was evaluated for all possible combinations of the extreme points of the uncertain parameters. This required $2^7 = 128$ cost function evaluations. The maximum value of the cost function found was 9.3341 and the corresponding worst-case combination of uncertain parameters is given by

$$x_\delta = \begin{bmatrix}-1 & -1 & -1 & +1 & -1 & +1 & +1\end{bmatrix} \tag{21}$$

These results can be considered as the current industrial benchmark for this type of problem, (Fielding *et al.*, 2002). Note that the exponential increase in computation time places severe limits on the number of uncertain parameters that can be considered under this approach. In addition, since only the vertices of the uncertain parameter space are checked, worst-cases that occur in the interior of the parameter space are guaranteed to be missed a priori.

3.2 Euler Lagrange Optimisation Framework

In the classical Euler Lagrange framework, the augmented cost is given by

$$J = \int_{t_0}^{t_f} g^T g + \lambda^T[f - \dot{x}] + \lambda_\delta^T[0 - \dot{x}_\delta]dt \tag{22}$$

where $\tilde{\lambda}$ and λ_δ are the Lagrange multipliers. Taking the first variation of this cost, δJ, gives the following adjoint system: [3]

$$\dot{\lambda} = -\left(\frac{\partial f}{\partial x}\right)^T \lambda - \left(\frac{\partial g}{\partial x_2}\right)^T y \tag{23a}$$

$$\dot{\lambda}_\delta = -\left(\frac{\partial f}{\partial x_\delta}\right)^T \lambda \tag{23b}$$

with boundary conditions:

$$\lambda(t_f) = 0 \tag{24a}$$

$$\lambda_\delta^T(t_f) = 0 \tag{24b}$$

Practically, to obtain (23) could be very lengthy and tedious process. Thus, δJ becomes:

$$\delta J = \lambda_\delta^T(t_0)\delta x_\delta \tag{25}$$

The initial value of λ_δ can, therefore, be interpreted as the gradient of the cost function with respect to the uncertain parameters, i.e.,

$$\lambda_\delta^T(t_0) = \left.\frac{\partial J}{\partial x_\delta}\right|_{t=t_0} \tag{26}$$

$\lambda_\delta(t_0)$ can be obtained at each numerical iteration by backward integration of the adjoint system, (23), with the final condition, $\lambda_\delta(t_f) = 0$. Considering the Lagrange multiplier term by term

$$\lambda_{\delta_i}(t_0) = \left.\frac{\partial J}{\partial x_{\delta_i}}\right|_{t=t_0} \tag{27}$$

for $i = 1, 2, \ldots, p$ there are three possible values for the initial condition:

$$x_{\delta_i}^* = \begin{cases} x_{\delta_i} = \bar{x}_{\delta_i} \text{ and } \lambda_{\delta_i}(t_0) > 0 \\ x_{\delta_i} = \underline{x}_{\delta_i} \text{ and } \lambda_{\delta_i}(t_0) < 0 \\ \underline{x}_{\delta_i} \le x_{\delta_i} \le \bar{x}_{\delta_i} \text{ where } \lambda_{\delta_i} = 0 \end{cases} \tag{28}$$

[3] The gradient is row vector in this paper.

More details of the Euler Lagrange framework can be found in (Bryson *et al.*, 1975).

The original update law used in (Tierno *et al.*, 1995), (Gregory and Tierno, 1996), and (Tierno, 1996) is given by

$$x_{\delta_i}^{\text{uncnst}} = x_{\delta_i}^{\text{current}} + \lambda_{\delta_i}(t_0) \tag{29}$$

for $i = 1, 2, \ldots, r$, so that

$$x_{\delta_i}^{\text{updated}} = \begin{cases} \underline{x}_{\delta_i}, & \text{for } x_{\delta_i}^{\text{uncnst}} < \underline{x}_{\delta_i} \\ \bar{x}_{\delta_i}, & \text{for } x_{\delta_i}^{\text{uncnst}} > \bar{x}_{\delta_i} \\ x_{\delta_i}^{\text{uncnst}}, & \text{otherwise} \end{cases} \tag{30}$$

for $i = 1, 2, \ldots, p$. Since the update law, (29), is in the steepest ascent direction, it will have all the disadvantages of steepest direction methods, such as slow convergence along a long smooth hill (Bertsekas, 1999). Indeed, very slow convergence of the original algorithm was reported in (Ledegang, 1999), where the algorithm was applied to find the worst-case uncertain parameter combination for a Cessna Citation aircraft model. To avoid this problem, the x_δ^{uncnst} can be updated in the following way:

$$x_{\delta_i}^{\text{uncnst}} = x_{\delta_i}^{\text{current}} + \gamma^k \lambda_{\delta_i}^k(t_0) \tag{31}$$

where k is the iteration step, and γ^k is the step-size to be determined. The step size γ^k can be chosen using the *Maximisation Rule* [4] or *Armijo's Rule*. In this paper the Maximisation rule is adopted and the algorithm is given by

$$\gamma^k = \arg \max_{\gamma \in (0,\, \gamma_{\max}]} J\left[x_\delta^k + \gamma \lambda_\delta^k(t_0)\right] \tag{32}$$

where $\gamma_{\max}$ is a positive constant. Also, the following successive step-size reduction formulation is used:

$$\gamma^k = \psi^m \gamma_{\max} \tag{33}$$

where ψ is equal to $1/2$, $\gamma_{\max}$ is equal to 1 and m is the smallest integer such that

$$J\left[x_\delta^{\text{current}} + \gamma^k \lambda_\delta^k(t_0)\right] \ge J\left(x_\delta^{\text{current}}\right) \tag{34}$$

The line search is completed with respect to γ at each iteration. More details of this scheme can be found in (Bertsekas, 1999). Finally, the stop condition is given by

$$\left| J\left(x_\delta^{\text{updated}}\right) - J\left(x_\delta^{\text{current}}\right) \right| \le \epsilon \left| \lambda_\delta^{\text{initial}} \right| \tag{35}$$

where ϵ is equal to 10^{-6}.

3.3 Sequential Quadratic Programming

Sequential Quadratic Programming (SQP) is a powerful nonlinear programming method which has been applied to a wide variety of problems. In each iteration of SQP, the nonlinear optimisation problem is approximated as a Quadratic Programming (QP) problem and the QP is solved

[4] In minimisation problems this is known as the minimisation rule.

Table 4 The cost function values for 100 trials

Method	Min	Max	Average	Standard Deviation
Gridding	1.4643	9.3341	3.5131	2.2837
Euler Lagrange	2.0608	9.3691	8.4046	1.4522
SQP	1.8996	9.3909	8.6654	2.1254

Table 5 Number of cost function evaluations for 100 trials

Method	Min	Max	Average	Standard Deviation	Total
Gridding	N/A	N/A	$2^7 = 128$	N/A	N/A
Euler Lagrange	45	759	340.4	200.9	34041
SQP	53	910	221.5	181.7	22154

using the Lagrange method, active set method, etc (Fletcher, 1981).

The QP problem corresponding to (13) is given by (Fletcher, 1980)

$$\min_{s^k \in \Re^p} \frac{1}{2} {s^k}^T H^k s^k + \left. \frac{\partial J}{\partial x_\delta} \right|_{x_\delta = x_\delta^k} s^k \qquad (36)$$

where H^k is the Hessian to be approximated and the gradient of $J(x_\delta)$ is computed using a finite-difference approximation. Then, the solution for s^k is determined using some QP algorihtm. Since the search direction is determined in the above equation, the x_δ is updated as follows:

$$x_{\delta_i}^{\text{uncnst}} = x_{\delta_i}^{\text{current}} + \gamma^k s^k \qquad (37)$$

where γ^k is determined by the same maximisation rule as was used in the previous section. Finally, the Hessian is approximated using the BFGS formula, which can be found in (Fletcher, 1980). The stop condition is the same as (35).

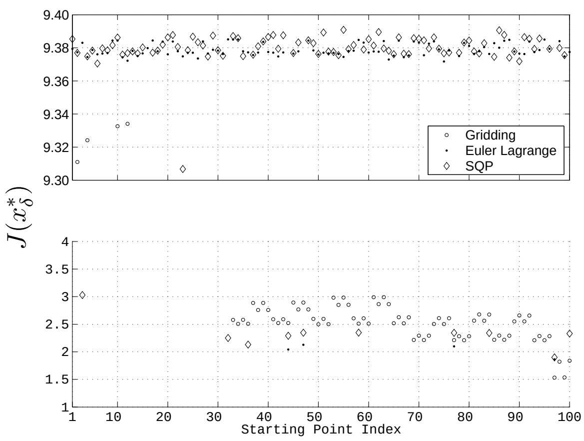

Fig. 2. Cost function values for 100 trials for each method

4. RESULTS AND DISCUSSION

Results for both optimisation schemes, for 100 trials with random initial guesses for the parameters, are given in Table 4 and 5. From the tables, it is clear that all three methods are almost equally good in terms of finding the maximum value of the cost function. As shown in Figure 2, however, there exist significant differences in the computational performance of each algorithm.

For the gridding approach, only a few of the 128 points are close to the maximum value of the cost function. The much smaller number of function evaluations required by the gridding approach is also only due to the small number of uncertain parameters considered, i.e., 7. This advantage immediately vanishes when the number of parameters increases, for example, for 15 uncertain parameters the number of function evaluations required is 32,768.

The Euler Lagrange approach provides a mathematically attractive formulation of the problem, but is heavily reliant on having a fast and accurate numerical integration algorithm for the adjoint system. As shown in Table 4, the maximum value found with the Euler Lagrange approach is greater than the one with the gridding approach, but still smaller than the one found with the SQP method. Also, the Euler Lagrange approach requires 10,000 more function evaluations in total than the SQP. [5] This is again due mainly to numerical errors arising in the integration of the differential equations for the adjoint system. Because of the inaccuracy in the integration, the gradients point in the wrong direction and the resulting convergent points are scattered in various region of the uncertain parameters space. The SQP gave the best results among three methods, as it found the maximum value of the cost function and for most of the 100 trials converged to points very close to the maximum value. The corresponding worst case uncertain parameter combination for the maximum is as follows:

$$x_\delta^* = \begin{bmatrix} -0.9 & -0.9 & -1.0 & +1.0 & -1.0 & +1.0 & +1.0 \end{bmatrix} \qquad (38)$$

Note that ΔU^* and Δa^*, are not on the boundary of their possible range of variation. This type of solution can never be found by a gridding approach unless it has very fine gridding, which is computationally prohibitive in practice.

With the initial conditions for the system given in (20), in the absence of any control input the nominal system goes into a limit cycle. When the control input given by (11) is applied, the response converges to the origin as shown in Figure 3. If the system model is subsequently changed to those values of the uncertain parameters given in (38),

[5] In addition, the integration of the adjoint system takes significantly more time than is required for integration of the original system.

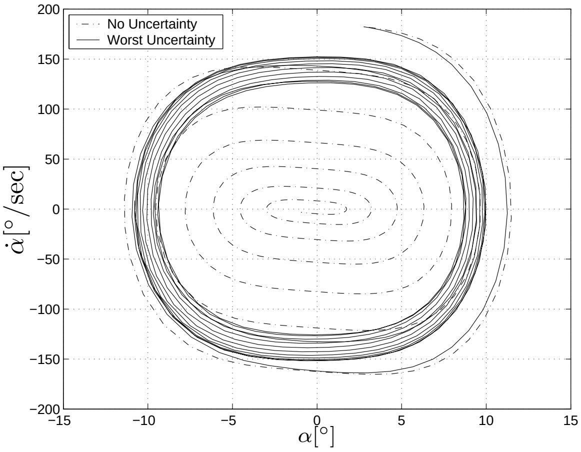

Fig. 3. Angle of attack Phase plot for the nominal and worst-case systems

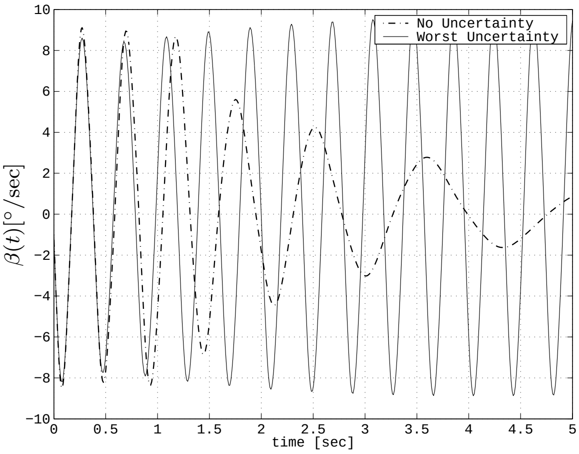

Fig. 4. Flap deflection angle histories for the nominal and worst-case systems

however, a limit cycle is once again generated in the response, as also shown in Figure 3. Time histories of the control signal for both cases are shown in Figure 4.

5. CONCLUSION

The problem of analysing the worst-case performance of a nonlinear aeroelastic system was formulated as an optimisation problem and solved using gradient-based local optimisation methods. Two different approaches were considered. The first formulates the optimisation problem in the classical Euler-Lagrange setting and computes the gradient by backward integration of the resulting adjoint system. The second uses a Sequential Quadratic Programming (SQP) method which solves a Quadratic Programming (QP) subproblem at each iteration. The study highlighted some practical problems with applying the Euler Lagrange optimisation theory, in particular the sensitivity of the approach to numerical errors when integrating the adjoint system. The SQP method on the other hand produced promising results, both in terms of numerical accuracy, reliability and computational cost.

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