A NEW APPROACH FOR MINIMUM TIME MOTION PLANNING PROBLEM OF WHEELED MOBILE ROBOTS

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Abstract: We present a new method to plan minimum time trajectories for wheeled mobile robots. The problem is known to be complex in particular when dynamics is taken into account. Our approach is based on a simultaneous search for the robot path and the time evolution on this path. The whole problem is formulated in such a way that all geometric, kinematic and dynamic constraints are handled sequentially which makes more effective the use of stochastic methods.

Keywords: Mobile Robot, Minimum Time, Trajectory Planning, Stochastic Optimization.

1. INTRODUCTION

Mobile robots are, in general, devoted to achieve various tasks in large workspaces with a large autonomy. This makes the motion planning phase very important and requires enormous attention. This importance is naturally reflected in the number and the variety of research activities. In general efforts are oriented towards two main topics: *path planning* and *motion planning.* The former concerns the search of a continuous sequence of configurations between two limit configurations. The later concerns the time history of such a sequence taking into account kinematics and dynamics. Papers treating the first topic are numerous while for the second they are relatively few. All these works can be grouped according to:

- the dimension of the robot environment;
- the model adopted for the robot;
- the optimization criteria;
- the method proposed.

Planning the robot motion in a 2D structured space can be considered as the reference or basic problem. Although it is relatively simplified, it is still complex (Latombe, 1991; Yamamato, *et al.*, 1999). Increasing the space dimension intends generally to allow the inclusion of irregular terrains. In this case, the problem becomes more complex because of stability complications (Siméon, 1991; Cherif, *et al.*, 1994).

Early works treated the problem of mobile robot motion planning from a purely geometric point of view (Cherif, *et al*., 1994). The principal aim was to avoid collisions in a structured environment. These works leaded to a large number of techniques of diverse complexity and demonstrated that the problem of motion planning for a Wheeled Mobile Robot (WMR) is NP hard (Latombe, 1991). More recently, due to the constant need to increase the productivity in various industries, others approaches have been proposed to solve the Minimum Time Motion Planning Problem (MTMPP) for WMR while considering kinematic constraints (David, *et al*., 1994; Renaud and Fourquet, 1997; Jiang, *et al.* 1997).

It is clear however that dynamics is also important, particularly in the case of high velocities or when there is a need to compute correctly inputs of the system control. Few are works that treat the MTMPP while handling dynamics. In studies that consider the WMR as a mass–point (Shiller, 1991), the corresponding dynamic model might be insufficient to describe the robot real behavior. Such models do not take into account nonholonomic constraints that are an essential characteristic of the WMR.
On the other hand, there are several works that model the WMR as a multi-body system and adopt, in general, a Hamiltonian formulation for the dynamics coupled with several optimal control techniques such as the phase plane method (Yamamato, *et al.*, 1999).

However, this type of approaches suffers from the fact that for any new constraint or new cost function the whole problem formulation must be reviewed.

Here, we focus our interest on solving one of the most important problems in mobile robotics: how to plan trajectories while taking into account constraints on:

- geometry (obstacle avoidance and bounds on joint positions);
- kinematics (nonholonomic constraints and bounds on joint velocities and accelerations,);
- dynamics (stability constraints and bounds on joint torques).

The approach we propose here is an iterative process that tries to improve the solution by a searching for the robot path and, simultaneously, for the time evolution on this path. The key idea consists of defining two normalized scales: one for the path and the other for time. With these scales, the problem can be reformulated so that all constraints are treated sequentially and a given cost function minimized by a stochastic technique. The implementation of this approach using, for example, a Simulated Annealing (SA) algorithm allows treating complex situations and leads to competitive results when compared to those already published (Yamamato, *et al.*, 1999). An equivalent method for robotic manipulators has been tested successfully (Chettibi and Lehtihet, 2002; Chettibi, *et al*., 2004).

2. DYNAMIC MODEL

The WMR considered in this study, can be defined as a platform supplied with wheels and that is capable of moving autonomously in a 2D plan environment. The main part is a rigid chassis that is commonly provided with electrically motorized wheels and passive wheels (Fig. 1).

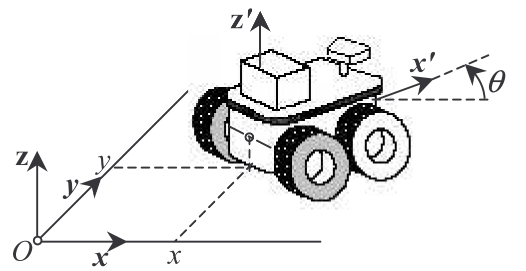

Fig. 1. A Wheeled Mobile Robot

Let $\Re$=(o, $\mathbf{x}$, $\mathbf{y}$, $\mathbf{z}$) be the fixed frame of the world coordinates and $\Re'$ = (o', $\mathbf{x}'$, $\mathbf{y}'$, $\mathbf{z}'$) the moving frame attached to the platform. A configuration of the WMR is completely defined by the vector $\boldsymbol{q} = [x \quad y \quad \theta \quad \boldsymbol{q_a} \quad \boldsymbol{q_p}]^T$ of n dimensions where:

- (x, y, θ) specifies the position/orientation of the main body in $\Re$;
- $\boldsymbol{q}_a$ is the vector of n_a active-joints parameters,
- $\boldsymbol{q}_p$ is the vector of n_p passive-joints parameters,

so that: $n = 3 + n_a + n_p$

We adopt the following assumptions in modeling the WMR system:

- There is no slipping between the wheel and the floor. The robot cannot move sidelong to maintain the nonholonomic constraint.
- The motion of the platform is confined to the (O, $\boldsymbol{x}$, $\boldsymbol{y}$) plan and no translations are allowed along the global $\boldsymbol{z}$ axis.

Nonholonomic constraints are formulated as first-order non integrable differential equations containing time-derivatives (i.e. velocity-level) of generalized coordinates $\dot{\boldsymbol{q}}$. They are, generally represented in a compact form as follows (Campion, *et al.*, 1996):

$$\boldsymbol{A}(\boldsymbol{q}) \cdot \dot{\boldsymbol{q}} = 0 \qquad (1)$$

Using Lagrange's formulation, a dynamic model that includes (1), can be written in the following form (Guy, *et al.*, 1996):

$$\boldsymbol{M}(\boldsymbol{q}) \cdot \ddot{\boldsymbol{q}} + \boldsymbol{c}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \boldsymbol{B}(\boldsymbol{q}) \cdot \boldsymbol{\tau}_a + \boldsymbol{A}^T(\boldsymbol{q}) \cdot \boldsymbol{\eta} \qquad (2)$$

where $\boldsymbol{M}(\boldsymbol{q})$ is the inertia moment matrix, $\boldsymbol{c}(\boldsymbol{q},\dot{\boldsymbol{q}})$ is the centrifugal and Coriolis vector force, $\boldsymbol{B}(\boldsymbol{q})$ is the input transformation matrix, $\boldsymbol{\eta}$ is the vector of Lagrange multipliers and $\boldsymbol{\tau}_a$ is the vector of active joint torques.

3. PROBLEM STATEMENT

The mobile robot is required to move freely, without following a specified path, from initial to final states $\boldsymbol{X}_0$ and $\boldsymbol{X}_f$ (Fig.2) given by:

$$\boldsymbol{X}_0 = [x_0 \quad y_0 \quad \theta_0]^T, \quad \boldsymbol{X}_f = [x_f \quad y_f \quad \theta_f]^T$$

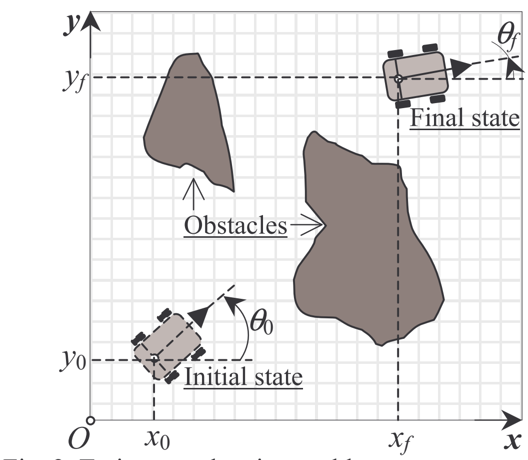

Fig. 2. Trajectory planning problem

In addition to the vector $\boldsymbol{\tau}_a(t)$ of actuator efforts and the final time T, we must find the motion defined by $\boldsymbol{X}(t) = [x(t) \quad y(t) \quad \theta(t)]^T$ such as the initial and final states are matched, constraints are respected and a cost function is optimized.

The following boundary conditions inherent to the achievement of the desired task must be taken into account:

- *Position/Orientation*

$$\begin{cases} \boldsymbol{X}(0) = \boldsymbol{X}_0 \\ \boldsymbol{X}(T) = \boldsymbol{X}_f \end{cases} \qquad (3a).$$

- *Velocity*

$$\begin{cases} \dot{\boldsymbol{X}}(0) = \vec{0} \\ \dot{\boldsymbol{X}}(T) = \vec{0} \end{cases} \qquad (3b).$$

When obstacles are present in the workspace, the following constraints will hold during the motion:

$$C(\boldsymbol{X}(t)) = false \qquad (0 < t < T) \qquad (3c).$$

The Boolean function C indicates whether or not the robot at configuration $\boldsymbol{X}$ is in collision with an obstacle.

Additional constraints that may have to be included represent other physical limitations such as those imposed on:

- *active-joint velocities:*

$$\left|\dot{q}_{ai}(t)\right| \le \dot{q}_{ai}^{\max} \qquad i = 1\ldots, n_a \qquad (4a),$$

- *active-joint accelerations:*

$$\left|\ddot{q}_{ai}(t)\right| \le \ddot{q}_{ai}^{\max} \qquad i = 1\ldots, n_a \qquad (4b),$$

- *active-joint torques:*

$$\left|\tau_{ai}(t)\right| \le \tau_{ai}^{\max} \qquad i = 1\ldots, n_a \qquad (4c).$$

Of course, non–symmetrical bounds on the above physical quantities can also be handled without any new difficulties.

The goal function F_{obj} to be minimized represents the traveling cost between initial and final states. It is generally an expression containing significant physical parameters related to the WMR behavior and also to the productivity of the system. For simplicity, we will limit the discussion to the case of MTMPP where $F_{\text{obj}} \equiv T$ but the same method is still applicable for other more general forms of the cost function.

4. PROPOSED METHOD

At each iteration of the optimization process the WRM motion $\boldsymbol{X}(t)$ is defined in two main steps:

***step* 1** : specify the robot path $\boldsymbol{X}(\lambda)$.

***step* 2** : specify the motion profile $\lambda(\xi)$ on this path.

The first parametric form $\boldsymbol{X}(\lambda)$, $\lambda \in [0, 1]$, describes the geometry of the robot path in the (O, $\boldsymbol{x}$, $\boldsymbol{y}$) plane while the second form $\lambda(\xi)$, $\xi \in [0, 1]$, determines the time evolution along this path. Here, ξ represents a normalized time scale: $\xi = t / T$.

Hence, the problem is transformed to a parametric optimization problem. One of the parameters is the unknown traveling time T. The other parameters are two sets, S_P and S_C, of free discretisation nodes. The set S_P is composed of N_P control points in the robot workspace (Fig. 3) while S_C consists of N_C collocation points in the (ξ, λ) plane (Fig. 4).

With S_P, we can define a path $\boldsymbol{X}(\lambda)$ using parametric functions, such as B-spline, that takes into account constraints (3a) and (3c). Note that $\theta(\lambda)$ is deduced directly from $x(\lambda)$ and $y(\lambda)$ using the nonholonomic constraint :

$$-x'(\lambda)\cdot Sin\big(\theta(\lambda)\big) + y'(\lambda)\cdot Cos\big(\theta(\lambda)\big) = 0 \qquad (5).$$

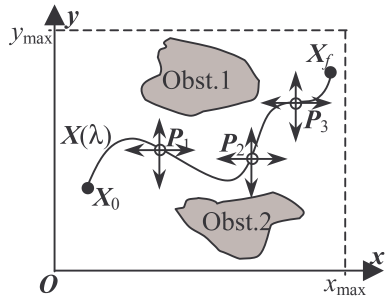

Fig. 3. A path $\boldsymbol{X}(\lambda)$ through N_P free control points.

With S_C, we can define a motion profile $\lambda(\xi)$ using, for example, a clamped cubic spline interpolation that takes into account constraints (3b).

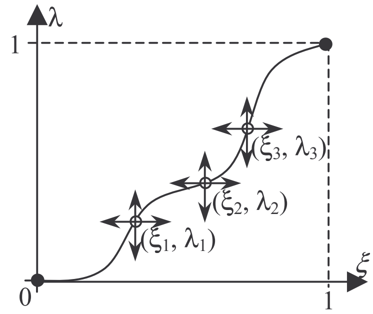

Fig. 4. A motion profile $\lambda(\xi)$ with N_C free collocation points.

The method adopted here uses a stochastic process that scans simultaneously the available solution space of both sets S_P and S_C to propose candidate trajectory profiles $\boldsymbol{X}(\xi) \equiv \boldsymbol{X}(\lambda(\xi))$ for a global minimization of F_{obj}. Note that the generalized coordinate vector $\boldsymbol{q}$ and its derivatives with respect to ξ are deduced directly from $\boldsymbol{X}(\xi)$ using (1). Thereafter, active joint torques $\boldsymbol{\tau}_a(\xi)$ are evaluated from (2).

4.1 Constraints

Given a candidate $\boldsymbol{X}(\xi)$, most constraints simply translate to bounds on admissible values of the optimal traveling time T_X of that candidate. Here, constraints are conveniently grouped in three categories (Chettibi and Lehtihet, 2002). They will be used as a sequence of rejection tests applied systematically on any proposed candidate.

The first category concerns geometric constraints. They will not yield any restriction on T_X. For example, in the case of the obstacle constraint (3*c*), only the path $\boldsymbol{X}(\lambda)$ is actually relevant. Any candidate $\boldsymbol{X}(\lambda)$ that already infringes (3*c*) will be rejected early in the process of selection since it will not lead to a feasible trajectory.

The second category concerns kinematic constraints. They translate to an explicit lower bound on T_X. For example, velocity constraints (4*a*) become:

$$T_X \geq T_v \quad \text{where} \quad T_v = \frac{\max_{\xi \in [0,1]} \left| q'_{ai}(\xi) \right|}{\dot{q}_{ai}^{\max}} \qquad (6a).$$

Acceleration and torque constraints (4*b* and 4*c*) when treated in a similar way will yield two new lower bounds noted, respectively, T_A and T_τ:

$$T_X \geq T_A \qquad (6b),$$

$$T_X \geq T_\tau \qquad (6c).$$

The optimal traveling time T_X of candidate $\boldsymbol{X}(\xi)$ is therefore given by $T_X = \max(T_v, T_A, T_\tau)$.

4.2 Details of implementation

The method is initialized with N_M milestones points in the workspace. These points, illustrated as crosses in figure 5, can be found by cell decomposition, visibility graph or probabilistic roadmaps methods (Latombe, 1991; Kavaraki, et *al.* 1996). Their role is essentially to define a corridor between obstacles. The convergence of the method does not depend on the precise position of these points but calculations will be restricted to the selected corridor. Hence, the method will need to be reinitialized using another set of milestone points to test a different corridor.

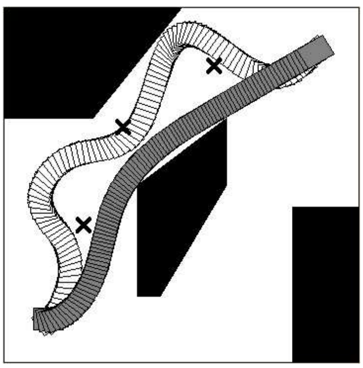

Fig.5. Milestone points, trial path and final path.

Indeed, once a corridor is chosen, it will constitute a search space in which elements of set S_p will be randomly selected to produce trial paths $\boldsymbol{X}(\lambda)$. One such a path is shown in Figure 5. Although, initially this path might present undesirable distortions, they will be attenuated progressively as the process converges. Here, the optimization is performed using a simulated annealing method which is known for its efficiency when exploring large solutions spaces (Hajek, 1985, Kirkpatrick, *et al.* 1983).

5. NUMERICAL EXAMPLE

This section gives numerical results concerning minimum–time trajectories for a WMR constituted of a platform and two independently driven wheels (Yamamoto *et al.*, 1999). The centre of gravity is in the middle of the wheel axis and the inertia moment of the platform is considered only around **z**' axis of the WMR coordinate system $\Re$'.

Table 1: Parameters of the platform

$L = 0.75\ m$	$m_1 = 50.0\ kg$	$I_{z1} = 26.04\ kg.m^2$
$b = 2.00\ m$	$m_2 = 1.00\ kg$	$I_{z2} = 0.0025\ kg.m^2$
$r = 0.10\ m$		$I_y = 0.0050\ kg.m^2$

m_1, m_2 are the mass of the platform and the wheel, I_{z1} is the inertia moment of the platform around **z'**-axis, I_{z2} and I_y are inertia moments of the wheel around **z**'-axis and ***y***'-axis in the $\Re'$ coordinate system respectively, *r* is radius of the wheel. The platform is considered as a $(2L\times b)$ rectangle.

Constraints on driving torques are given as follows:

$$-1.0 \leq \tau_1, \tau_2 \leq 1.0 \quad (N.m)$$

5.1 Problem 1

The workspace consists of a $24m\times 24m$ flat floor with three obstacles (Fig. 6). The task to be achieved is characterized by null limit velocities. The motion starts at $\boldsymbol{X}_0 = (3, 3, 0)$ and ends at $\boldsymbol{X}_f = (21, 21, \pi/6)$.

Numerical solutions calculated for two different corridors are shown in Figure 6. The corresponding time history of joint torques is illustrated in Figure 7. These results are quite similar to those given in (Yamamoto, *et al.* 1999). The calculated traveling times are of the same order (17.82 vs. 18.94 *sec* and 18.06 vs. 18.73*sec*).

For this problem we have adopted for $\boldsymbol{X}(\lambda)$ a fourth – order B–spline model with $N_p = 6$ control points and for $\lambda(\xi)$ a clamped cubic spline model with $N_C = 4$ interpolation points. The required runtime was about 4 minutes on a 2.4 GHz P4.

5.2 Problem 2

This problem concerns another example given in (Yamamoto, *et al*. 1999). The workspace consists of a $36m\times 36m$ flat floor with five obstacles (Fig. 8). The motion begins at (3, 3, 0) and ends at (33, 33, $\pi/4$). Four different candidate corridors have been tested. The best solution is shown in Figure 8*a*. The motion is executed in 22.71 *sec* (vs. 23.54 *sec* in the cited reference). The time evolution of joint torques is depicted on Fig.8*b*.

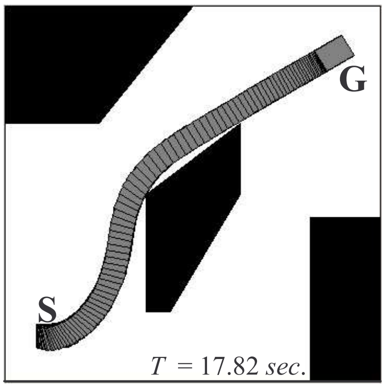

Fig. 6*a* : Simulation result (Problem 1, path *a*)

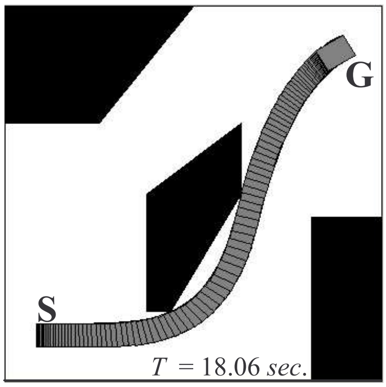

Fig. 6*b* : Simulation result (Problem 1, path *b*)

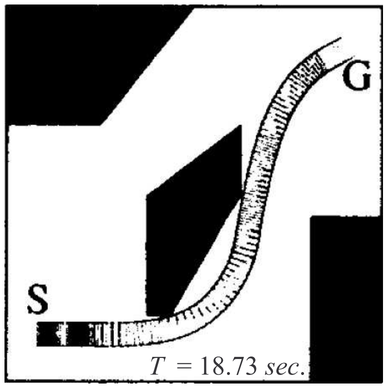

Fig. 6*b*': Simulation result (Problem 1 path *b* (Yamamoto, *et al.* 1999)

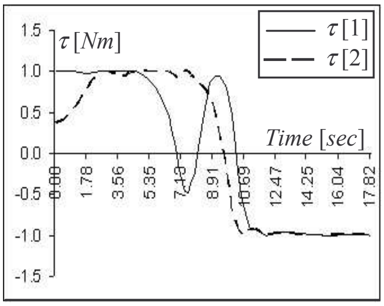

Fig. 7*a*: Time evolution of joint torques (path *a*).

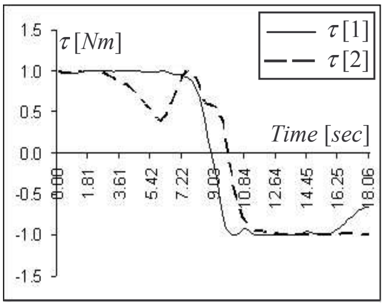

Fig. 7*b*: Time evolution of joint torques (path. *b*).

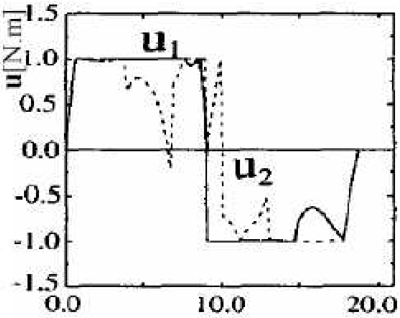

Fig. 7*b'*: Time evolution of joint torques (path. *b*) (Yamamoto, *et al.,* 1999).

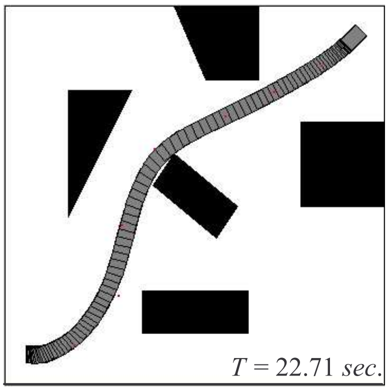

Fig. 8*a*. Simulation result (Problem 2).

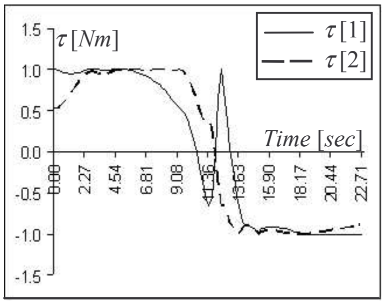

Fig. 8*b* : Time evolution of joint torques (Problem 2).

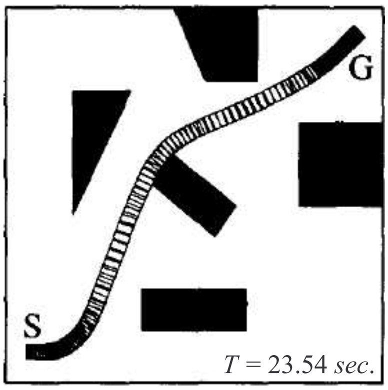

Fig. 8*c* : Simulation result (Problem 2) (Yamamoto, *et al.,* 1999).

6. CONCLUSION

We have proposed a versatile motion planner for wheeled mobile robots. The problem treated here consists of defining the optimal time history of a continuous sequence of configurations between two limit configurations, while considering kinodynamic constraints. The proposed method is an iterative process that tries to improve solutions by searching simultaneously for the robot path and for the time evolution on this path. The use of parametric functions to model the path and the motion transforms the MTMPP, which is a complex optimal control problem, to a constrained parametric optimization problem. Furthermore, all kinodynamic constraints have been conveniently translated to bounds on admissible values of the traveling time and have been treated sequentially. Additional constraints can be handled similarly without inducing any modification of the method.

Numerical results obtained here are comparable to those already published for the MTMPP. In contrast, our approach is easily extensible to problems involving other types of cost function that include energy, actuator efforts, etc. In fact, the same method is applicable even in the case of discontinuous dynamic models (e.g.: frictions). These extensions will be treated elsewhere.

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