ROBUST ℓ_1 SYNTHESIS WITH ERROR QUANTIFICATION FOR COPRIME FACTOR NOMINAL SYSTEM

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Abstract: A priori information for optimal robust synthesis includes a nominal system and sets of perturbations and disturbances. This paper addresses the problem of optimal robust synthesis in the ℓ_1 setup when the nominal system is given but upper bounds for norm-bounded perturbations and exogenous disturbance are not known to controller designer. We consider coprime factor nominal systems and show that the worst-case norm of the system output is a linear fractional function of the induced norms of system transfer functions and the norms of perturbations and disturbance. Iterative alternate optimization of this function over the Youla parameters and the norms of perturbations and disturbance that are not falsified by data is used for synthesis of robust controller. Efficiency of the method is illustrated by simulations.

Keywords: robust control, ℓ_1 optimization, error analysis, model validation, unknown-but-bounded disturbance.

1. INTRODUCTION

The ℓ_1 optimal control is associated with magnitude bounded exogenous disturbances and the ℓ_∞ signal space (Dahleh & Diaz-Bobillo, 1995; Barabanov, 1996). This paper addresses the problem of ℓ_1 optimal robust synthesis. Basic results on stability and performance robustness for systems with structured time-varying uncertainty were obtained in Khammash and Pearson (1991, 1993). A standard approach to complicated non-convex problem of optimal robust synthesis is to solve a family of auxiliary problems, each is to guarantee a prespecified upper bound for the control criterion. Then the one-parametric search of approximately minimal upper bound gives a suboptimal controller. Khammash, Salapaka, & Vanvoorhis (2001) proposed a solution to the auxiliary problem for linear time-invariant (LTI) nominal systems with structured uncertainty and bounded exogenous disturbance. This solution is based on searching over a mesh in a set of scaling diagonal matrices. In order to get an approximate solution to the auxiliary problem with an ε tolerance, the number of linear programming problems to be solved is in the order of $\left(\frac{1}{\varepsilon}\right)^{n+1}$ where n is the number of independent perturbation blocks.

A priori information for suboptimal robust synthesis includes not only the description of the nominal system but the set of admissible uncertainties as well. This paper addresses the problem of suboptimal robust synthesis under insufficient a priori information about uncertainties. We assume that upper bounds for the norm of exogenous disturbance and the induced norm of perturbations are very conservative (or unknown) and must be estimated on the basis of finite measurement data. This estimation problem is known as error quantification. It implies a solution of model valida-

[1] Supported by the Russian Fund of Basic Research.

tion problem to determine whether specific upper bounds for uncertainties are not falsified by data. In H_∞ setup, the model validation problem was studied since early 90th (see, e.g., Smith & Doyle (1992) and Poolla, Khargonekar, Tikku, Krause, & Nagpal (1994) for the time-domain approach) and remains incompletely solved (see, e.g., Smith, Dullerud, & Miller (2000) and Dullerud & Smith (2002) for recent progress). In contrast to the H_∞ setup, the model validation problem in the ℓ_1 setup is much easier . Its solution for LTI nominal models with structured uncertainties is reduced to the test of certain inequalities produced by data (Sokolov, 2003).

The problem of optimal robust synthesis under unknown uncertainty level involves the joint minimization of the control criterion over the Youla parameter of controller and over the upper bounds for uncertainties that are not falsified by data. It means, in particular, that the error quantification problem is considered in the optimal setup where the control criterion is treated as the identification criterion. Then the searching method proposed in Khammash, Salapaka, & Vanvoorhis (2001) for minimization over the Youla parameter needs not only the additional one-parametric search of minimal upper bound for the control criterion but a multi-parametric search of optimal non-falsified upper bounds for uncertainties as well. This may make the problem of optimal robust synthesis computationally intractable.

To make this problem computationally tractable, we consider coprime factor nominal models and show that they have following significant merits. First, the control criterion for these models turns out to be a linear fractional function both with respect to the induced norms of system transfer matrices and with respect to the upper bounds of the norms of perturbations and exogenous disturbance. Second, the inequalities in the model validation test become linear in the upper bounds. Then the minimization over the Youla parameter comes to a linear programming problem and no additional search is required. Linear fractional structure of control criteria and reducibility of the optimal robust synthesis to linear programming were exploited in Sokolov (2002,2001) in the regulation problem and in Yamada, & Funahashi (2002) in a specific tracking problem. Furthermore, the optimization of the control criterion over non-falsified upper bounds comes also to a linear programming problem. Then iterative alternate minimizations over the Youla parameter and the upper bounds is a natural heuristic method for solution of the problem under consideration. Although the convergence to the global minimum is not proved, the iterative scheme performs well in simulations. At least, this method provides nonconservative non-falsified estimates of the upper bounds and the optimal robust controller associated with these estimates. Efficiency of the method is illustrated by simulations.

Notation

$|x|_\infty := \max_i |x_i|$ for the vector $x = (x_1, \ldots, x_n)^* \in \mathbb{R}^n$. ℓ^n is the space of real vector sequences $x = (x(0), x(1), \ldots)$ with elements $x(k) \in \mathbb{R}^n$, $x_0^t := (x(0), x(1), \ldots, x(t))$. ℓ_∞^n is the space of real bounded vector sequences with the norm $\|x\|_\infty := \sup_k \, |x(k)|_\infty$. ℓ_1 is the space of real summable sequences with the norm $\|x\|_1 := \sum_k |x(k)|$. A map $H : \ell^p \to \ell^q$ is said to be ℓ_∞-stable (stable, for short), if it is causal, maps ℓ_∞^p into ℓ_∞^q, and

$$\|H\| := \sup_{x \neq 0} \frac{\|H(x)\|_\infty}{\|x\|_\infty} < +\infty \qquad (1)$$

($\|H\|$ is the gain of H for nonlinear H). The terms of system and map are understood as equivalent. Any linear time-invariant causal system $H : \ell^p \to \ell^q$ can be defined by the convolution

$$Hx(t) := \sum_{k=0}^{t} H(k)x(t-k) \,, \quad H(k) := (H_{ij}(k)),$$

where the same notation H is used for the $q \times p$ matrix of impulse responses $H_{ij} \in \ell$. The system H is ℓ_∞-stable iff $H_{ij} \in \ell_1$ for all i, j. The induced ℓ_∞-norm of the stable LTI system H is defined by (1) and equals

$$\|H\| = \max_{1 \leq i \leq q} \sum_{j=1}^{p} \|H_{ij}\|_1 \,.$$

The matrix valued function $H(\lambda) := \sum_{k=0}^\infty H(k)\lambda^k$ of complex variable λ is called the transfer function of the system H and $\|H(\lambda)\| := \|H\|$.

2. PROBLEM STATEMENT

Consider a discrete-time system shown on Fig. 1.

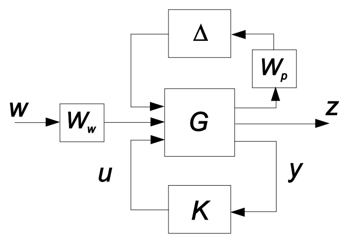

Fig. 1. Control system with uncertainties

G, K, z, y, u and w are the generalized LTI nominal system, the LTI controller, the regulated output, the measured output, the control input, and the exogenous disturbance, respectively. Inner model uncertainty is represented by the perturbation block Δ which is restricted to lie in the set $\underline{\Delta}$ of admissible structured perturbations:

$$\Delta \in \underline{\Delta} := \{\Delta = \text{diag}\,(\Delta_1, \cdots, \Delta_n) \mid$$

$\Delta_i : \ell_\infty^{q_i} \to \ell_\infty^{p_i}$ is strictly causal and $\|\Delta_i\| \leq 1\}$.
Here n is the number of independent perturbation blocks Δ_i. The LTI blocks W_w and W_p represent the weights of exogenous disturbance and perturbations.

Let the control criterion be

$$J(K, G, W_w, W_p) := \sup_{\Delta \in \underline{\Delta}} \sup_{\|w\|_\infty \leq 1} \|z\|_\infty$$

The system is called robustly stable, if J is finite. *The problem of optimal robust synthesis* is stated as follows

$$\min_K J(K, G, W_w, W_p) \qquad (2)$$

In the particular case $\Delta = 0$, this problem is the problem of the ℓ_1 optimal control (Dahleh & Diaz-Bobillo, 1995; Barabanov, 1996).

A standard approach to approximate solution of this problem is to solve the auxiliary problems

$$J(K, G, W_w, W_p) \leq \gamma \qquad (3)$$

and to search minimal reachable γ.

In problem (2), the nominal system G and the weights W_w and W_p are assumed to be known to controller designer. We shall consider a problem where the weights W_w and W_p are unknown and must be estimated from the measured data (z_0^T, y_0^T, u_0^T). Denote by W_{NF} the set of pairs (W_w, W_p) that are not falsified by the data (see subsection 3.2 for details on non-falsification) and consider the following *problem of optimal robust synthesis under unknown uncertainty level*

$$\min_{(W_w, W_p) \in W_{NF}} \min_K J(K, G, W_w, W_p) . \qquad (4)$$

Problem (4) implies the solution of the model validation problem (that is to describe the set of non-falsified weights W_{NF}) and the error quantification in the optimal setup where the control criterion is treated as the estimation one.

Problem (4) concerns the area of control-oriented identification or identification for robust control. Although the nominal system G is given and only the weights (W_w, W_p) are to be estimated, the problem seems to be computationally intractable (much more difficult problem of optimal estimation of the nominal system is beyond the scope of the present paper). Most advanced problems of error quantification are usually formulated as follows. Let $W_w = \delta_w I$ and $W_p = \delta_p I$, where $\delta_w \geq 0$ and $\delta_p \geq 0$ are the unknown upper bounds for the norm of exogenous disturbance and the induced norm of perturbations, respectively. Then the problem of error quantification is typically stated as

$$\min\{ \delta_w \mid (W_w, W_p) \in W_{NF} , \ \delta_p \leq \delta_p^{max} \}$$

where δ_p^{max} is a prescribed upper bound providing the robust stability (Poolla, Khargonekar, Tikku, Krause, & Nagpal, 1994; Smith & Doyle, 1992).

In order to make problem (4) computationally tractable, we restrict our consideration to LTI nominal systems under coprime factor perturbations. The weight W_p is assumed to be a diagonal real matrix and $W_w = \delta_w I$. We show that the control criterion J for such systems becomes a linear fractional function both with respect to the induced norms of the system transfer matrices and with respect to the weights while the set of non-falsified weights W_{NF} is described by linear inequalities produced by data. Then the minimization of the control criterion J both over the Youla parameter of controller and over the unknown weights comes to linear programming problems. This allows to propose the iterative alternate minimizations over the weights and the Youla parameter as a heuristic method for solution of problem (4). Simulations presented in section 4 shows the efficiency of this iterative scheme.

3. NOMINAL SYSTEM UNDER COPRIME FACTOR PERTURBATIONS

3.1 Approximate optimal robust synthesis under known uncertainty level

Consider the closed loop control system

$$\begin{array}{c}(\tilde{M} - \Delta_1 W_y) y = (\tilde{N} + \Delta_2 W_u) u + \delta_w w \ , \\ u = Ky\end{array} \qquad (5)$$

where $\tilde{M}(q^{-1})$ and $\tilde{N}(q^{-1})$ are left coprime polynomial matrices in the backward shift operator q^{-1}, $\det \tilde{M}(0) \neq 0$, and K is a rational transfer matrix of the controller. The signal w represents the normalized exogenous disturbance with the weight δ_w and Δ_1 and Δ_2 are normalized perturbations with the weights

$$W_y = diag\ (\delta_y^1, \ldots, \delta_y^{n_y}),\ W_u = diag\ (\delta_u^1, \ldots, \delta_u^{n_u})\,.$$

All the weights are nonnegative without loss of generality. We consider perturbations of two kinds:

a) structured uncertainty

$$\Delta = \begin{bmatrix} \Delta_1 & 0 \\ 0 & \Delta_2 \end{bmatrix} , \quad \|\Delta_1\| \leq 1 \, , \quad \|\Delta_2\| \leq 1 \, ,$$

b) unstructured uncertainty

$$\Delta = [\Delta_1, \Delta_2] \ , \quad \| \, [\Delta_1, \Delta_2] \, \| \leq 1 \ .$$

For systems under coprime factor perturbations, the case b) is usually considered while the case a) of independent perturbations in the output and control is more realistic and may provide less conservative error quantification.

Let NM^{-1} be the right coprime factorization of the nominal system $\tilde{M}^{-1}\tilde{N}$ and polynomial matrices X and Y be the solutions to the Bezout equation

$$\tilde{M}X - \tilde{N}Y = I \ .$$

Then all stable transfer functions G_{yd} and G_{ud} from the total disturbance $d := \Delta_1 W_y y + \Delta_2 W_u u + \delta_w w$ to the output y and control u associated with stabilizing rational controllers K are of the form

$$G_{yd} = X - NQ\ , \quad G_{ud} = Y - MQ$$

where Q is the Youla parameter (see, e.g., Barabanov, 1996). In order to represent the system (5) in the form of Fig. 1 define

$$z = y\ , \quad W_w := \delta_w I\ , \quad W_p := \begin{bmatrix} W_y & 0 \\ 0 & W_u \end{bmatrix} ,$$

Theorem 1. For the system (5) with zero initial conditions

$$J(K, G, W_w, W_p) = \frac{\delta_w \|G_{yd}\|}{1 - \|W_y G_{yd}\| - \|W_u G_{ud}\|}$$

in the case a) of structured uncertainty and

$$J(K, G, W_w, W_p) = \frac{\delta_w \|G_{yd}\|}{1 - \max\{\ \|W_y G_{yd}\|\ , \|W_u G_{ud}\|\ \}}$$

in the case b) of unstructured uncertainty. The system is robustly stable if and only if the denominator in the respective formula is positive.

Proof. The proof is omitted.

Note that the relatively simple linear fractional formulae for the control criterion J does not hold if the weights W_y or W_u are not diagonal or the perturbations are weighted on theirs outputs.

Consider the problem of optimal robust synthesis (2) in the case a) of structured uncertainty. Due to Theorem 1 the auxiliary problem takes the form

$$\frac{\delta_w \|G_{yd}\|}{1 - \|W_y G_{yd}\| - \|W_u G_{ud}\|} \le \gamma$$

and can be rewritten as

$$\frac{\delta_w}{\gamma} \|G_{yd}\| + \|W_y G_{yd}\| + \|W_u G_{ud}\| \le 1\ . \qquad (6)$$

Problem (6) is the standard mixed sensitivity problem of the ℓ_1 optimization. Its approximate solution by the scaled-Q method proposed in (Khammash, 2000) comes to a finite linear programming with respect to the coefficients $Q(k)$ of the polynomial Youla parameter Q. To get an approximate solution with an ϵ tolerance, it is necessary to solve one linear programming problem for the polynomial matrix Q of sufficiently high degree. At the same time, suboptimal robust synthesis by the method proposed in Khammash, Salapaka, & Vanvoorhis (2001) needs the solution of $1/\epsilon^3$ linear programming problems of the same complexity. In the case b) of unstructured uncertainty, similar reduction of the auxiliary problem to a linear programming is possible (although the reformulated problem is not the standard mixed sensitivity problem). The method proposed in Khammash, Salapaka, & Vanvoorhis (2001) needs in this case the solution of $1/\epsilon^2$ linear programming problems.

3.2 Optimal error quantification for fixed controller

Consider the following *problem of optimal error quantification*

$$\min_{(W_w, W_p) \in W_{NF}} J(K, G, W_w, W_p)\ . \qquad (7)$$

A solution to problem (7) provides optimal estimates of the weights when the robust controller K for the given nominal system is fixed. This problem is auxiliary for our main problem (4) but seems to be useful in itself. Indeed, the weights of uncertainties may be different in different operational conditions for the same control system. Then the solution to problem (7) provides the best non-falsified estimates of weights in terms of the control criterion J. It must be emphasized, that the observed real magnitude of the system output is typically less than its worst-case value J and may not serve as a strict and logical criterion for evaluation of the given nominal model and the used controller (this is illustrated by Simulation 1 in section 4).

Now we formalize the notion of non-falsified weights and show that problem (7) for system (5) is a linear programming problem with respect to all weights of uncertainties.

Definition. Given a nominal system $P = \tilde{M}^{-1}\tilde{N}$ and the data (y_0^T, u_0^T), the weights (W_w, W_p) are said to be not falsified by the data, if there exist a disturbance w, $\|w\|_\infty \le 1$, and a perturbation Δ, $\Delta \in \underline{\Delta}$, such that the upper equation in (5) is satisfied on the time interval $[0, T]$.

Lemma 1. The weights (W_w, W_p) are not falsified by the data (y_0^T, u_0^T) if and only if for all $\tau = 0, 1, \cdots, T$

$$|(\tilde{M}y)(\tau) - (\tilde{N}u)(\tau)|_\infty \le \delta_w + \max_{s<\tau} |W_y y(s)|_\infty + \max_{s<\tau} |W_u u(s)|_\infty$$

in the case a) of structured uncertainty and

$$|(\tilde{M}y)(\tau) - (\tilde{N}u)(\tau)|_\infty \le \delta_w + \max_{s<\tau} \max\{|W_y y(s)|_\infty\ , |W_u u(s)|_\infty\}$$

in the case b) of unstructured uncertainty.

Proof. The proof of Lemma 1 follows obviously from the upper equation in (5) and Lemma 4 in Khammash & Pearson (1991).

Lemma 2. The problem of optimal estimation of weights (7) for the system (5) is a linear programming problem with respect to the weights $\delta_y^1, \ldots, \delta_y^{n_y}$, $\delta_u^1, \ldots, \delta_u^{n_u}$, δ_w.

Proof. In view of Theorem 1 the cost function in the problem (7) is linear fractional in the weights. The set of unfalsified weights W_{NF} is described by linear inequalities due to Lemma 1. Finally, any linear fractional problem under linear constraints is reducible to a linear programming problem (see, e.g., Schaible, 1974).

3.3 Iterative robust synthesis under unknown uncertainty level

Since problems (2) and (7) for system(5) are reduced to linear programming, a natural heuristic method for approximate solution of problem (4) is to solve both problems iteratively.

Step 1. Set $i := 1$. Define $K^0 := K$ and arbitrary $(W_w^0, W_p^0) \in W_{NF}$.

Step 2. On the i*th* iteration

$$K^i := \operatorname*{argmin}_K \; J(K, G, W_w^{i-1}, W_p^{i-1}),$$

$$(W_w^i, W_p^i) := \operatorname*{argmin}_{(W_w, W_p) \in W_{NF}} J(K^i, G, W_w, W_p).$$

Step 3. If

$$J(K^i, G, W_w^i, W_p^i) < (1-2\varepsilon) J(K^i, G, W_w^{i-1}, W_p^{i-1}),$$

then $i := i + 1$ and go to step 2; otherwise stop the algorithm.

Here $\epsilon > 0$ is the tolerance of approximate solution of optimal problems on step 2.

The iterative algorithm provides non-falsified estimates of the unknown weights and a decrease of the control criterion on each iteration. Although the convergence to global or local minimum in problem (4) is not guaranteed in theory, simulations in section 4 show the efficiency of the algorithm.

4. SIMULATIONS FOR SISO SYSTEM

Consider the discrete-time SISO system

$$\left(1 + \frac{13}{6}q^{-1} + \frac{17}{8}q^{-2} + \frac{3}{4}q^{-3}\right) y(t) =$$

$$\left(0.3q^{-1} + 0.8q^{-2} - 0.3q^{-3}\right) u(t) + d(t).$$

The nominal system $a(\lambda)/b(\lambda)$ is unstable and non-minimum phase with the poles $-3/2$ and $-2/3 \pm 2/3i$ and the zeros 3 and $-1/3$. The total disturbance d is simulated as follows

$$\begin{aligned} d(t) = \delta_w \xi_1(t) + 0.5\, \delta_y y(t-3) + 0.5\, \delta_y \xi_2(t) y(t-4) \\ +0.5\, \delta_u u(t-3) + 0.5\, \delta_u \xi_3(t) u(t-4) \end{aligned} \tag{8}$$

where ξ_1, ξ_2, and ξ_3 are independent uniformly distributed on [-1,1] and unknown weights are

$$[\, \delta_w \,, \delta_y \,, \delta_u \,] = [\, 0.1 \,, 0.05 \,, 0.15 \,].$$

Thus, the structured uncertainty in the system is represented by unmodeled deterministic and stochastic dynamics in d.

The solution of problem (2) with 0.001 relative tolerance gives the optimal robust controller characterized by the pair

$$(\|G_{yd}^{opt}\|, \|G_{yd}^{opt}\|) = (7.339, 1.9322).$$

Note that the optimal robust controller depends only on the weight δ_u (see Sokolov (2000) for details). Therefore, the iterative algorithm was tested for 10 different initial estimates of δ_u.

Simulation 1. The upper graph in Fig. 2 presents the outputs y_0^{100} produced by the ℓ_1 optimal controller associated with zero perturbations. For all, except zero, initial estimates of δ_u, the iterative algorithm of subsection 3.3 gives a controller associated with the pair $(\|G_{yd}\|, \|G_{yd}\|) = (6.4384, 2.9213)$. For comparison, the lower graph presents the outputs produced by the optimal robust controller, which is unknown to controller designer.

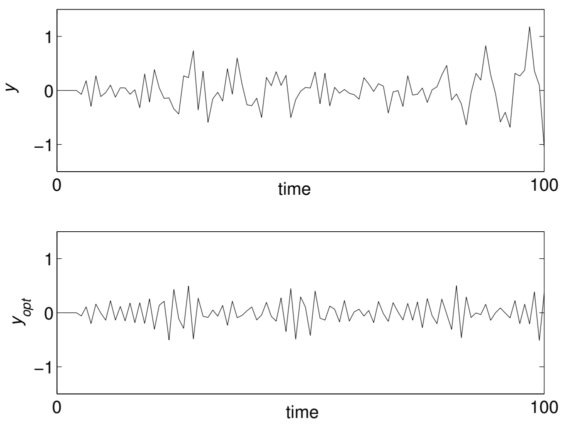

Fig. 2. Outputs: y – for the ℓ_1 optimal controller, y_{opt} – for the actual optimal robust controller

Simulation 2. The upper graph in Fig. 3 presents the outputs y_0^{100} produced by the controller obtained in Simulation 1. For all, except zero, initial estimates of δ_u, the iterative algorithm of subsection 3.3 applied to the joint data of Simulations 1 and 2 gives the same controller as in Simulation 1. For comparison, the lower graph presents the outputs produced by the optimal robust controller.

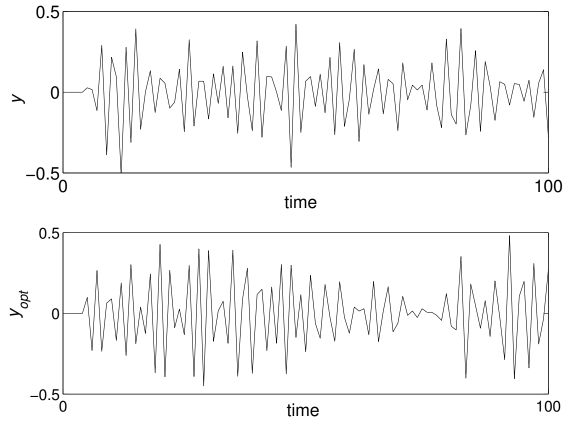

Fig. 3. Outputs: y – for the controller provided by Simulation 1, y_{opt} – for the actual optimal robust controller

Simulation 3. Note that the ℓ_1-optimal controller is characterized by the pair $(\|G_{yd}\|, \|G_{ud}\|) = (4.1714, 17.649)$ and does not ensure robust stability of the closed loop system. Although uncertainty (8) is not worst-case, potential instability of the system closed by the ℓ_1-optimal controller is illustrated on Fig. 4. In this case the iterative algorithm of robust synthesis gives exactly the optimal robust controller.

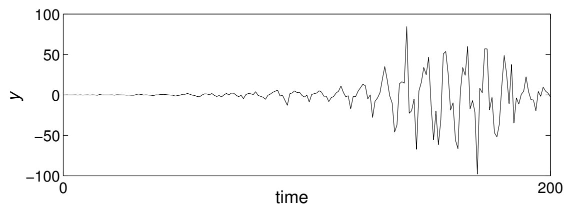

Fig. 4. The output y for the ℓ_1 optimal controller

5. CONCLUSION

The problem of suboptimal robust synthesis under unknown uncertainty level was discussed for coprime factor nominal systems. Proposed iterative algorithm allows to avoid the search over a grid in the space of upper bounds for perturbations and exogenous disturbance.

References

Barabanov, A.E. (1996). *Synthesis of minimax controllers.* Saint-Petersburg: Saint-Petersburg University (in Russian).

Dahleh, M.A., & Diaz-Bobillo, I.J. (1995). *Control of uncertain systems: a linear programming approach.* Englewood Cliffs, NJ: Prentice-Hall.

Dullerud, G. & Smith, R. (2002). A Nonlinear Functional Approach to LFT Model Validation. *Systems and Control Letters 47*, 1-11.

Khammash, M.H. (2000). A new approach to the solution of the ℓ_1 control problem: the scaled-Q method. *IEEE Trans. Automat. Control 45*, 180-187.

Khammash, M.H. (1995) Robust steady-state tracking. *IEEE Trans. Autom. Control 40*, 1872-1880.

Khammash, M. H., & Pearson, J.B. (1993). Analysis and design for robust performance with structured uncertainty. *Systems and Control Letters 20*, 179-187.

Khammash, M.H., & Pearson, J.B. (1991). Performance Robustness of Discrete-Time Systems with Structured Uncertainty. *IEEE Trans. Automat. Control 36* (4), 398-412.

Khammash, M.H., Salapaka, M.V., & Vanvoorhis, T. (2001). Robust Synthesis in ℓ_1: A Globally Optimal Solution. *IEEE Trans. Automat. Control 46* (11), 11744-1754.

Khammash, M. H., & Pearson, J.B. (1993). Analysis and design for robust performance with structured uncertainty. *Systems and Control Letters 20*, 179-187.

Khammash, M.H., & Pearson, J.B. (1991). Performance Robustness of Discrete-Time Systems with Structured Uncertainty. *IEEE Trans. Automat. Control 36* (4), 398-412.

Poolla, K., Khargonekar, P., Tikku, A., Krause, J., & Nagpal, K. (1994). A Time-Domain Approach to Model Validation. *IEEE Trans. Autom. Control 39* 951-959.

Schaible, S. (1974). Parameter-free convex equivalent and dual programs of fractional programming problems, *Zeitschrift für Operations Research 18*, 187-196.

Smith, R.C., and Doyle, J.C. (1992). Model Validation: A Connection Between Robust Control and Identification. *IEEE Trans. Automat. Control 37* 942-952.

Smith, R., Dullerud, G., & Miller, S. (2000). Model validation for nonlinear feedback systems. In *Proc. 39th IEEE Conf. on Decision and Control*, Sydney, Australia, 1232-1236.

Sokolov, V.F. (2003). Robust Control in ℓ_1-formulation: Model Validation and Perturbation Weight Estimation. *Automation and Remote Control 64* (11), 1769-1781.

Sokolov, V.F. (2002). l1 suboptimal robust controllers for MIMO plants under coprime factor perturbations. *Proc. 15th IFAC World Congress*, Barselona, Spain, 879-884.

Sokolov, V.F. (2000). ℓ_1 Optimal Robust Controller for SISO Plant under Coprime Factor Perturbations, *IEEE Trans. Autom. Control 45*, (12) 2339-2345.

Yamada, M. & Funahashi, Y. (2002) Minimization of the Worst-Case Steady-State Error for Multivariable Deadbeat Control Systems with Time-Varying Uncertainty. In *Proceedings of the 41st IEEE Conference on Decision and Control*, Las Vegas, USA, 3324-3329.