AUTONOMOUS OSCILLATION GENERATION IN THE BOOST CONVERTER

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Abstract: In this paper we introduce a control method for generation of oscillations in electronic converters. The main novelty of the new control strategy is the lack of reference signals. This is accomplished by a feedback law associated to a Lyapunov function which guarantees the stability and robustness of the system. The method is illustrated by means of a boost converter system. Simulations show the performance of the closed-loop systems.

Keywords: Energy shaping, nonlinear oscillations, power converters, boost converter.

1. INTRODUCTION

In electronic converters with AC output the control objective can be seen as the generation of a stable limit cycle with given amplitude and frequency for which voltages and currents present a sinusoidal behaviour with pre-specified phase shift. If a control law is able to produce such a limit cycle, the generation of alternating current will be accomplished without the need for introducing any time dependent reference signal (tracking method). The generation of limit cycles for producing self-oscillations has been successfully applied to electro-mechanical systems (Aracil *et al.*, 2002; Gómez-Estern *et al.*, 2002; Gordillo *et al.*, 2002; Aracil *et al.*, 2004). In Aracil and Gordillo (2002) this idea was used to generate oscillations in a three-phase DC/AC converter, which was modelled using a linear model. Here we apply the same idea to a nonlinear converter such as the boost converter. It is shown that the same control idea can be applied by changing variables and neglecting high harmonics.

This paper is organized as follows. Section 2 briefly describes the theory of oscillation generation by energy shaping. In Section 3, the boost model is analyzed taking into account its internal dynamics. Section 4 deals with the problem of generating oscillations in a boost converter circuit using the approach of Section 2. Section 5 presents simulation results.

2. AUTONOMOUS OSCILLATION GENERATION

The normalized form of a non-linear oscillator can be expressed as (Aracil *et al.*, 2002)

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\omega^2 x_1 - k x_2 \Gamma \end{cases} \tag{1}$$

where $\Gamma = \omega^2 x_1^2 + x_2^2 - \mu$ since the solutions of (1) are $x_1 = A \sin \omega t$ and $x_2 = A\omega \cos \omega t$ with $A = \sqrt{\frac{\mu}{\omega^2}}$. It is easy to see that system (1) has the Lyapunov function

$$V = \frac{\omega}{4}\Gamma^2. \tag{2}$$

In effect

$$\dot{V} = -kx_2^2\omega^2\Gamma^2 \leq 0. \tag{3}$$

The minimum of function V is reached at the set $\omega^2 x_1^2 + x_2^2 - \mu = 0$. This set is a closed curve (an ellipse) that is a clear candidate to be a limit cycle. We can define a dynamical system as such in which this closed curve is its limit cycle. This can be reached adopting V as a Hamiltomian function and defining the following dynamical system

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & \frac{1}{\Gamma}\\ -\frac{1}{\Gamma} & -k\end{bmatrix}\begin{bmatrix}\omega^2 x_1\Gamma\\ x_2\Gamma\end{bmatrix}. \tag{4}$$

This last expression is an alternative way of writing Eq. (1). Variables ω and μ are design parameters for the frequency and amplitude of the desired behavior while k defines the speed of the transient response of

$$\ddot{x} + k\Gamma\dot{x} + \omega^2 x = 0. \tag{5}$$

The close curve $\Gamma = 0$ divides the state space into two regions. In what follows consider $k > 0$. When $\Gamma > 0$, the damping term of (5), $k\Gamma\dot{x}$, is positive and for $\Gamma < 0$ is negative. In the first case, the effect is that of an attenuator and in the other case of an amplifier. The system has only one equilibrium point given by $(\bar{x}_1, \bar{x}_2) = (0, 0)$ that is stable if $\Gamma > 0$. In the second case ($\Gamma < 0$) it is unstable. It is worth noting that the desired long term behavior takes place at the boundary between the two regions of different damping behavior (Fig. 1), indicating the presence of a stable limit cycle.

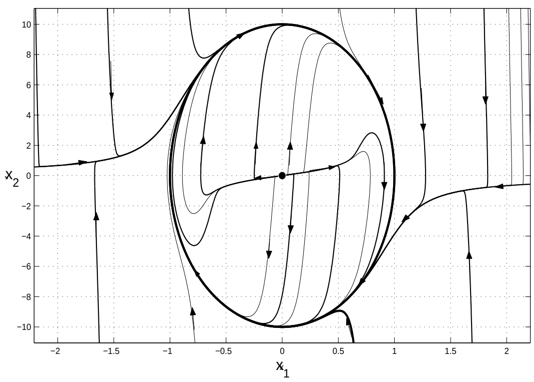

Fig. 1. State space (x_1, x_2) of system (4), for $\mu = 100$, $\omega = 10$ and $k = 1$, showing a stable limit cycle.

Also notice that for $k = 0$ or $\Gamma = 0$ system (4) reduces to

$$\begin{cases}\dot{x}_1 = x_2\\ \dot{x}_2 = -\omega^2 x_1\end{cases} \tag{6}$$

that is the well-know harmonic oscillator $\ddot{x} = -\omega^2 x$ where the equilibrium $(\bar{x}_1, \bar{x}_2) = (0, 0)$ is a center.

There is an alternative method for obtaining an oscillating system. This is based on sliding modes. This consists in considering a second order system similar to (1), but with Lyapunov function

$$V = \frac{|\Gamma|}{2} \tag{7}$$

instead of (2). We have then,

$$\nabla_x V = \begin{bmatrix}D_{x_1}V\\ D_{x_2}V\end{bmatrix} = \begin{bmatrix}2\omega^2 x_1 \mathrm{sgn}\Gamma\\ 2x_2\mathrm{sgn}\Gamma\end{bmatrix} \tag{8}$$

Adopting V as a Hamiltonian function, we can obtain the following dynamical system

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & \frac{1}{\mathrm{sgn}\Gamma}\\ -\frac{1}{\mathrm{sgn}\Gamma} & -k\end{bmatrix}\begin{bmatrix}\omega^2 x_1\mathrm{sgn}\Gamma\\ x_2\mathrm{sgn}\Gamma\end{bmatrix} \tag{9}$$

which can be written as

$$\begin{cases}\dot{x}_1 = x_2\\ \dot{x}_2 = -\omega^2 x_1 - kx_2\mathrm{sgn}\Gamma\end{cases} \tag{10}$$

Expressions (7) and (10) can be compared with (2) and (1), respectively.

3. MODEL OF THE BOOST CONVERTER

The boost converter is usually used as a DC-DC converter when the desired output voltage is higher than the input voltage. Here, the objective is to obtain an oscillating signal for this output (Biel *et al.*, 1999; Fossas and Olm, 2002; Sira-Ramírez and Prada-Rizzo, 1993). Due to the characteristics of the converter, the output signal can not cross through zero and, therefore, the desired signal has to present an offset in such a way that it is always positive. An application of this idea is to combine two of such converters by connecting the load differentially in order to obtain an alternating current without an offset (Cáceres and Barbi, 1999).

Figure (2) shows a schematic diagram of the converter, which consists of an input inductance L, a set of switch composed by a diode and a MOSFET transistor and an output capacitor C. We consider that all the elements are ideal and that the converter operates in Continuous Conversion Mode (CCM). The constraints for the state variables are $\xi_1 \geq 0$, $\xi_2 \geq 0$, where ξ_1 is the inductor current and ξ_2 is the capacitor voltage V_c. In the circuit of Fig. 2, R represents the load resistance; $E > 0$ is a DC voltage source and $V_{\mathrm{out}} = V_c$ is the output voltage that we want to regulate.

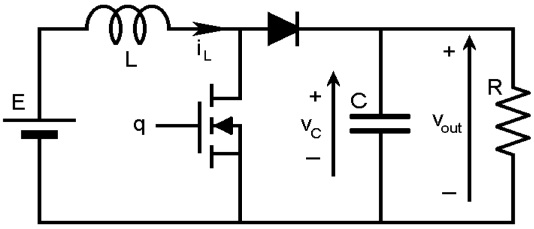

Fig. 2. Ideal model for the boost converter.

The instantaneous model can be written as

$$\begin{cases} L\dot{\xi}_1 = -u\xi_2 + E \\ C\dot{\xi}_2 = u\xi_1 - \dfrac{1}{R}\xi_2 \end{cases} \tag{11}$$

where $u = 1-q$ is the control action and $q \in \{0,1\}$ is the discrete state of the switch. On the other hand, if we consider the average values model, the control action will be $u = 1 - d$ where d is the duty ratio of the Pulse-Width Modulation (PWM) signal applied to the switch (Cunha and Pagano, 2002).

3.1 *Normalized model*

In order to simplify the study of the boost converter, the following change of variables is applied

$$\begin{aligned} x_1 &= \frac{1}{E}\sqrt{\frac{L}{C}}\xi_1 \\ x_2 &= \frac{\xi_2}{E,} \end{aligned} \tag{12}$$

resulting in

$$\begin{aligned} \dot{x}_1 &= -\frac{1}{\sqrt{(LC)}}ux_2 + \frac{1}{\sqrt{(LC)}} \\ \dot{x}_2 &= \frac{1}{\sqrt{(LC)}}ux_1 - \frac{1}{RC}x_2. \end{aligned} \tag{13}$$

Defining

$$\tilde{t} = \omega_0 t, \tag{14}$$

as a new time variable with

$$\omega_0 = \frac{1}{\sqrt{LC}}, \tag{15}$$

the resulting normalized model is

$$\begin{aligned} \dot{x}_1 &= 1 - ux_2 \\ \dot{x}_2 &= -ax_2 + ux_1 \end{aligned} \tag{16}$$

where $a = \frac{1}{R}\sqrt{\frac{L}{C}}$.

3.2 *Internal dynamic analysis*

In this Section, the internal dynamics of the boost model is analyzed. From (16), eliminating u we have that

$$x_1(1 - \dot{x}_1) = x_2(\dot{x}_2 + ax_2). \tag{17}$$

This equation puts in evidence a differential relation between the state variables and its derivatives without the action control u. Eq. (17) represents the internal dynamics of the system. Making $\dot{x}_1 = 0$ and $\dot{x}_2 = 0$, the equilibria manifold $x_1 = ax_2^2$ can be obtained. In such a way, the internal dynamics of system (16) given by (17) acts as a constrain on the state of the system.

It can be proved (Fossas and Olm, 2002) from Eq. (17) that by merely controlling x_1 the desired signal in x_2 can be obtained thus maintaining the stability of the system.

Therefore we can obtain the desired behavior of the state variables when a desired trajectory ${x_1}^*$ is imposed in Eq. (17). Since the control objective is in terms of x_2, the desired behaviour for x_1 has to be computed.

4. CONTROLLER DESIGN

The control objective is to generate a stable limit cycle with amplitude and frequency for which voltage and current of the boost circuit present a sinusoidal behavior.

In order to apply the ideas of Section (2), first it is necessary to obtain an analytical expression of the desired objective curve in the plane (x_1, x_2). Assume that the desired time evolution for x_2 is

$${x_2}^* = A\sin(\omega t) + B. \tag{18}$$

where A, B, ω take pre-specified values that would be obtained from the desired evolution for ξ_1 and ξ_2 using (12), (14) and (15). Assume that the steady state for x_1 that gives this desired value for x_2 can be approximated by (Olm, 2004)

$$x_1^* = a\alpha_0 + \alpha_1\cos\omega t + \beta_1\sin\omega t. \tag{19}$$

Substituting (18) and (19) in (17), we obtain

$$\begin{aligned} & a\alpha_0 + a\alpha_0\,\alpha_1\,\sin\omega t\omega - a\alpha_0\,\beta_1\,\cos\omega t\omega \\ & + \alpha_1\,\cos\omega t + 1/2\,{\alpha_1}^2\omega\,\sin 2\,\omega t - \alpha_1\,\beta_1\,\omega\,\cos 2\,\omega t \\ & + \beta_1\,\sin\omega t - 1/2\,{\beta_1}^2\omega\,\sin 2\,\omega t \\ & = 1/2\,A^2\omega\,\sin 2\,\omega t + 1/2\,A^2 a - 1/2\,A^2 a\cos 2\,\omega t \\ & + 2\,A\sin\omega t a B + BA\cos\omega t\omega + B^2 a \end{aligned} \tag{20}$$

Neglecting the terms $\sin 2\omega t$ and $\cos 2\omega t$ and equating the bias, $\sin\omega t$ and $\cos\omega t$ the following equations are obtained:

$$\begin{aligned} a\alpha_0 &= A^2 a + \frac{1}{2}B^2 a \\ -a\alpha_0\beta_1\omega + \alpha_1 &= AB\omega \\ a\alpha_0\alpha_1\omega + \beta_1 &= 2aAB. \end{aligned}$$

Solving this system of equations for α_0, α_1 and β_0 yields

$$\alpha_0 = A^2 + \frac{1}{2}B^2 \quad (21)$$

$$\alpha_1 = \frac{4AB\omega(2a^2A^2 + 1 + a^2B^2)}{4 + 4a^2\omega^2A^4 + a^2\omega^2A^2B^2 + a^2\omega^2B^4} \quad (22)$$

$$\beta_1 = \frac{-2aAB(2\omega^2A^2 + \omega^2B^2 - 4)}{4 + 4a^2\omega^2A^4 + 4a^2\omega^2A^2B^2 + a^2\omega^2B^4} \quad (23)$$

In this way an approximated expression for x_1^* is obtained in the form $x_1^* = a\alpha_0 + \alpha_1 \cos\omega t + \beta_1 \sin\omega t$ corresponding to x_2^*. This result is approximate since the second order harmonics has been neglected. In order to validate this assumption, it is possible to calculate the difference between the two sides of Eq.(17). Using the approximation of x_1 given by Eq.(19) and x_2^* as variables, we obtain

$$e(t) = g(t) - h(t) \quad (24)$$

where

$$h(t) = x_1^*(1 - \dot{x}_1^*)$$

and

$$g(t) = x_2^*(\dot{x}_2^* + ax_2^*).$$

The functions $h(t)$ and $g(t)$ are shown in Fig.(3) and the error $e(t)$, resulting from neglecting the double frequency terms, is shown in Fig.(4).

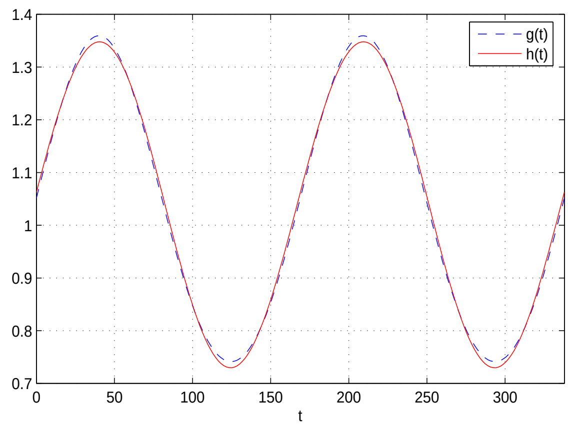

Fig. 3. $h(t)$ and $g(t)$ functions.

As can be seen, in Figs.(3) and (4), the error resulting from neglecting the terms $\sin 2\omega t$ and $\cos 2\omega t$ is not relevant with respect to the aim of AC generation.

We need another change of variables in order to obtain a model that permits the matching with (1). For this, define

$$\begin{aligned} y_1 &= \frac{x_1^2 + x_2^2}{2} \\ y_2 &= x_1 - ax_2^2 + y_{20} \end{aligned} \quad (25)$$

where y_{20} represents an offset term that will be a tuning parameter. It is easy to see that

$$\begin{aligned} \dot{y}_1 &= y_2 - y_{20} \\ \dot{y}_2 &= 1 + 2a^2x_2^2 - x_2(1 + 2ax_1)u \end{aligned} \quad (26)$$

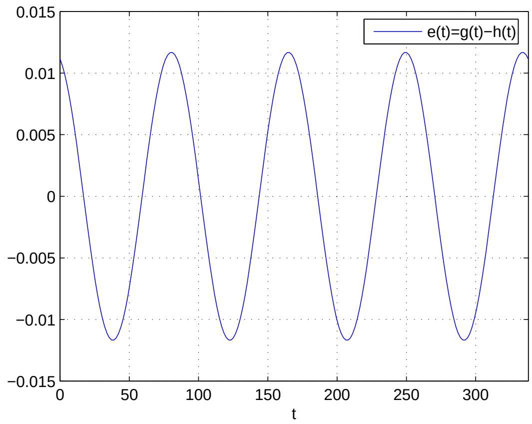

Fig. 4. Resulting error $e(t)$ due to neglecting the double frequency terms.

where

$$\begin{aligned} x_1 &= f(y_1, y_2) = \frac{\sqrt{1 + 8a^2y_1 + 4a(y_2 - y_{20})} - 1}{2a} \\ x_2 &= \sqrt{\frac{x_1 - y_2 - y_{20}}{a}}. \end{aligned} \quad (27)$$

As it will be seen later, the objective for variables y_1, y_2 is also an ellipse but not necessarily centered at the origin. It is easy to see that the system

$$\begin{aligned} \dot{y}_1 &= y_2 - y_{20} \\ \dot{y}_2 &= -\omega_1^2(y_1 - y_{10})^2 - k\Gamma(y_2 - y_{20}) \end{aligned} \quad (28)$$

with $\Gamma = \omega_1^2(y_1 - y_{10})^2 + (y_2 - y_{20})^2 - \mu$, $\mu > 0$ presents as limit set the ellipse $\Gamma = 0$. Notice the similarity between Eq. (28) and (1). An alternative form can be obtained using Eq.(10).

The control law u that matches systems (26) and (28) is

$$u = \frac{1 + 2a^2x_2^2 + k\,\Gamma\,(y_2 - y_{20}) + \omega^2(y_1 - y_{10})}{x_2(1 + 2ax_1)} \quad (29)$$

or

$$u = \frac{1 + 2a^2x_2^2 + k\,\mathrm{sgn}\Gamma\,(y_2 - y_{20}) + \omega^2(y_1 - y_{10})}{x_2(1 + 2ax_1)} \quad (30)$$

where k is a tuning parameter that defines the convergence rate towards the desired ellipse.

The only questions that remains now is to show that the desired behavior for y_1 and y_2 is an ellipse and to define the ellipse parameters (ω_1, y_{10}, y_{20} and μ) in terms of the desired behavior for x_2 – which depends on the desired behavior for ξ_2. For this it is necessary to obtain the desired evolution for y_1 and y_2 applying the change of variables (25) to (18) and (19):

$$\begin{aligned} y_1 &= \frac{1}{2}[(a\alpha_0 + \alpha_1 \sin\omega t \\ &+\beta_1 \cos\omega t)^2 + (A\sin\omega t + B)^2] \\ y_2 &= a\alpha_0 + \alpha_1 \sin\omega t \\ &+\beta_1 \cos\omega t - a(A\sin\omega t + B)^2 + y_{20} \end{aligned} \quad (31)$$

Expanding these expressions in Fourier terms yields

$$\begin{aligned} y_1 &= y_1^{(0)} + y_1^{(11)} \cos \omega t + y_1^{(12)} \sin \omega t + \\ & y_1^{(21)} \cos 2\omega t + y_1^{(22)} \sin 2\omega t \\ y_2 &= y_2^{(0)} + y_2^{(11)} \cos \omega t + y_2^{(12)} \sin \omega t \\ & + y_2^{(21)} \cos 2\omega t + y_2^{(22)} \sin 2\omega t, \end{aligned}$$

with

$$\begin{aligned} y_1^{(0)} &= \frac{2a^2\alpha_0^2 + \alpha_1^2 + \beta_1^2 + A^2 + 2B^2}{4} \\ y_1^{(11)} &= a\alpha_0\alpha_1 \\ y_1^{(12)} &= a\alpha_0\beta_1 + AB \\ y_1^{(21)} &= \frac{\alpha_1^2 - \beta_1^2 - A^2}{4} \\ y_1^{(22)} &= \frac{\alpha_1\beta_1}{2} \\ y_2^{(0)} &= y_{20} \\ y_2^{(11)} &= \alpha_1 \\ y_2^{(12)} &= \beta_1 - 2aAB \\ y_2^{(21)} &= \frac{aA^2}{2} \\ y_2^{(22)} &= 0. \end{aligned}$$

Assuming that the double frequency terms

$$y_1^{(21)}, y_1^{(22)}, y_2^{(21)}, y_2^{(22)}$$

can be neglected, these expressions can be approximated by an ellipse in the plane (y_1, y_2) since, using (21)–(23),

$$\begin{aligned} \omega y_1^{(11)} &= -y_2^{(12)} \\ \omega y_1^{(12)} &= y_2^{(11)}. \end{aligned}$$

The parameters of this ellipse are given by

$$\begin{aligned} \omega_1 &= \omega \\ y_{10} &= y_1^{(0)} \\ y_{20} &= y_2^{(0)} \\ \mu &= \omega^2((y_1^{(11)})^2 + (y_1^{(12)})^2). \end{aligned}$$

In control law (29), the parameter y_{10} allows us to tune the mean value of x_1; y_{20} has not any effect on the system (x_1, x_2); μ can be used in order to adjust the amplitude of x_1 and k defines the speed of convergence to the desired cycle limit.

5. SIMULATION RESULTS

Considering $L = 18mH$, $C = 220\mu F$, $E = 50V$ and $R = 10\Omega$ as the boost parameters, it is calculated $a = 0.9045$ (Olm, 2004). The desired sinusoidal output voltage of the boost circuit is defined as

$$\xi_2{}^* = 135 + 15 \sin 2\pi 50t. \qquad (32)$$

Applying the change of variables (12) we obtain

$$x_2{}^* = 2.7 + 0.3 \sin 0.6252t \qquad (33)$$

in the normalized variables x_1, x_2.

Defining $y_{20} = 10$ and using the above formulae the following parameter values are obtained:

$$\begin{aligned} y_1^{(0)} &= 25.7089 \\ y_1^{(11)} &= 2.3995 \\ y_1^{(12)} &= 0.5785 \\ y_1^{(21)} &= 0.0099 \\ y_1^{(22)} &= -0.0063 \\ y_2^{(0)} &= 10 \\ y_2^{(11)} &= 0.3617 \\ y_2^{(12)} &= 1.5002 \\ y_2^{(21)} &= 0.0407 \\ y_2^{(22)} &= 0. \end{aligned}$$

Notice the validity of the double harmonic terms neglect. Then, the ellipse parameters in y_1, y_2 are:

$$\begin{aligned} \omega_1 &= 0.6252 \\ y_{10} &= 25.7089 \\ y_{20} &= 10 \\ \mu &= 2.3814. \end{aligned}$$

The control law is defined introducing these parameter values in (29) and choosing the damping parameter value. In the following $k = 0.1$ is used.

Figure 5 shows the results of a simulation. Figure 6 presents the corresponding behavior when the PWM is included in the system. A switching frequency for the PWM equal to $10kHz$ has been used.

Simulation results show that the system reaches an oscillatory time response that corresponds to the desired output voltage.

6. CONCLUSIONS

In this paper, a control strategy to generate autonomous oscillations in electronic converters has been presented. This approach avoids the need for reference signals in order to generate AC voltages, as used in tracking control methods. The proposed methodology has been applied to a boost converter. Output voltage control of boost converter is performed indirectly through the inductor current. Since this control objective is defined in

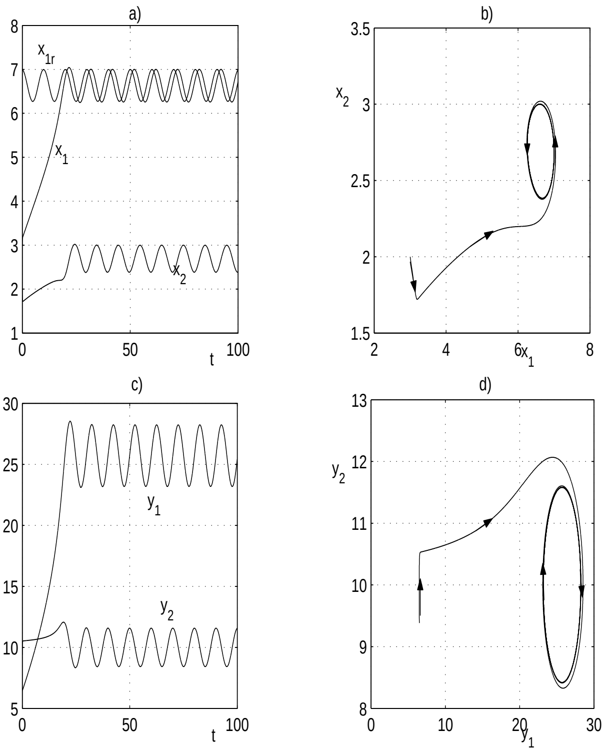

Fig. 5. a) Time response for $x_1(t)$, $x_2(t)$; b) state space (x_1, x_2); c) Time response for $y_1(t)$, $y_2(t)$; d) state space (y_1, y_2).

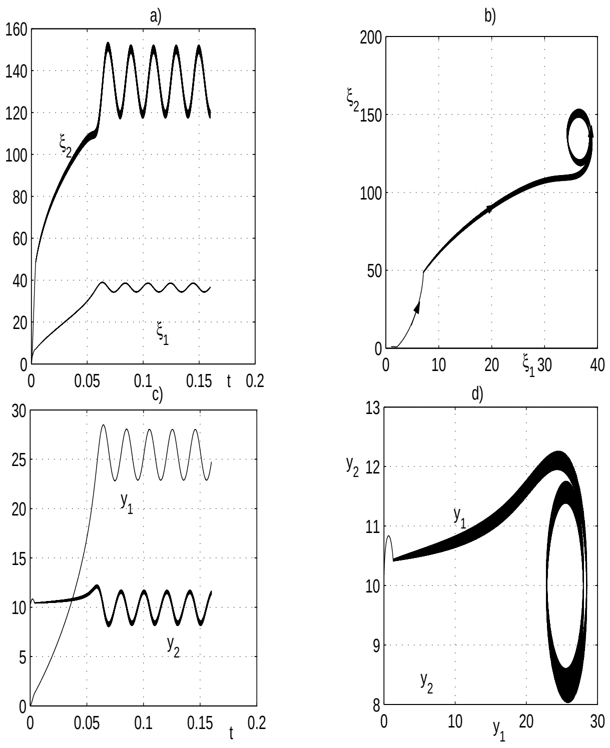

Fig. 6. a) Time response for $\xi_1(t)$, $\xi_2(t)$; b) state space (ξ_1, ξ_2); c) time response for $y_1(t)$, $y_2(t)$; d) state space (y_1, y_2).

terms of the voltage output, the desired behaviour of the inductor current has to be computed in an approximate form. This approach allows for sensitivity to load changes, which can be removed by means of an adaptive strategy though this is not shown here. The validity of the laws has been checked by simulation. This methodology can be extended to other electronic converters such as the double boost inverter.

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