CONSTRAINED DECENTRALIZED FLOW CONTROL OF COMMUNICATION NETWORKS [1]

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Abstract: We present a decentralized nonlinear flow control scheme for a class of communication networks with physical constraints. Through a detailed analysis, we demonstrate that nonlinear system theory can be applied to cope with saturation nonlinearity and unknown disturbances in network flow control problems. We solve the constrained queue regulation problem against traffic interferences with control input and state saturation, under two explicitly identified conditions, namely a "PE" condition and a Lipschitz-like condition. Asymptotic regulation is achieved for both a single-node system and large-scale networks, for all feasible initial states. The trade-offs of various control parameter settings are revealed through our analysis. Computer simulations confirm the effectiveness of our nonlinear network flow control scheme. *Copyright ©2005 IFAC*

Keywords: Flow control, Saturation, Asymptotic regulation, Nonlinear control.

1. INTRODUCTION

Recently, linear and nonlinear analysis and control design tools prove effective in network flow control problems (Alpcan and Basar, 2003; Wen and Arcak, 2004; Srikant, 2004; Low *et al.*, 2002; Quet and Ozbay, 2004). Many previous results are based on linearization ideas or linear robust control theory. Saturation constraints, especially on nonlinear models have not been systematically addressed. Thus, the application of nonlinear control theory to handle hard nonlinearities in large-scale networks deserves further investigation.

We focus on nonlinear flow control problems, in particular, under capacity saturation constraints and nonlinear disturbances. In recent literature, the authors of (Pitsillides *et al.*, 2001) proposed a nonlinear regulator for a buffer management model. Using feedback linearization and robust adaptive control ideas, the authors gave solution to bounded regulation against unknown time varying traffics. However, the impact of saturation constraints on control system performance is not analyzed in (Pitsillides *et al.*, 2001)

Our work is inspired in part by the above discussion with particular interest to address the saturation and to achieve asymptotic regulation against disturbances. Besides a single-node system, decentralized regulators for networks with interconnected nodes are also proposed. Due to the presence of saturation nonlinearities, extending the robust adaptive control scheme in (Pitsillides *et al.*, 2001) to large-scale networks is nontrivial.

In this paper, we first propose a new control law which achieves either asymptotic or practical queue regulation of the single-node system. Quantitative bounds for the controller parameters are explicitly provided to guide us tune the performance of the closed-loop system. Then, the decentralized control of large-scale networks is addressed, with the inter-node traffic being treated as subsystem interconnection. We identify the conditions under which the constrained regulation problem is solvable, namely a "persistent excitation" (PE) (Khalil, 2002) condition and a saturation-based Lipschitz-like condition. Our ideas are in part inspired by recent advances in decentralized control for large-scale nonlinear systems (Jiang, 2004). The analysis shows that the control design for the single-node system is scalable to interconnected networks. The large-scale network achieves either asymptotic or practical regulation, by appropriately tuning control parameters.

The contributions of our paper are:

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1. We address the issue of saturation constraints on the control input and the state variables, because of their importance in control engineering applications. As it is clearly understood in the present literature (see (Hu and Lin, 2001) and numerous references therein), the available solutions for the stabilization of linear systems with control input saturation already require involved analysis and synthesis techniques. For nonlinear systems with state and input constraints, the stabilization problem has received much less attention and requires further study. In particular, we give a solution to the constrained regulation of a class of large-scale networks, for all feasible initial conditions. In this sense, global regulation is achieved.
2. Unlike (Pitsillides *et al.*, 2001) where only bounded regulation was addressed for a single-node system, here we consider both a single-node system and a network with interconnected nodes. Moreover, we achieve asymptotic regulation as opposed to the bounded regulation. For the single-node system, the uncertain traffic is treated as non-vanishing disturbances, while for the large-scale network, the unknown traffic is separated into structured (vanishing) and unstructured (non-vanishing) uncertainties to enhance the performance.
3. As compared to our previous "low gain" control design in (Fan and Jiang, 2004), we use "high gain" feedback in this paper to improve the performance of the closed-loop system, namely to achieve faster convergence and stronger disturbance rejection.
It should be mentioned that our proposed solution is application-oriented and does employ the physical characteristics of the system in question. For instance, the asymptotic regulation of queue length is achievable by means of a feedback control law, provided that a "PE" (persistency of excitation) condition is met for the incoming (disturbance) traffic. As it will be clear later, there is no need to activate the feedback control law (or assign additional capacity) if this PE condition is not fulfilled, or physically speaking if there is not enough incoming (to-be-rejected disturbance) traffic. The Lipschitz-like condition is related to the physical constraints such as power constraint and capacity constraint.

2. SINGLE-NODE SYSTEM DESIGN

By "node", we refer to a router/switch in the network throughout the rest of the paper. The following model is first introduced in (Agnew, 1976) and is considered by several authors recently (Tipper and Sundareshan, 1990)(Pitsillides *et al.*, 2001)(Guffens and Bastin, 2003). The model uses the conservation law to establish the dynamic equation of the buffer queue length:

$$\dot{x}(t) = -\frac{x(t)}{1+x(t)} \cdot C(t) + \lambda(t) \quad (1)$$

$$x(t) \in [0, x_{buffer}] \quad (2)$$

$$C(t) \in [0, C_{server}] \quad (3)$$

In the above equation, queue size x is taken as the state variable. C represents the to-be-assigned capacity. It is taken as the control input. These variables are subject to physical constraints (2)-(3) with x_{buffer} denoting size of the buffer and C_{server} the maximum available capacity. λ represents the average incoming traffic rate, which is considered as a disturbance input. By conservation law, the first term in the above equation represents the average outgoing traffic rate. The validity of using $\mu(x) = \frac{x}{1+x}C$ to represent the average traffic departure rate has been verified by Filipiak and Tipper through simulations, respectively in (Filipiak, 1984) and (Tipper and Sundareshan, 1990).
x_{ref} is introduced as the given reference queue length. It should be chosen such that the node is sufficiently utilized while preserving certain capability to handle additional traffic bursts. In practice, an empty or extremely small steady state queue usually leads to link under-utilization and is thus undesired. We suppose that the reference value x_{ref} satisfies:

$$\underline{\epsilon} \leq x_{ref} < x_{buffer}.$$

The lower bound $\underline{\epsilon} > 0$ could be an arbitrary positive value. Our control law does not bear singularity when $x_{ref} \to 0$. The assumption $x_{ref} \geq \underline{\epsilon}$ is due to physical considerations. For later reference, $\bar{x} \doteq x - x_{ref}$ stands for the regulation error.
We are more interested in asymptotic regulation for the queue length (i.e., $\bar{x} \to 0$ or, $x \to x_{ref}$) than (merely) bounded regulation as in (Pitsillides *et al.*, 2001).
The following saturated controller is proposed for achieving asymptotic regulation, in the presence unknown but bounded disturbances.

$$C(x) = \max\{0, C_{server} \cdot sat[\alpha\bar{x} + \beta sgn(\bar{x})]\} \quad (4)$$

where α, β are design parameters. "*sat*" is the commonly used saturation function defined as $sat(y) = \min\{|y|, 1\} sgn(y)$, where "*sgn*" is the standard signum function. Since it is unnecessary to assign additional capacity when the node is under-utilized, we take $C = 0$ when $x < x_{ref}$.
We now analyze the performance of the closed-loop system under the control law (4). In this section, assume that the average rate of the incoming traffic (λ) satisfies the following hypothesis.

Assumption 1. $\lambda(t)$ belongs to $[0, b]$ for all $t \geq 0$, with b a constant satisfying that

$$0 < b \leq \frac{x_{ref}}{1+x_{ref}} C_{server}. \quad (5)$$

Furthermore, let $t_0 \geq 0$ be any initial time instant, $x_{ref} < \int_{t_0}^{\infty} \lambda(t)dt \leq \infty$.

Remark 1. $x_{ref} < \int_{t_0}^{\infty} \lambda(t)dt \leq \infty$ is a "persistent excitation"(PE) requirement. Under this assumption, queue length $x(t)$ will reach the reference value in finite-time if $x(t_0) < x_{ref}$. Physically, this implies that the node is sufficiently utilized in the long run. Otherwise if $x(t) < x_{ref}$ for $\forall t \in [t_0, \infty)$, we consider the node as being under-utilized and it is not necessary to assign (additional) capacity. Note that no regularity assumption is made on λ.

We will show that if $\lambda(t)$ satisfies Assumption 1 and if the control parameters are properly tuned, the trajectories of the closed-loop system converge to x_{ref} asymptotically under the control law (4). The system can also be tuned such that x is ultimately confined to an arbitrarily small neighborhood around x_{ref}. The trade-offs between different parameter settings will be revealed through the analysis and the remark that follows.
For notational convenience, set $\epsilon = b(1 + x_{ref})/C_{server}x_{ref} \leq 1$.

Theorem 1. Consider the closed-loop system composed of (1) and (4), where $\alpha > 0, \beta \geq 0$. Suppose $\lambda(t)$ satisfies Assumption 1. For all $x(t_0) \in [0, x_{buffer}]$ with $t_0 \geq 0$ being the initial time instant, if $\beta \geq \epsilon$, $x(t)$ converges to x_{ref} asymptotically. For any $0 \leq \beta < \epsilon$, if α is chosen large enough, the system achieves practical regulation in the sense that $x(t)$ satisfies (2) and $x(t)$ is ultimately confined within an arbitrarily small neighborhood of x_{ref}.

Proof. Under the "PE" condition in Assumption 1, there exists $t_0 < t_1 < \infty$ such that $x(t_1) \geq x_{ref}$ if $x(t_0) < x_{ref}$. $x(t) \geq x_{ref}, \forall t \geq t_1$ by observing (1) and (4). Due to such observations, we assume without loss of generality that $x(t_0) \geq x_{ref}$. This implies that $x(t) \geq x_{ref}, \forall t \geq t_0$. We analyze separately for the three cases when $\beta \geq 1$, $\epsilon \leq \beta < 1$ or when $0 \leq \beta < \epsilon$. In the first two cases, asymptotic regulation is achieved; In the third case, practical regulation is achieved if α is chosen to be sufficiently large.
Case 1. When $\beta \geq 1$, $\alpha\bar{x} + \beta sgn(\bar{x}) \geq 1$ and $C(x) = C_{server}$ for all $x > x_{ref}$. The controller in this case is of the "bang-bang" type. For all $x > x_{ref}$,

$$\dot{x}(t) < -\frac{x_{ref}}{1 + x_{ref}} C_{server} + b \leq 0.$$

It follows that $x(t) \to x_{ref}$ as $t \to \infty$.
Case 2. When $\epsilon \leq \beta < 1$[4], we show that $x(t)$ converges to x_{ref} asymptotically for all $\alpha > 0$ as follows. We consider the following two cases when $\alpha \in \left(\frac{1-\beta}{x_{buffer} - x_{ref}}, \infty\right)$ or when $\alpha \in \left(0, \frac{1-\beta}{x_{buffer} - x_{ref}}\right]$.
Case 2.a If $\alpha > \frac{1-\beta}{x_{buffer} - x_{ref}}$, it can be shown that, there exists finite $t^* \geq t_0$ such that $x(t^*) \in [x_{ref}, x^*]$ and $x(t) \in [x_{ref}, x^*], \forall t \geq t^*$. In fact, in this case, the following lemma holds, whose proof is given in the Appendix.

Lemma 1. Consider the closed-loop system composed of (1) and (4). Suppose Assumption 1 holds and $0 \leq \beta < 1$, $\alpha > \frac{1-\beta}{x_{buffer} - x_{ref}}$. For all $x(t_0) \in [0, x_{buffer}]$, $x(t)$ satisfies (2) for all $t \geq t_0$. Furthermore, there exists some finite $t^* \geq t_0$ such that $\forall t \geq t^*$, $x_{ref} \leq x(t) \leq x^*$, where $x^* = \frac{1-\beta}{\alpha} + x_{ref}$.

On $[x_{ref}, x^*]$, $C(x)$ is unsaturated, leading to

$$\dot{x}(t) = -\frac{x(t)}{1 + x(t)} C_{server} \left[\alpha\bar{x} + \beta sgn(\bar{x})\right] + \lambda(t).$$

Consider the function $V = \frac{1}{2}\bar{x}^2$. For all $t \geq t^*$,

$$\begin{aligned} \dot{V} &= \bar{x} \left(-\frac{x}{1+x} C_{server} \left[\alpha\bar{x} + \beta sgn(\bar{x})\right] + \lambda(t) \right) \qquad (6) \\ &\leq -\frac{x_{ref}}{1 + x_{ref}} C_{server} \alpha \bar{x}^2 - \left(\frac{x_{ref}}{1 + x_{ref}} C_{server} \beta - b \right) |\bar{x}|. \end{aligned}$$

Thus $\dot{V} \leq 0$. By Barbălat's Lemma (Khalil, 2002), we can conclude that $\bar{x}(t) \to 0$ as $t \to \infty$.
Case 2.b If $0 < \alpha \leq \frac{1-\beta}{x_{buffer} - x_{ref}}$, it holds that $\alpha\bar{x} + \beta sgn(\bar{x}) \leq 1$ for all $x \in [x_{ref}, x_{buffer}]$. $C(x)$ is unsaturated. By similar analysis as we did for Case 2.a, we can arrive at that $\lim_{t \to \infty} \bar{x}(t) \to 0$.
In Case 1 and Case 2, if it holds that $\epsilon < 1$ and $\beta > \epsilon$, $\bar{x}(t)$ converges to zero in finite time.
Case 3. If $0 \leq \beta < \epsilon$, for α large enough, $x(t)$ converges to an arbitrarily small neighborhood of x_{ref}. Indeed, for all $\alpha > \frac{1-\beta}{x_{buffer} - x_{ref}}$, by Lemma 1, $x(t)$ satisfies (2) and is ultimately confined to $\{x | x_{ref} \leq x \leq x^*\}$. The set can be made arbitrarily small if α is chosen to be large enough, i.e., $x^* \to x_{ref}$ as $\alpha \to \infty$. □

Remark 2. The controller is discontinuous for any $\beta > 0$. In the case when $\beta = 0$, the controller is continuous and practical regulation is achieved if α is sufficiently large.

3. LARGE-SCALE SYSTEM SYNTHESIS

In this section, we study a large-scale network composed of n interconnected nodes. For each node $i = 1, ..., n$, we use the following model to describe its dynamics.

$$\dot{x}_i = -\frac{x_i}{1 + x_i} \cdot C_i + \lambda_i(t, x_1, ..., x_n), \qquad (7)$$

$$x_i \in [0, x^{[i]}_{buffer}], \qquad (8)$$

$$C_i \in \left[0, C^{[i]}_{server}\right]. \qquad (9)$$

[4] In this case, $\epsilon < 1$.

For every ith subsystem, the variables have the same physical meanings as those of (1). In a large-scale network where the nodes are interconnected, the interfering traffic is affected by the activities of the interfering nodes, reflected in part by their queue lengths. We use a nonlinear function $\lambda_i : [0, \infty) \times \Re^n_+ \to \Re_+$ to denote the average incoming traffic rate from other interconnected nodes, which is considered as a disturbance input. Let x_i^{ref} denote a desired reference queue length for node i. It is assumed that the reference value $x_i^{ref}, \forall i \in [1, n]$, satisfy:

$$\underline{\epsilon} \le x_i^{ref} < x_{buffer}^{[i]}.$$

The lower bound $\underline{\epsilon} > 0$ could be an arbitrary positive value. Denote $\bar{x}_i \doteq x_i - x_i^{ref}$ the regulation error between the queue state and the reference value. Before addressing the asymptotic queue regulation, we first introduce assumptions and some interesting preliminary results.

Hypothesis and Preliminary Results.

Assumption 2. $\forall i = 1, ..., n$, λ_i satisfies the following Lipschitz-like condition [5]:

$$|\lambda_i(t, y_1, ..., y_n) - \lambda_i(t, y_1', ..., y_n')| \le \sum_{j=1, j\neq i}^{n} \gamma_{ij} sat\left\{\sigma_{ij}(|\tilde{y}_j|)\right\}, \tag{10}$$

$$\lambda_i(t, 0, ..., 0) = 0, \qquad \forall t \ge 0 \tag{11}$$

where $\tilde{y}_j = y_j - y_j'$, $\sigma_{ij}(\cdot) : \Re^+ \to \Re^+$ are continuously differentiable functions that satisfies $\sigma_{ij}(0) = 0, \forall i, j \in \{1, ..., n\}$. γ_{ij}s are positive constants that satisfy

$$\sum_{j=1, j\neq i}^{n} \gamma_{ij} < \frac{x_i^{ref}}{1 + x_i^{ref}} C_{server}^{[i]}. \tag{12}$$

Furthermore, for each i and for any fixed $t_0 \ge 0$, the following inequality holds where $X(t) = [x_1(t), ..., x_n(t)]^T$:

$$x_i^{ref} < \int_{t_0}^{\infty} \lambda_i(t, X(t))dt \le \infty. \tag{13}$$

Remark 3. The Assumption is motivated by physical characteristics of networks. The function "*sat*" is adopted here to highlight the impact of capacity constraints on the interfering traffic. Equation (11) means there are no traffic interferences for a network with empty queue.

Except for limited service capacity, other physical factors also affect the interference intensity among the nodes, such as the distance between nodes i, j, the power constraint (in wireless network) and the connectivity conditions. We use constant coefficients γ_{ij} to represent the impacts of these physical factors on the disturbance traffic from node j to node i.

We now propose the following control law, then introduce a lemma to discover interesting properties of the closed-loop system. For all $i = 1, ..., n$,

$$C_i(x_i) = C_{server}^{[i]} \cdot sat\left[\alpha_i \bar{x}_i + \beta_i sgn(\bar{x}_i)\right],$$
$$\text{where } \alpha_i > \frac{1 - \beta_i}{x_{buffer}^{[i]} - x_i^{ref}}, \ 0 \le \beta_i < 1.$$

Lemma 2. Consider the closed-loop large-scale system composed of (7) and (14). Suppose Assumption 2 holds.

For all $x_i(t_0) \in [0, x_{buffer}^{[i]}]$, $x_i(t)$ satisfies (8), $\forall i \in \{1, ..., n\}$. Furthermore, there exists some finite $T \ge t_0$ such that $\forall t \ge T$,

$$X(t) \in \Omega \doteq \left\{X \in \Re^n | x_i^{ref} \le x_i \le x_i^*, \forall i = 1, ..., n\right\},$$

where $x_i^* = \frac{1-\beta_i}{\alpha_i} + x_i^{ref} > x_i^{ref}$.

The proof of Lemma 2 is based on Lemma 1. It is omitted due to space limitation.

Main Result.

We now analyze the closed-loop system performance under the control law (14) with α_i and β_i satisfying (14). For notational conveniences, denote P the $n \times n$ matrix with elements $p_{ij}, i, j = 1, ..., n$,

$$p_{ii} = \frac{x_i^{ref}}{1 + x_i^{ref}} C_{server}^{[i]} \alpha_i, \ p_{ij} = -\frac{1}{2}\left(\bar{\gamma}_{ij} + \bar{\gamma}_{ji}\right),$$
$$\bar{\gamma}_{ij} = \max_{0 \le \bar{x}_j \le x_j^* - x_j^{ref}} \frac{\gamma_{ij} sat\{\sigma_{ij}(\bar{x}_j)\}}{\bar{x}_j}.$$

$\bar{\gamma}_{ij}$ is finite because of the definition of "*sat*" function and the property of function σ_{ij}. For all $i = 1, ..., n$, let

$$\upsilon_i \doteq \sum_{j \neq i}^{n} \gamma_{ij} sat\left\{\sigma_{ij}(x_j^{ref})\right\}, \quad \epsilon_i \doteq \frac{\upsilon_i(1 + x_i^{ref})}{C_{server}^{[i]} x_i^{ref}}.$$

Theorem 2. Consider closed-loop system composed of (7) and (14) where α_i, β_i satisfy (14) and α_is are chosen such that P is positive definite. Suppose that the interfering traffic λ_i satisfies Assumption 2. If $\beta_i \in [\epsilon_i, 1), \forall i \in \{1, ..., n\}$, the queue length x_i of every node converges to x_i^{ref} asymptotically for all $x_i(t_0) \in [0, x_{buffer}^{[i]}]$. If for some $i \in \{1, ..., n\}$, $\beta_i \in [0, \epsilon_i)$, the closed-loop system achieves practical regulation if α_is are chosen sufficiently large.

[5] Technically, the R.H.S. can be relaxed as $\sum_{j=1}^{n} \gamma_{ij} sat\left\{\sigma_{ij}(\cdot)\right\}$. From the physical meaning of this problem, we only consider the case for $j \neq i$, which means the upper bound of the interfering traffics from other nodes to node i is not dependent on the state of node i.

Proof. We first show that if all β_is satisfy $\beta_i \in [\epsilon_i, 1)$, the system achieves asymptotic regulation. By definition of υ_i,

$$\upsilon_i \leq \sum_{j=1,j\neq i}^{n} \gamma_{ij} < \frac{x_i^{ref}}{1+x_i^{ref}} C_{server}^{[i]}, i = 1, ..., n.$$

Thus by the definition of ϵ_i, there exists β_i such that $\epsilon_i \leq \beta_i < 1$ for every i. By the definition of P, α_is can be chosen such that $P > 0$. According to Lemma 2, the trajectories the closed-loop system satisfy (8), and after some finite $T \geq t_0$, are ultimately confined within Ω. We now analyze the system for all $t \geq T$. By applying (10) and (11), it holds

$$|\lambda_i(t, x_1, ..., x_n) - \lambda_i(t, x_1^{ref}, ..., x_n^{ref})| \qquad (14)$$
$$\leq \sum_{j=1,j\neq i}^{n} \gamma_{ij} sat\left\{\sigma(\bar{x}_j)\right\}.$$
$$\lambda_i(t, x_1^{ref}, ..., x_n^{ref}) \leq \upsilon_i. \qquad (15)$$

leading to

$$\lambda_i(t, x_1, ..., x_n) \leq \sum_{j=1,j\neq i}^{n} \gamma_{ij} sat\left\{\sigma_{ij}(\bar{x}_j)\right\} + \upsilon_i.$$

Since $x_i(t) \in [x_i^{ref}, x_i^*]$, $\alpha_i \bar{x}_i(t) + \beta_i sgn(\bar{x}_i(t)) \leq 1, \forall i = 1, ..., n$,

$$C_i(x_i(t)) = C_{server}^{[i]}\left[\alpha \bar{x}_i(t) + \beta_i sgn(\bar{x}_i(t))\right].$$

Consider the function $V = \frac{1}{2}\sum_{i=1}^{n} \bar{x}_i^2$. It holds, by differentiating V along the closed-loop system trajectory and using the definitions of P and $\bar{\gamma}_{ij}$,

$$\begin{aligned}
\dot{V} &= \sum_{i=1}^{n} \bar{x}_i \Big(-\frac{x_i}{1+x_i} C_i + \lambda_i \Big) \\
&\leq \sum_{i=1}^{n} \bar{x}_i \Big[-\frac{x_i^{ref}}{1+x_i^{ref}} C_{server}^{[i]} \left[\alpha_i \bar{x}_i + \beta_i sgn(\bar{x}_i)\right] \\
&\quad + \sum_{j=1,j\neq i}^{n} \gamma_{ij} sat\left\{\sigma_{ij}(\bar{x}_j)\right\} + \upsilon_i \Big] \\
&\leq -\sum_{i=1}^{n} \frac{x_i^{ref} C_{server}^{[i]}}{1+x_i^{ref}} \alpha_i \bar{x}_i^2 + \sum_{i=1}^{n} \bar{x}_i \sum_{j=1,j\neq i}^{n} \bar{\gamma}_{ij} \bar{x}_j \\
&\quad - \sum_{i=1}^{n} \left(\frac{x_i^{ref}}{1+x_i^{ref}} C_{server}^{[i]} \beta_i - \upsilon_i \right) |\bar{x}_i| \\
&\leq -\bar{X}^T P \bar{X} - \sum_{i=1}^{n} \left(\frac{x_i^{ref}}{1+x_i^{ref}} C_{server}^{[i]} \beta_i - \upsilon_i \right) |\bar{x}_i|,
\end{aligned}$$

where $\bar{X} = [\bar{x}_1, ..., \bar{x}_n]^T$.
Therefore, $\dot{V} \leq 0$. By Barbălat's Lemma (Khalil, 2002), we conclude that

$$\lim_{t\to\infty} \sum_{i=1}^{n} |\bar{x}_i(t)| = 0.$$

If for some $i \in \{1, ..., n\}$, $\beta_i \in [0, \epsilon_i)$, we show that practical regulation is achieved if α_is are sufficiently large. Indeed, by Lemma 2, $x_i(t)$ satisfies (8) $\forall t \geq t_0, \forall i \in \{1, ..., n\}$ and $X(t)$ is ultimately confined within Ω. By definition of x_i^*, if $\alpha_i \to \infty$, $x_i^* \to x_i^{ref}$. In other words, Ω can be made arbitrarily small if α_is are chosen to be sufficiently large.□

We now present our simulation results for both a single node system and a three-node interconnected system. We choose the following parameters for the single network node: $C_{server} = 5, x_{buffer} = 30, x_{ref} = 5, x(t_0) = 30$. The controller parameters are set as: $\alpha = 0.018, \beta = 0.62$. We use a sine wave bounded by $b = 2.6$ to represent the uncertain traffic disturbances. For the three-node interconnected system, the parameters of the three nodes and their respective controller parameters are shown in Table 1. The inter-node traffic λ_i is modelled by a sine waves. $\gamma_{12} = 2.04, \gamma_{13} = 2.42, \gamma_{21} = 2.34, \gamma_{23} = 2.65, \gamma_{31} = 1.4, \gamma_{32} = 2.8$. We take σ_{ij}s as $\tan^{-1}$ functions. As confirmed by Figure 1, the regulation errors of both systems converge to zero asymptotically.

Table 1. Simulation parameters for a three-node network

	node 1	node 2	node 3
$x_{buffer}^{[i]}$	35	28	36
$x_i(t_0)$	35	28	36
x_i^{ref}	10	8	7
$C_{server}^{[i]}$	7	8	8
α_i	0.05	0.062	0.04
β_i	0.17	0.16	0.25

4. CONCLUSIONS

Through theoretic analysis and simulations, we have shown that we achieve either asymptotic regulation or practical regulation of a class of networks against unknown traffics, for all feasible initial queue lengths, as opposed to the bounded regulation in (Pitsillides *et al.*, 2001). Explicit conditions are identified under which the constrained global regulation problem is solvable. We decompose the disturbances in large-scale networks into structured (vanishing) and unstructured (unvanishing) uncertainties to improve the performance. The control scheme is scalable and the system can accommodate newly appended nodes.

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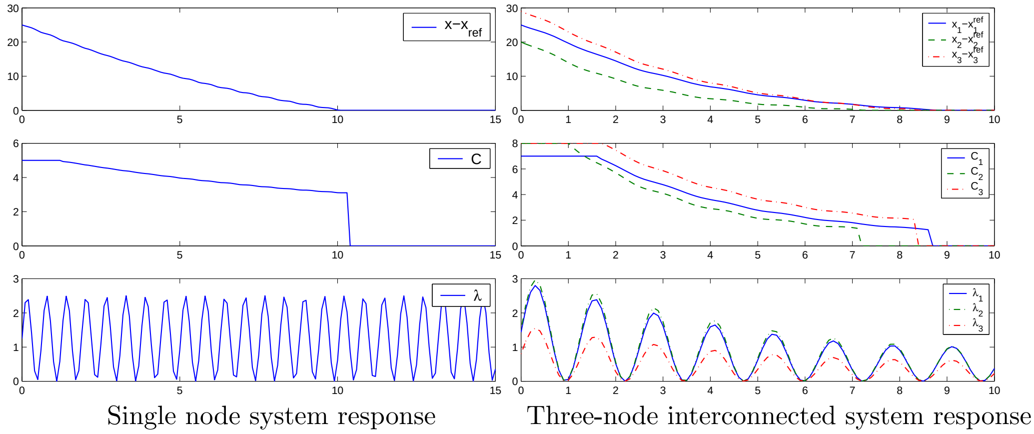

Fig. 1. Closed-loop performance of a single-node system and a three-node interconnected system

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5. APPENDIX

5.1 Proof of Lemma 1.

It can be directly checked that $x_{ref} < x^* < x_{buffer}$. We first analyze the case when $x(t_0) < x_{ref}$. When $x(t) < x_{ref}$, it holds $\dot{x}(t) = \lambda(t) \geq 0$. Thus for any $x(t_0) < x_{ref}$, under the "PE" condition in Assumption 1, there exists finite $t' > t_0$ such that $x_{ref} \leq x(t') \leq x^*$.

We then analyze the case when $x(t_0) > x^*$. For all $x(t) \in (x^*, x_{buffer}]$, $\alpha\bar{x}(t) + \beta sgn(\bar{x}(t)) > 1$, thus $C(x(t)) = C_{server}$. It holds, by applying Assumption 1,

$$\dot{x}(t) < -\frac{x_i^*}{1 + x_i^*} C_{server} + b < 0.$$

It follows that for any $x(t_0) > x^*$, there exists some finite $t^{''} > t_0$ such that $x_{ref} \leq x(t^{''}) \leq x^*$. The above analysis leads to that for all $x(t_0) \in [0, x_{buffer}]$, there must exists finite $t^* \geq t_0$ such that $x_{ref} \leq x(t^*) \leq x^*$. We now prove by contradiction that $x_{ref} \leq x(t) \leq x^*, \forall t \geq t^*$. In fact, suppose $x(t) > x^*$ or $x(t) < x_{ref}$ for some $t > t^*$, there must exists some $t_1 \geq t^*$ such that either one of the following two statements is true:

S1. $x(t_1) = x^*$ and $x(t_1 + \tau) > x^*$ for some $\tau > 0$,

S2. $x(t_1) = x_{ref}$ and $x(t_1 + \tau) < x_{ref}$ for some $\tau > 0$.

When $x = x^*$, $\alpha\bar{x} + \beta sgn(\bar{x}) = 1$, thus $C(x^*) = C_{server}$. It holds:

$$\dot{x} < -\frac{x_{ref}}{1 + x_{ref}} C_{server} + b \leq 0.$$

This contradicts S1. When $x = x_{ref}$, $C(x_{ref}) = 0$. Thus $\dot{x} = \lambda \geq 0$, which contradicts S2. We can conclude that $x_{ref} \leq x(t) \leq x^*, \forall t \geq t^*$. In other words, $\{x | x_{ref} \leq x \leq x^*\}$ is an attractive invariant set. The above analysis has revealed that $x(t)$ satisfies (2) for all $t \geq t_0$.