ROBUST FDF FOR LINEAR UNCERTAIN SYSTEMS OF THE POLYTOPIC TYPE

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Abstract: In this paper, the robust fault detection filter (RFDF) design problem for linear discrete-time systems with both unknown inputs and polytopic type uncertainties is studied. The main contributions include the H_∞-filtering formulation of RFDF design problem, the extension of an H_∞-filtering approach to the polytopic type RFDF problem, the derivation of sufficient conditions in terms of linear matrix inequalities (LMIs), and the parameterization of parameter-independent RFDF solutions. A numerical example is given to illustrate the effectiveness of the proposed method.

Keywords: Fault detection, filter, linear matrix inequality, polytopic type uncertainty

1. INTRODUCTION

In this paper, we consider the robust fault detection filter (RFDF) problem for a kind of uncertain linear discrete time systems described by

$$x(k+1) = Ax(k) + Bu(k) + B_d d(k) + B_f f(k) \quad (1)$$

$$y(k) = Cx(k) + Du(k) + D_d d(k) + D_f f(k) \quad (2)$$

for $k = 0, 1, 2, \ldots$, where $x(k) \in \mathbb{R}^n$ is the state, $u(k) \in \mathbb{R}^p$ the control input, $y(k) \in \mathbb{R}^q$ the measurement output, $d(k) \in \mathbb{R}^m$ the unknown input, $f(k) \in \mathbb{R}^l$ the fault to be detected. It is assumed that $d(k)$ and $f(k)$ are l_2-norm bounded. C, D, B_d, B_f, D_d, D_f are known matrices with appropriate dimensions. A and B are matrices with uncertainties and obey the real convex polytopic model $\left[\, A \;\, B \,\right] \in \Omega$ which is denoted by

$$\Omega \triangleq \left\{ \left[\, A \;\, B \,\right] = \sum_{i=1}^{N} \zeta_i \left[\, A_i \;\, B_i \,\right], \quad \sum_{i=1}^{N} \zeta_i = 1, \quad \zeta_i > 0 \right\}$$

for $i = 1, 2, \cdots, N$, the A_i and B_i are constant matrices with appropriate dimensions, ζ_i denote time-invariant uncertainties.

Robustness of an FDI system involves two aspects: robustness to modelling errors, disturbances and sensitivity to faults (Chen and Patton, 1999; Frank *et al.*, 2000; Gertler, 1998; Kinnaert, 2003; Mangoubi and Edelmayer, 2000). It is often the nature of industrial systems that the effects of the possible faults and disturbances are coupled and that modelling error are unavoidable. The performance of an FDI system should therefore be measured by a suitable trade-off between the robustness and sensitivity, see for instance the parity space method , eigenstructure assignment, the H_∞/H_∞ or H_∞/H_- based optimization, the linear matrix inequality (LMI) approach, and the H_∞-filtering formulation of robust FDI (Chen and Patton, 2000; Ding *et al.*, 2000; Ding *et al.*, 2001; Gertler and DiPierro, 1997; Patton and Chen, 2000; Zhong *et al.*, 2001).

In this paper, a parameter-independent RFDF is developed for systems described by (1)–(2). The design problem of RFDF is formulated in the sense of H_∞-filtering formulation and, based on this, a sufficient condition for the solvability of this problem is derived in terms of LMIs by applying H_∞-filtering techniques in (Geromel *et al.*, 2000). A simulation example is given to illustrate the effectiveness of the proposed method.

Notations. Throughout this paper, the superscript T denotes matrix transposition, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. $\mathbb{R}^{n\times m}$ is the set of all $n\times m$ real matrices. For a real symmetric matrix M, we use $M>0$ (<0) to denote its positive (negative) definiteness. All matrices, if their dimensions are not explicitly stated, are assumed to be compatible. I denotes identity matrix with approximate dimensions. l_2 denotes the space of square summable sequences. For $w\in l_2$, $\|w\|_2$ denotes the l_2-norm.

2. PROBLEM FORMULATION

In this paper, the following RFDF is considered:

$$\eta(k+1) = A_\eta\eta(k) + M_1u(k) + B_\eta y(k) \quad (3)$$
$$r(k) = C_\eta\eta(k) + M_2u(k) + D_\eta y(k) \quad (4)$$

where $\eta(k)\in\mathbb{R}^n$ is a vector, $r(k)\in\mathbb{R}^l$ denotes the generated residual. $A_\eta\in\mathbb{R}^{n\times n}$, $B_\eta\in\mathbb{R}^{n\times q}$, $C_\eta\in\mathbb{R}^{l\times n}$, $D_\eta\in\mathbb{R}^{l\times q}$, $M_1\in\mathbb{R}^{n\times p}$, $M_2\in\mathbb{R}^{l\times p}$ are parameter matrices to be determined. The main attention is paid on finding suitable matrices $A_\eta, B_\eta, C_\eta, D_\eta, M_1$ and M_2 such that system (3)–(4) is asymptotically stable and, under zero initial condition, make $\gamma>0$ small in the feasibility of

$$\sup_{A,B\in\Omega,\|w\|_2\neq 0}\frac{\left\|r-\hat{f}\right\|_2}{\|w\|_2} < \gamma \quad (5)$$

where $w(k) = \left[\, u^T(k)\ d^T(k)\ f^T(k)\,\right]^T$, $\hat{f}(z) = W_f(z)f(z)$, $W_f(z)$ is a given weighting matrix which is used to limit the frequency range of interested fault.

Suppose the state space realization of $\hat{f}(z) = W_f(z)f(z)$ is

$$x_f(k+1) = A_{Wf}\ x_f(k) + B_{Wf}\ f(k) \quad (6)$$
$$\hat{f}(k) = C_{Wf}\ x_f(k) + D_{Wf}\ f(k) \quad (7)$$
$$x_f(0) = 0 \quad (8)$$

Denote $r_e(k) = r(k)-\hat{f}(k)$. From (1)–(4) and (6)–(8), the overall dynamics of RFDF are obtained as following:

$$x(k+1) = Ax(k) + Bu(k) + B_dd(k) + B_ff(k) \quad (9)$$
$$\eta(k+1) = B_\eta Cx(k) + A_\eta\eta(k) + (M_1+B_\eta D)u(k) + B_\eta D_dd(k) + B_\eta D_ff(k) \quad (10)$$
$$x_f(k+1) = A_{Wf}\ x_f(k) + B_{Wf}\ f(k) \quad (11)$$
$$r_e(k) = D_\eta Cx(k) - C_{Wf}\ x_f(k) + C_\eta\eta(k) + (M_2+D_\eta D)u(k) + D_\eta D_dd(k) + (D_\eta D_f - D_{Wf})f(k) \quad (12)$$

Thus, performance index (5) is equivalent to

$$\sup_{A,B\in\Omega}\|G_{r_ew}(z)\|_\infty < \gamma \quad (13)$$

where

$$G_{r_ew}(z) = \left[\, D_\eta C\ \ C_\eta\ \ -C_{Wf}\,\right] \times\left(zI - \begin{bmatrix} A & 0 & 0\\ B_\eta C & A_\eta & 0\\ 0 & 0 & A_{Wf}\end{bmatrix}\right)^{-1} \times\begin{bmatrix} B & B_d & B_f\\ M_1+B_\eta D & B_\eta D_d & B_\eta D_f\\ 0 & 0 & B_{Wf}\end{bmatrix} + \left[\,(M_2+D_\eta D)\ \ D_\eta D_d\ \ (D_\eta D_f - D_{Wf})\,\right]$$

For the sake of simplicity, we further rewrite (9)–(12) into the following augmented system:

$$\xi(k+1) = \tilde{A}\xi(k) + \tilde{B}_ww(k) \quad (14)$$
$$r_e(k) = \tilde{C}\xi(k) + \hat{D}_ww(k) \quad (15)$$

where

$$\xi = \left[\, x^T\ \eta^T\ x_f^T\,\right]^T,\quad w = \left[\, u^T\ d^T\ f^T\,\right]^T \quad (16)$$
$$\tilde{A} = \begin{bmatrix} A & 0 & 0\\ B_\eta C & A_\eta & 0\\ 0 & 0 & A_{Wf}\end{bmatrix} \quad (17)$$
$$\tilde{B}_w = \begin{bmatrix} B & B_d & B_f\\ M_1+B_\eta D & B_\eta D_d & B_\eta D_f\\ 0 & 0 & B_{Wf}\end{bmatrix} \quad (18)$$
$$\tilde{C} = \left[\, D_\eta C\ \ C_\eta\ \ -C_{Wf}\,\right] \quad (19)$$
$$\tilde{D}_w = \left[\, M_2+D_\eta D\ \ D_\eta D_d\ \ D_\eta D_f - D_{Wf}\,\right] \quad (20)$$

Now the RFDF problem can be re-formulated as to find A_η, B_η, C_η, D_η, M_1 and M_2 such that, for all $\left[\, A\ B\,\right]\in\Omega$, system (14)–(15) is asymptotically stable and (13) is satisfied.

Remark 1. The background of our study is the recent development in the field of continuous-time system H_∞-filtering formulation of fault detection

(Chen and Patton, 2000; Niemann and Stoustrup, 2001; Zhong *et al.*, 2003). The introducing of weighting matrix $W_f(z)$ is used to limit the frequency interval, in which the fault should be identified. While the fault f is not necessary to be exactly known.

3. MAIN RESULTS

The following lemmas are required to derive the main results of this paper.

Lemma 1. (de Oliveira et al.*, 2002)*Consider LTI system

$$\begin{aligned} x(k+1) &= Ax(k) + Bd(k) \\ y(k) &= Cx(k) + Dd(k) \\ x(0) &= x_0 \end{aligned}$$

where $x(k)$, $y(k)$, $d(k)$ are defined as in (1)–(2). A, B, C and D are known matrices with appropriate dimensions. For given $\gamma > 0$, the system is asymptotically stable and satisfies $\|G_{yd}(z)\|_\infty < \gamma$, if and only if there exists matrix $P > 0$ satisfying LMI

$$\begin{bmatrix} P & AP & B & 0 \\ PA^T & P & 0 & PC^T \\ B^T & 0 & \gamma^2 I & D^T \\ 0 & CP & D & I \end{bmatrix} > 0 \tag{21}$$

Lemma 2. (de Oliveira et al.*, 2002)* LMI (21) is feasible, if and only if there exist matrices $P > 0$ and $\mathcal{G}$ such that the LMI

$$\begin{bmatrix} P & A\mathcal{G} & B & 0 \\ \mathcal{G}A^T & \mathcal{G}+\mathcal{G}^T-P & 0 & \mathcal{G}^T C^T \\ B^T & 0 & \gamma^2 I & D^T \\ 0 & C\mathcal{G} & D & I \end{bmatrix} > 0$$

is satisfied.

We are now in the position to state the main result of this paper.

Theorem 3. For given $\gamma > 0$, the RFDF problem is solvable, if there exist matrices P_i, J_i, H_i, Z, Y, F, R, L, Q, S and $P_f > 0$ such that LMIs

$$\begin{bmatrix} P_i & J_i & Z^T A_i & Z^T A_i & Z^T B_{wi} & 0 & 0 & 0 \\ J_i^T & H_i & \phi_{23,i} & \phi_{24,i} & \phi_{25,i} & 0 & 0 & 0 \\ A_i^T Z & \phi_{23,i}^T & \phi_{33,i} & \phi_{34,i} & 0 & \phi_{36,i} & 0 & 0 \\ A_i^T Z & \phi_{24,i}^T & \phi_{34,i}^T & \phi_{44,i} & 0 & \hat{C}_0^T R^T & 0 & 0 \\ B_{wi}^T Z & \phi_{25,i}^T & 0 & 0 & \gamma^2 I & \phi_{56,i} & \hat{B}_{Wf}^T & 0 \\ 0 & 0 & \phi_{36,i}^T & R\hat{C}_0 & \phi_{56,i}^T & I & 0 & -C_{Wf}P_f \\ 0 & 0 & 0 & 0 & \hat{B}_{Wf} & 0 & P_f & A_{Wf}P_f \\ 0 & 0 & 0 & 0 & 0 & -P_f C_{Wf}^T & P_f A_{Wf}^T & P_f \end{bmatrix} > 0 \tag{22}$$

$$\begin{bmatrix} P_i & J_i \\ J_i^T & H_i \end{bmatrix} > 0 \tag{23}$$

are feasible for $i = 1, 2, \ldots, N$, where

$$\begin{aligned} \phi_{23,i} &= Q + Y^T A_i + F\hat{C}_0, \quad \phi_{24,i} = Y^T A_i + F\hat{C}_0 \\ \phi_{25,i} &= Y^T B_{w,i} + F D_{yw}, \quad \phi_{33,i} = Z + Z^T - P_i \\ \phi_{34,i} &= Z^T + Y + S^T - J_i, \quad \phi_{36,i} = L^T + \hat{C}_0^T R^T \\ \phi_{44,i} &= Y + Y^T - H_i, \quad \phi_{56,i} = D_{\eta w}^T + D_{yw}^T R^T \end{aligned}$$

$$B_{wi} = \begin{bmatrix} B_i & B_d & B_f \end{bmatrix}, \quad \hat{B}_{Wf} = \begin{bmatrix} 0 & 0 & B_{Wf} \end{bmatrix} \tag{24}$$

$$\hat{C}_0 = \begin{bmatrix} C \\ 0 \end{bmatrix}, \quad D_{yw} = \begin{bmatrix} D & D_d & D_f \\ I & 0 & 0 \end{bmatrix} \tag{25}$$

$$D_{\eta w} = \begin{bmatrix} 0 & 0 & -D_{Wf} \end{bmatrix} \tag{26}$$

In this case, the parameter matrices of RFDF are given by

$$A_\eta = \left(V^T\right)^{-1} Q S^{-1} V^T \tag{27}$$

$$\begin{bmatrix} B_\eta & M_1 \end{bmatrix} = \left(V^T\right)^{-1} F \tag{28}$$

$$C_\eta = L S^{-1} V^T, \quad \begin{bmatrix} D_\eta & M_2 \end{bmatrix} = R \tag{29}$$

where $V \in \mathbb{R}^{n \times n}$ is arbitrary inverse matrix.

Proof. Given scalar $\gamma > 0$, from Lemma 1 it is known that system (14)–(15) is asymptotically stable and the H_∞ norm constraint (13) is satisfied for all $\begin{bmatrix} A & B \end{bmatrix} \in \Omega$, if there exists matrix $\tilde{P} = \begin{bmatrix} \mathcal{P} & 0 \\ 0 & P_f \end{bmatrix} > 0$ such that LMI

$$\begin{bmatrix} \tilde{P} & \tilde{A}\tilde{P} & \tilde{B}_w & 0 \\ \tilde{P}\tilde{A}^T & \tilde{P} & 0 & \tilde{P}\tilde{C}^T \\ \tilde{B}_w^T & 0 & \gamma^2 I & \tilde{D}_w^T \\ 0 & \tilde{C}\tilde{P} & \tilde{D}_w & I \end{bmatrix} > 0 \tag{30}$$

holds, where $\tilde{A}, \tilde{B}_w, \tilde{C}, \tilde{D}_w$ are defined in (16)–(20). Moreover, LMI (30) can be re-written into

$$\begin{bmatrix} \mathcal{P} & \hat{A}\mathcal{P} & \hat{B}_w & 0 \\ \mathcal{P}\hat{A}^T & \mathcal{P} & 0 & \mathcal{P}\hat{C}^T \\ \hat{B}_w^T & 0 & \gamma^2 I & \tilde{D}_w^T \\ 0 & \hat{C}\mathcal{P} & \tilde{D}_w & I \\ 0 & 0 & \hat{B}_{Wf} & 0 \\ 0 & 0 & 0 & -P_f C_{Wf}^T \end{bmatrix}$$

$$\left.\begin{matrix} 0 & 0 \\ 0 & 0 \\ \hat{B}_{Wf}^T & 0 \\ 0 & -C_{Wf}P_f \\ P_f & A_{Wf}P_f \\ P_f A_{Wf}^T & P_f \end{matrix}\right] > 0 \tag{31}$$

where $\hat{B}_{Wf}$ is defined in (24), and

$$\hat{A} = \begin{bmatrix} A & 0 \\ B_\eta C & A_\eta \end{bmatrix}, \quad \hat{C} = \begin{bmatrix} D_\eta C & C_\eta \end{bmatrix}$$

$$\hat{B}_w = \begin{bmatrix} B & B_d & B_f \\ M_1 + B_\eta D & B_\eta D_d & B_\eta D_f \end{bmatrix}$$

From Lemma 2, LMI (31) is feasible if and only if there exist matrices $\mathcal{P} > 0$, $P_f > 0$ and $\mathcal{G}$ such that

$$\begin{bmatrix} \mathcal{P} & \hat{A}\mathcal{G} & \hat{B}_w & 0 & 0 & 0 \\ \mathcal{G}\hat{A}^T & \mathcal{G}+\mathcal{G}^T-\mathcal{P} & 0 & \mathcal{G}^T\hat{C}^T & 0 & 0 \\ \hat{B}_w^T & 0 & \gamma^2 I & \tilde{D}_w^T & \hat{B}_{Wf}^T & 0 \\ 0 & \hat{C}\mathcal{G} & \tilde{D}_w & I & 0 & -C_{Wf}P_f \\ 0 & 0 & \hat{B}_{Wf} & 0 & P_f & A_{Wf}P_f \\ 0 & 0 & 0 & -P_f C_{Wf}^T & P_f A_{Wf}^T & P_f \end{bmatrix} > 0 \tag{32}$$

holds. Inspired by (Geromel *et al.*, 2000), we define non-singular matrices U, V and $S = V^T U Z$, introduce non-singular matrices

$$\mathcal{G} = \begin{bmatrix} Z^{-1} & * \\ U & * \end{bmatrix}, \quad \mathcal{G}^{-1} = \begin{bmatrix} Y & * \\ V & * \end{bmatrix} \tag{33}$$

$$\mathcal{T} = \begin{bmatrix} Z & Y \\ 0 & V \end{bmatrix} \tag{34}$$

and non-linear transformation

$$\begin{bmatrix} Q & F \\ L & R \end{bmatrix} = \begin{bmatrix} V^T & 0 \\ 0 & I \end{bmatrix} \times \begin{bmatrix} A_\eta & \begin{bmatrix} B_\eta & M_1 \end{bmatrix} \\ C_\eta & \begin{bmatrix} D_\eta & M_2 \end{bmatrix} \end{bmatrix} \begin{bmatrix} UZ & 0 \\ 0 & I \end{bmatrix} \tag{35}$$

$$\begin{bmatrix} P & J \\ J^T & H \end{bmatrix} = \mathcal{T}^T \mathcal{P} \mathcal{T} \tag{36}$$

It is obtained that

$$\mathcal{T}^T \hat{A}\mathcal{G}\mathcal{T} = \begin{bmatrix} Z^T A & Z^T A \\ Y^T A + F\hat{C}_0 + Q & Y^T A + F\hat{C}_0 \end{bmatrix} \tag{37}$$

$$\mathcal{T}^T \hat{B}_w = \begin{bmatrix} Z^T B_w \\ Y^T B_w + F D_{yw} \end{bmatrix} \tag{38}$$

$$\hat{C}\mathcal{G}\mathcal{T} = \begin{bmatrix} R\hat{C}_0 + L & R\hat{C}_0 \end{bmatrix} \tag{39}$$

$$\tilde{D}_w = D_{\eta w} + R D_{yw} \tag{40}$$

$$\mathcal{T}^T(\mathcal{G}+\mathcal{G}^T-\mathcal{P})\mathcal{T} = -\begin{bmatrix} P & J \\ J^T & H \end{bmatrix} + \begin{bmatrix} Z + Z^T & Z^T + Y + S^T \\ Z + Y^T + S & Y + Y^T \end{bmatrix} \tag{41}$$

where $\hat{C}_0$, $D_{\eta w}$, D_{yw} are defined in (24)–(26). It follows from (32)–(41) that

$$\Sigma^T \Xi \Sigma = \Phi$$

where

$$\Sigma = \mathrm{diag}[\mathcal{T}, \mathcal{T}, I, I, I, I]$$

$$\Xi = \begin{bmatrix} \mathcal{P} & \hat{A}\mathcal{G} & \hat{B}_w & 0 & 0 & 0 \\ \mathcal{G}\hat{A}^T & \mathcal{G}+\mathcal{G}^T-\mathcal{P} & 0 & \mathcal{G}^T\hat{C}^T & 0 & 0 \\ \hat{B}_w^T & 0 & \gamma^2 I & \tilde{D}_w^T & \hat{B}_{Wf}^T & 0 \\ 0 & \hat{C}\mathcal{G} & \tilde{D}_w & I & 0 & -C_{Wf}P_f \\ 0 & 0 & \hat{B}_{Wf} & 0 & P_f & A_{Wf}P_f \\ 0 & 0 & 0 & -P_f C_{Wf}^T & P_f A_{Wf}^T & P_f \end{bmatrix}$$

$$\Phi = \begin{bmatrix} P & J & Z^T A & Z^T A & Z^T B_w & 0 & 0 & 0 \\ J^T & H & \phi_{23} & \phi_{24} & \phi_{25} & 0 & 0 & 0 \\ A^T Z & \phi_{23}^T & \phi_{33} & \phi_{34} & 0 & \phi_{36} & 0 & 0 \\ A^T Z & \phi_{24}^T & \phi_{34}^T & \phi_{44} & 0 & \hat{C}_0^T R^T & 0 & 0 \\ B_w^T Z & \phi_{25}^T & 0 & 0 & \gamma^2 I & \phi_{56} & \hat{B}_{Wf}^T & 0 \\ 0 & 0 & \phi_{36}^T & R\hat{C}_0 & \phi_{56}^T & I & 0 & -C_{Wf}P_f \\ 0 & 0 & 0 & 0 & \hat{B}_{Wf} & 0 & P_f & A_{Wf}P_f \\ 0 & 0 & 0 & 0 & 0 & -P_f C_{Wf}^T & P_f A_{Wf}^T & P_f \end{bmatrix}$$

$$\phi_{23} = Q + Y^T A + F\hat{C}_0, \quad \phi_{24} = Y^T A + F\hat{C}_0$$

$$\phi_{25} = Y^T B_w + F D_{yw}, \quad \phi_{33} = Z + Z^T - P$$

$$\phi_{34} = Z^T + Y + S^T - J, \quad \phi_{36} = L^T + \hat{C}_0^T R^T$$

$$\phi_{44} = Y + Y^T - H, \quad \phi_{56} = D_{\eta w}^T + D_{yw}^T R^T$$

Obviously, if there exist matrices P_i, J_i, H_i , Z, Y, F, R, L, Q, S and $P_f > 0$ such that LMIs (22)–(23) are feasible, then $\Phi > 0$ holds true for all $\begin{bmatrix} A & B \end{bmatrix} \in \Omega$ and

$$P = \sum_{i=1}^{N} (\zeta_i P_i), \quad J = \sum_{i=1}^{N} (\zeta_i J_i)$$

$$H = \sum_{i=1}^{N} (\zeta_i H_i), \quad \sum_{i=1}^{N} \zeta_i = 1$$

which implies that LMI (32) and further (30) are feasible also. Moreover, for any inverse matrix $V \in \mathbb{R}^{n\times n}$, the parameter matrices of RFDF can be derived as in (27)–(29) from (35). □

Remark 2. Given $\gamma > 0$, Theorem 3 gives a sufficient condition for the existence of polytopic type uncertain linear discrete-time system RFDF in terms of LMIs. In order to achieve an RFDF with γ made as small as possible in terms of the feasibility of (13), a repeated application of Theorem 3 is also required.

4. NUMERICAL EXAMPLE

To illustrate the proposed method, following linear discrete-time polytopic type unknown systems are considered

$$\begin{aligned} x(k+1) &= \sum_{i=1}^{3} \zeta_i A_i x(k) + \sum_{i=1}^{3} \zeta_i B_i u(k) \\ &\quad + B_f f(k) + B_d d(k) \\ y(k) &= Cx(k) + D_d d(k) \end{aligned}$$

where

$$A_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & -0.01 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -0.1 & 0 \\ 0.1 & -0.2 \end{bmatrix}$$
$$A_3 = \begin{bmatrix} 0.5 & 0.1 \\ 0 & 0.015 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.2 \\ 0.5 \end{bmatrix}$$
$$B_2 = \begin{bmatrix} 0.1 \\ 0.8 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix}$$
$$B_d = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \quad B_f = \begin{bmatrix} 0.5 \\ 0.3 \end{bmatrix}$$
$$C = \begin{bmatrix} 1 & 1 \end{bmatrix}, \quad D_d = 0.1, \quad \sum_{i=1}^{3} \zeta_i = 1$$

The weighting matrix is supposed to be $W_f(z) = \frac{0.5z}{z-0.5}$, with state space realization

$$\begin{aligned} x_f(k+1) &= 0.5x_f(k) + 0.25f \\ \hat{f}(k) &= x_f(k) + 0.5f \end{aligned}$$

The obtained results are:

$$\gamma = 1.01, \quad A_\eta = \begin{bmatrix} 0.4072 & 0.2094 \\ -0.4540 & -0.2369 \end{bmatrix}$$
$$B_\eta = \begin{bmatrix} -0.0154 \\ 0.0166 \end{bmatrix}, \quad M_1 = \begin{bmatrix} 0.0226 \\ -0.0271 \end{bmatrix}$$
$$C_\eta = \begin{bmatrix} -0.0164 & -0.0037 \end{bmatrix}$$
$$D_\eta = 0.0009, \quad M_2 = 0.0057$$

For $k = 0, 1, \ldots, 100$, suppose the control input is unit step signal, unknown input is white noise with power 0.05, the fault is set up 1 over $k \in [10, 30]$ and $k \in [60, 80]$ (and is zero otherwise). When $\zeta_1 = 0.3$, $\zeta_2 = 0.3$ and $\zeta_3 = 0.4$, Figure 1 to Figure 3 show the time response of the residual of case1 to case 3 respectively.

- case 1: $u(k) = 0,\ f(k) = 0$;
  case 2: $u(k) = 0,\ f(k) \neq 0$;
  case 3: $u(k) \neq 0,\ f(k) \neq 0$.

The simulation results show that the residual is robust to unknown input, sensitive to fault, while the influence of control input remains large. Under the assumption of control input being online known, the appeared fault can be detected efficiently.

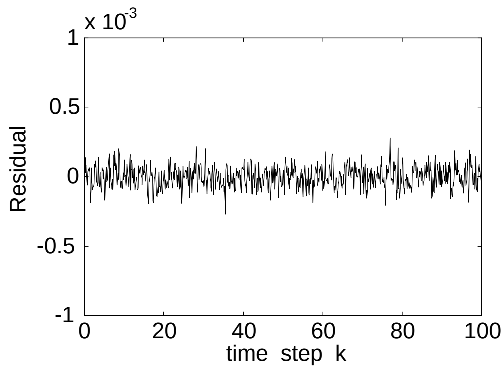

Figure 1. Case 1: $u = 0,\ f = 0$

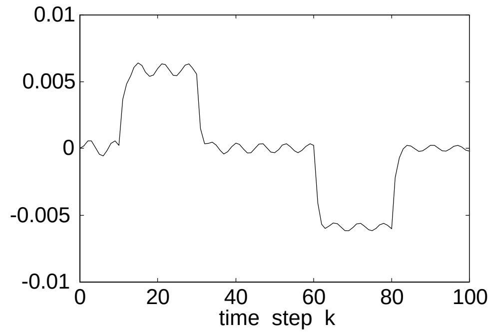

Figure 2. Case 2: $u = 0,\ f \neq 0$

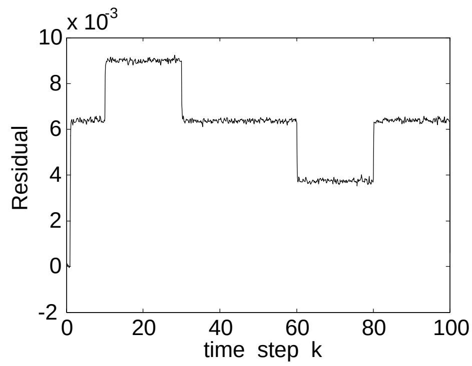

Figure 3. Case 3: $u \neq 0,\ f \neq 0$

5. CONCLUSION

Using a general parameter-independent RFDF as residual generator, the RFDF problem for linear discrete-time systems with polytopic type uncertainties has been formulated as an H_∞-filtering problem. A sufficient condition for the solvability of RFDF has been established in terms of LMIs. The final results of RFDF have been obtained by solving a set of LMIs, in which a free parameter matrix V is included. A simulation example has been given to demonstrated the effectiveness of the proposed method.

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REFERENCES

Chen, J. and R. J. Patton (1999). *Robust Model-Based Fault Diagnosis for Dynamic Systems*. Kluwer Academic Publishers. Boston.

Chen, J. and R. J. Patton (2000). Standard H_∞ filtering formulation of robust fault detection. In: *Proc. SAFEPROCESS'2000*. Budapest, Hungary. pp. 256–261.

de Oliveira, M. C., J. C. Geromel and J. Bernussou (2002). Extend H_2 and H_∞ norm characterizations and controller parameterizations for discrete-time systems. *Int. J. Contr.* **75**(9), 666–679.

Ding, S. X., M.-Y. Zhong and B. Y. Tang (2001). An LMI approach to the design of fault detection filter for time-delay LTI systems with unknown inputs. In: *Proc. Amer. Contr. Conf.*. Arlington, VA. pp. 2137–2142.

Ding, S. X., T. Jeinsch, P. M. Frank and E. L. Ding (2000). A unified approach to the optimization of fault detection systems. *Int. J. Adaptive Contr. Signal Processing* **14**(7), 725–745.

Frank, P. M., S. X. Ding and B. Koppen-Seliger (2000). Current developments in the theory of FDI. In: *Proc. SAFEPROCESS'2000*. Budapest, Hungary. pp. 16–27.

Geromel, José C., Jacques Bernussou, Germain Garcia and Maurício C. de Oliveira (2000). H_2 and H_∞ robust filtering for discrete-time linear systems. *SIAM J. Contr. Optim.* **38**(5), 1353–1368 (electronic).

Gertler, J. (1998). *Fault Detection and Diagnosis in Engineering Systems*. Marcel Dekker. USA.

Gertler, J. J. and G. DiPierro (1997). On the relationship between parity relations and parameter estimation. In: *Proc. SAFEPROCESS'1997*. Kingston Upon Hull, UK. pp. 468–473.

Kinnaert, Michel (2003). Fault diagnosis based on analytical models for linear and nonlinear systems-a tutorial. In: *Proc. SAFEPROCESS'2003*. Washington D. C., USA. pp. 37–50.

Mangoubi, R. S. and A. M. Edelmayer (2000). Model based fault detection: the optimal past, the robust present and a few thoughts on the future. In: *Proc. SAFEPROCESS'2000*. Budapest, Hungary. pp. 64–75.

Niemann, H. and J. Stoustrup (2001). Fault diagnosis for non-minimum phase systems using H_∞ optimization. In: *Proc. Amer. Contr. Conf.*. Arlington. pp. 4432–4436.

Patton, R. J. and J. Chen (2000). On eigenstructure assignment for robust fault diagnosis. *Int. J. Robust Nonlinear Contr.* **10**(14), 1193–1208.

Zhong, M., S. X. Ding and B. Tang (2001). An LMI approach to robust fault detection filter design for discrete-time systems with model uncertainty. In: *Proc. 40th Conf. Decis. Contr.*. Orlando, Florida USA. pp. 3613–3618.

Zhong, M., S. X. Ding, J. Lam and H Wang (2003). LMI approach to design robust fault detection filter for uncertain LTI systems. *Automatica* **39**(3), 543–550.