# AN ALGORITHM TO REDUCE THE TRACKING ERROR IN TS FUZZY MODELS: A NUMERICAL APPROACH [1]

**Jesus A. Meda-Campaña** *
**Bernardino Castillo-Toledo** *

* *Centro de Investigación y de Estudios Avanzados del IPN Unidad Guadalajara, México*

Abstract: We address the problem of tracking references generated by an exosystem when the plant is described by a Takagi-Sugeno (TS) fuzzy model. We propose the inclusion of a discontinuous term into the control law to improve the performance of the controller, and in those terms, we give a numerical algorithm based on the use of computer-aided design (CAD) tools, in order to reduce systematically the tracking error. 

Keywords: Fuzzy control, TS fuzzy models, Regulation theory, Discontinuous control, LMI techniques, CAD tools.

## 1. INTRODUCTION

As we all know, the tracking of reference signals, at least asymptotically, is a very important matter in system theory. In literature, we can find diverse approaches to perform this task. However, regulation theory provides a complete set of tools to accomplish this goal.

The regulator problem for a system affected by perturbation and reference signals, consists in finding a state or error feedback controller such that the equilibrium point of the closed-loop system with no external signals is asymptotically stable, and the tracking error goes to zero when the system is under the influence of the exosystem. In (Francis, 1977), the design of the linear regulator was given in terms of certain matrix equations (Francis equation), whose solution depends on the property of the exosystem signals to be observable for the system output. Isidori and Byrnes have shown that the nonlinear problem is solvable by means of some partial differential equations, named henceforth, Francis-Isidori-Byrnes (FIB) equations (Isidori and Byrnes, 1990). However this approach becomes impractical when is applied to complex nonlinear plants.

On the other hand, very often, we are unable to get a rigorous mathematical model. For this situation, Takagi and Sugeno proposed a fuzzy model which describes the dynamics of complex systems under the suitable selection of linear subsystems. These models allow us to extend linear results to the nonlinear field in a relatively easy way. For instance, the stability for the TS fuzzy models depends on the existence of a common definite positive matrix (Wang, 1997; Tanaka and Wang, 2001). This condition is relaxed in other papers, which propose searching for piece-wise quadratic Lyapunov functions (see e.g. (Johanson and Rantzer, 1997)).

A few years ago in (Xiao-Jun and Zeng-Qi, 2000) was presented an approach to design the fuzzy regulator based on local controllers exclusively. Nevertheless, as is mentioned in (Lee *et al.*, 2003)

[1] Work partially supported by Consejo Nacional de Ciencia y Tecnología, CONACYT, México, under grant No. 37687-A

and (Castillo-Toledo *et al.*, 2003), this technique only works in very particular cases.

In this work, we develop an algorithm to systematically reduce the tracking error. Our procedure is based on the addition of a discontinuous term to the overall fuzzy controller, LMI techniques and CAD tools.

The paper is organized as follows. In section 2 we review the basic results on output regulation. In section 3 the main result and the algorithm are presented while in section 4 a numerical simulation is carried out. The final comments are given in section 5.

## 2. BASIC RESULTS ON REGULATION THEORY

Considering the dynamical system

$$\dot{x} = f(x, u, w) \tag{1}$$

$$\dot{w} = s(w) \tag{2}$$

$$e = h(x, w); \tag{3}$$

with $x \in \mathbb{R}^n$, $w \in \mathbb{R}^p$, $u \in \mathbb{R}^m$, and $e \in \mathbb{R}^q$ as the state of the system, the state of exosystem, the input signal and the output tracking error, respectively; the *State Feedback Output Regulation Problem* **(SORP)** is defined as the problem of maintaining the closed-loop stability when the plant is not affected by the exosystem, and ensuring the reference tracking when the system is under the influence of the exosystem. More precisely, the **SORP**, consists in finding a controller

$$u(t) = \alpha(x, w) \tag{4}$$

such that, the following conditions hold:

**S)** (Stability) The equilibrium point $x = 0$ of the system

$$\dot{x} = f(x, \alpha(x, 0), 0)$$

is asymptotically stable.

**R)** (Regulation) The closed-loop system (1), (2) and (4) satisfies

$$\lim_{t\to\infty} e(t) = 0$$

On the other hand, the linear approximation for the system (1)-(3) around the equilibrium point $(x, w, u) = (0, 0, 0)$ is

$$\dot{x} = Ax + Bu + Pw \tag{5}$$

$$\dot{w} = Sw \tag{6}$$

$$e = Cx + Qw \tag{7}$$

Thus, if the pair $(A, B)$ is stabilizable, the solution for the **SORP** depends on the existence of nonlinear mappings $x_{ss} = \pi(w)$ and $u_{ss} = \gamma(w)$ satisfying the FIB equations (Byrnes *et al.*, 1997; Isidori, 1995)

$$\frac{\partial \pi(w)}{\partial w} s(w) = f(\pi(w), \alpha(\pi(w), w), w) \tag{8}$$

$$0 = h(\pi(w), w). \tag{9}$$

Roughly speaking, $x_{ss} = \pi(w)$ and $u_{ss} = \gamma(w)$ represent the steady state zero output submanifold and the steady state input which ensures the invariance of $\pi(w)$, respectively.

The resulting controller is

$$u = Kx + \gamma(w) - K\pi(w).$$

with $K$ such that $(A + BK)$ is Hurwitz.

For the linear case, equations (8)–(9) become (Isidori, 1995; Knobloch *et al.*, 1993)

$$\begin{aligned} \Pi S &= A\Pi + B\Gamma + P \\ 0 &= C\Pi + Q \end{aligned} \tag{10}$$

and the controller is $u = Kx + (\Gamma - K\Pi)w$.

## 3. THE NUMERICAL ALGORITHM

In this section, we propose to include a discontinuous term into the TS fuzzy controller. This additional element, is the basis of our approach and it is demonstrated that under certain conditions, a controller designed in this way reduces the steady state error obtained by the method of local regulators.

Let us consider the TS fuzzy model described by $r$ rules of the form

*Plant rule* $i$:

IF $z_1(t)$ is $M_{1i}$ and ... and $z_v(t)$ is $M_{vi}$

$$\text{THEN} \quad \sum_i : \begin{cases} \dot{x} = A_i x + B_i u + P_i w \\ \dot{w} = S_i w \\ e_i = C_i x + Q_i w, i = 1, ..., r \end{cases}$$

where $M_{ji}$ are the fuzzy sets, $z_1, ..., z_v$ are the corresponding premise variables which may coincide with $x$ or $w$, or even with a combination of these state vectors. The linear subsystems are not necessary obtained from linear approximation, instead they can be extracted from some knowledge of the process dynamics (Tanaka and Wang, 2001).

To simplify this analysis, we avoid the use of observers, i. e., we consider that the measurable variables include the whole information of both, the plant and the exosystem.

Thus, the overall TS fuzzy system is (Tanaka and Sugeno, 1992; Tanaka and Wang, 2001; Wang, 1997):

$$\dot{x} = \sum_{i=1}^{r} \mu_i A_i x + \sum_{i=1}^{r} \mu_i B_i u + \sum_{i=1}^{r} \mu_i P_i w, \quad (11)$$

$$\dot{w} = \sum_{i=1}^{r} \mu_i S_i w, \quad (12)$$

$$e = \sum_{i=1}^{r} \mu_i \left[ C_i x + Q_i w \right], \quad (13)$$

with $\mu_i$ as the normalized weight for each rule calculated from the membership functions of $z_j$ in $M_{ji}$ satisfying

$$\mu_i \geq 0$$
$$\sum_{i=1}^{r} \mu_i = 1, \qquad z = \left[ z_1, ..., z_v \right]^T.$$

Notice the TS fuzzy model is the result of singleton fuzzifier, product inference and center average defuzzifier.

To construct the fuzzy controller we could design local regulators by solving the following equations (Xiao-Jun and Zeng-Qi, 2000)

$$\begin{aligned} \Pi_i S_i &= A_i \Pi_i + B_i \Gamma_i + P_i \\ 0 &= C_i \Pi_i + Q_i \end{aligned} \quad (14)$$

for all $i = 1, \ldots, r$. Then, local controllers take the form

$$u = K_i x + L_i w,$$

with

$$L_i = \Gamma_i - K_i \left( \sum_{j=1}^{r} \mu_j \Pi_j \right),$$

and the overall nonlinear fuzzy controller would be

$$u = \left( \sum_{i=1}^{r} \mu_i K_i \right) x + \left( \sum_{i=1}^{r} \mu_i L_i \right) w. \quad (15)$$

Unfortunately, this regulator does not guarantee the asymptotical convergence of the error, in general (Lee *et al.*, 2003; Castillo-Toledo *et al.*, 2003). In fact, the latter analysis assumes

$$\hat{\pi}(w) = \left( \sum_{i=1}^{r} \mu_i \Pi_i \right) w,$$

$$\hat{\gamma}(w) = \left( \sum_{i=1}^{r} \mu_i \Gamma_i \right) w.$$

Nevertheless, in general, mappings $\hat{\pi}(w)$, $\hat{\gamma}(w)$ are not the exact solution of the FIB equations (8)–(9).

For the interested reader, the particular cases that are solved by means of $\hat{\pi}(w), \hat{\gamma}(w)$ are analyzed in (Castillo-Toledo *et al.*, 2003).

In the following, we propose to compensate the difference between $\hat{\pi}(w), \hat{\gamma}(w)$ and the exact mappings by means of a sliding mode term. Our motivation is that we may consider the existence of a nominal model for which the aggregate control (15) is exactly the equivalent control (Utkin *et al.*, 1999). In this sense, we take the TS fuzzy system as the disturbed version of such nominal model. The suggested switching function for this problem is

$$e(t) = \sum_{i=1}^{r} \mu_i C_i x(t) + \sum_{i=1}^{r} \mu_i Q_i w(t).$$

The rules for the fuzzy regulator have the form

*Controller rule i*:

IF $\quad z_1(t)$ is $M_{1i}$ and ..... and $z_p(t)$ is $M_{pi}$

THEN

$$u(t) = K_i(x(t) - \Pi_i w(t)) + \Gamma_i w(t),$$

and the final controller will be

$$u = u_{eq} + v(e) \quad (16)$$

where

$$u_{eq} = \left( \sum_{i=1}^{r} \mu_i K_i \right) x + \left( \sum_{i=1}^{r} \mu_i \Gamma_i - \sum_{i=1}^{r} \mu_i K_i \sum_{j=1}^{r} \mu_j \Pi_j \right) w$$

is the controller proposed in (Xiao-Jun and Zeng-Qi, 2000), and

$$v(e) = G sign(e)$$

is the additional discontinuous term.

Thus, the *Fuzzy Output Regulator Problem with Sliding Modes* **(FORPSM)** can be defined as the problem of finding a set of triplets $(K_i, \Pi_i, \Gamma_i)$ for $i = 1, .., r$ and $G$ such that the following conditions hold:

**FS)** (Fuzzy Stability) The equilibrium point $(x, w) = (0, 0)$ of the system

$$\dot{x} = \sum_{i=1}^{r} \mu_i A_i x(t) + \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i \mu_j B_i K_j x(t) + G sign(e)$$

is asymptotically stable.

**FR)** (Fuzzy Regulation) The solution of the closed-loop system (11)–(12)–(16) satisfies

$$\lim_{t \to \infty} e(t) = 0.$$

The following result states the conditions for the existence of such a controller.

*Theorem 1.* If matrices $S_i$ are neutrally stable for all $i = 1, \ldots, r$ and

**H1)** the pairs $(A_i, B_i)$ are stabilizable for all $i = 1, \ldots, r$,

**H2)** there exist matrices $\Pi_i$ and $\Gamma_i$ solving

$$\Pi_i S_i = A_i \Pi_i + B_i \Gamma_i + P_i \tag{17}$$

$$0 = C_i \Pi_i + Q_i \tag{18}$$

for all $i = 1, \ldots, r$,

**H3)** there exists matrices $K_i$ and $\mathbf{P}$ such that

$$N_{ii}^T \mathbf{P} + \mathbf{P} N_{ii} < 0$$

for $i = 1, ..., r$ and

$$\left(\frac{N_{ij} + N_{ji}}{2}\right)^T \mathbf{P} + \mathbf{P}\left(\frac{N_{ij} + N_{ji}}{2}\right) < 0$$

for all $i, j = 1, \ldots, r$ satisfying $\mu_i \mu_j \neq 0$ with

$$N_{ij} = (A_i + B_i K_j), \tag{19}$$

**H4)** there exist four real numbers $\alpha_1 > 0$, $\alpha_2 > 0$, $\alpha_3 > 0$ and $G$ such that $-\alpha_1 < G < 0$ and $G < -\alpha_2$

then the **FORPSM** is solvable. Moreover, the controller has the form

$$\begin{aligned} u = & \left(\sum_{i=1}^{r} \mu_i K_i\right) x \\ & + \left(\sum_{i=1}^{r} \mu_i \Gamma_i - \sum_{i,j=1}^{r} \mu_i \mu_j K_i \Pi_j\right) w + v(e), \end{aligned} \tag{20}$$

with $v(e)$ defined as above.

**Proof.** Due to the lack of space the proof is omitted, however it is given in (Meda-Campaña and Castillo-Toledo, 2005). ∎

From the latter theorem, it can be deduced that the regulation problem is solved when there exist $\alpha_1 > 0$ and $\alpha_2 > 0$ such that

$$-\alpha_1 < G < 0 \tag{21}$$

and

$$G < -\alpha_2 \tag{22}$$

are satisfied, where

$$\alpha_1 \equiv \sqrt{\frac{\left(\|\mathbf{Q}\| - \|\mathbf{P}\|^2 \left(\sum_{i=1}^{r} \|B_i\|\right)^2\right) \|x\|^2}{q}}. \tag{23}$$

with

$$\mathbf{Q} = \left(\sum_{i,j=1}^{r} \mu_i \mu_j N_{ij}^T \mathbf{P} + \mathbf{P} \sum_{i,j=1}^{r} \mu_i \mu_j N_{ij}\right),$$

$q$ as the dimension of the error, and $\alpha_2$ depending on $\pi(w)$ and $\gamma(w)$ which solve the following equations (Isidori, 1995)

$$\frac{\partial \pi}{\partial w} s(w) = \sum_{i=1}^{r} \mu_i A_i \pi(w) + \sum_{i=1}^{r} \mu_i B_i \gamma(w) + \sum_{i=1}^{r} \mu_i P_i w$$

and

$$0 = \sum_{i=1}^{r} \mu_i C_i \pi(w) + Q w.$$

Now, we present an algorithm based on LMI techniques, which provides a practical way to compute matrices $K_i$ and $\mathbf{P}$ in order to expand the stability region, such that the inclusion of the sliding mode term does not affect the stability property. For more details about LMIs, the reader is referred to (Boyd *et al.*, 1994), where a complete analysis of LMIs in control theory is presented.

We observe that assumption FH1) is satisfied and at the same time the existence of $\alpha_1 > 0 \in \mathbb{R}$ is guaranteed when the following LMIs are feasible

$$-\beta I > \mathcal{Q}_1 A_i^T + X_i^T B_i^T + A_i \mathcal{Q}_1 + B_i X_i + \lambda I \tag{24}$$

for $i = 1 \ldots r$, where $\mathcal{Q}_1$ and $X_i$ are the unknowns with $X_i = K_i \mathcal{Q}_1$ and $\mathcal{Q}_1 > 0$. The real number $\beta > 0$ is a design parameter that may be changed during the design process in order to obtain different values for $\alpha_1$. These LMIs ensure the stability for each subsystem, for the interpolation regions, i. e. FH3), we have to solve

$$\begin{aligned} -2\beta I > & \mathcal{Q}_1 A_i^T + X_j^T B_i^T + \mathcal{Q}_1 A_j^T + X_i^T B_j^T \\ & + A_i \mathcal{Q}_1 + B_i X_j + A_j \mathcal{Q}_1 + B_j X_i + 2\lambda^2 I \end{aligned} \tag{25}$$

for $i = 1 \ldots r-1$ and $i < j \leq r$. As before, the existence of $\alpha_1 > 0 \in \mathbb{R}$ is guaranteed, $\mathcal{Q}_1$ and $X_i$ are the unknowns with $X_i = K_i \mathcal{Q}_1$ and $\mathcal{Q}_1 > 0$, and the common matrix $\mathbf{P}$ is $\mathcal{Q}_1^{-1}$ (Tanaka and Wang, 2001).

From (23), (24) and (25), we notice $\alpha_1$ can be approximated by $\sqrt{\frac{\beta}{q}}$. On the other hand, we know that checking (22) is too complex because it involves the exact mappings $\pi(w)$ and $\gamma(w)$. Therefore, considering the great impact of computers into the control design field, we suggest the use of simulation tools in order to avoid the testing of (22). The controller design algorithm is as follows:

**Step 1:** Set the initial value $\beta = 0$ and any $\Delta\beta$ as increment.

**Step 2:** Solve LMIs (24) and (25). Construct the controller (20) by taking $G = -\sqrt{\frac{\beta}{q}}$.

**Step 3:** Evaluate the performance of the controller using any simulation tool. If the result is satisfactory then finish; otherwise, set $\beta = \beta + \Delta\beta$ and return to Step 2.

## 4. AN ILLUSTRATIVE EXAMPLE

Let us consider the fuzzy system (11)-(12)-(13) presented in (Xiao-Jun and Zeng-Qi, 2000) with

$$A_1 = \begin{pmatrix} 0 & 1 \\ a & 0 \end{pmatrix}; \; A_2 = \begin{pmatrix} 0 & 1 \\ \frac{2}{\pi}a & 0 \end{pmatrix}, \; B_1 = \begin{pmatrix} 0 \\ b \end{pmatrix},$$

$$B_2 = \begin{pmatrix} 0 \\ \alpha b \end{pmatrix}, \; C = \begin{pmatrix} 0 & 1 \end{pmatrix},$$

$$S_1 = S_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \; Q = \begin{pmatrix} 1 & 0 \end{pmatrix},$$

where $a = -\frac{Mgl}{Ml^2+I}$, $b = \frac{1}{Ml^2+I}$, $g = 9.81\ m/s^2$, $M = 20\ Kg$, $l = 0.5\ m$, $I = 0.8\ Kg \cdot m^2$, $\alpha = 2.5$, membership functions

$$\begin{aligned} \mu_1 [x_1(t)] &= \left[1 - \frac{1}{1 + e^{-7(x_1 - \pi/4)}}\right] \\ &\quad \times \left[\frac{1}{1 + e^{-7(x_1 + \pi/4)}}\right], \\ \mu_2 [x_1(t)] &= 1 - \mu_1 [x_1(t)], \end{aligned}$$

and, as can be seen, in this case $q = 1$.

The solutions for the linear subsystems are $\Pi_1 = \Pi_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, $\Gamma_1 = \begin{pmatrix} 0 & \frac{1+a}{b} \end{pmatrix}$, $\Gamma_2 = \begin{pmatrix} 0 & \frac{1+2a/\pi}{\alpha b} \end{pmatrix}$ thus, the overall fuzzy mappings are

$$\hat{\pi}(w) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}, \tag{26}$$

$$\hat{\gamma}(w) = \begin{pmatrix} 0 & \delta \end{pmatrix}, \tag{27}$$

with

$$\delta = \mu_1(w_2)\frac{1+a}{b} + \mu_2(w_2)\frac{1+2a/\pi}{\alpha b} \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}.$$

We can notice, $\hat{\pi}(w)$ and $\hat{\gamma}(w)$ do not solve the fuzzy regulation problem since they do not coincide with the exact solution

$$\begin{aligned} \pi(w) &= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}, \\ \gamma(w) &= \begin{pmatrix} 0 & \frac{1 + a\mu_1(w_2) + 2a\mu_2(w_2)/\pi}{\mu 1 b + \mu 2 \alpha b} \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}, \end{aligned}$$

which are clearly different from (26)–(27) when $\alpha \neq 1$. Therefore, we will try to compensate this difference by means of discontinuous control.

For this example we simply set $\beta = 0$ and $\Delta\beta = 2$. Using the LMI toolbox of MATLAB® we obtain the following controllers.

Start:

Controller 1 with $\beta = 0$:

$$\begin{aligned} K_1 &= \begin{pmatrix} -53.5904 & -20.9328 \end{pmatrix} \\ K_2 &= \begin{pmatrix} -36.4632 & -8.8450 \end{pmatrix} \\ G &= 0 \end{aligned}$$

Controller 2 with $\beta = 2$:

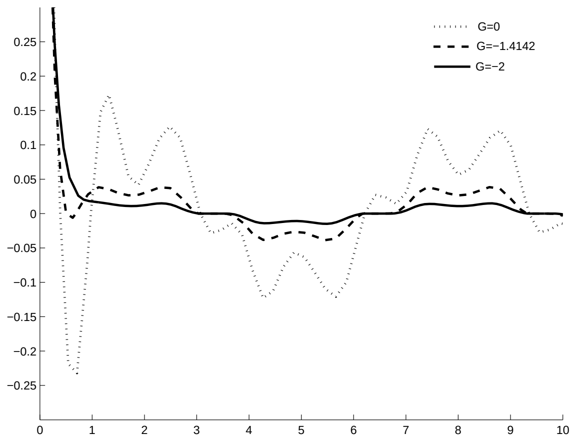


Fig. 1. Tracking error using different controllers.

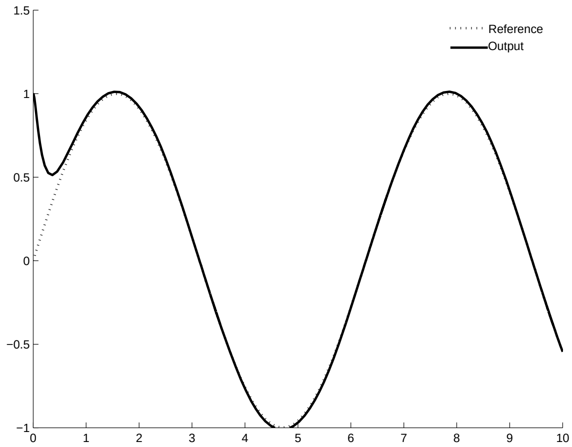


Fig. 2. Output vs reference with $G = -2$

$$\begin{aligned} K_1 &= \begin{pmatrix} -271.1921 & -69.5392 \end{pmatrix} \\ K_2 &= \begin{pmatrix} -127.2090 & -29.0904 \end{pmatrix} \\ G &= -\sqrt{2} \end{aligned}$$

Controller 3 with $\beta = 4$:

$$\begin{aligned} K_1 &= \begin{pmatrix} -787.8985 & -184.5593 \end{pmatrix} \\ K_2 &= \begin{pmatrix} -342.1423 & -76.9143 \end{pmatrix} \\ G &= -2 \end{aligned}$$

We use SIMULINK® to simulate the behavior of the plant under the action of the three controllers. The results are given in Figures 1, 2 and 3. Figure 1 compares the errors for the three regulators. As we can see, the controllers are improved as the algorithm progresses. Figure 2 shows the output of the plant and the reference signal when we apply controller 3, and Figure 3 presents the input signal at same conditions.

*Remark 2.* Observe that the input signal remains mainly smooth. A controller designed in this way demands less effort than those designed using discontinuous techniques exclusively.

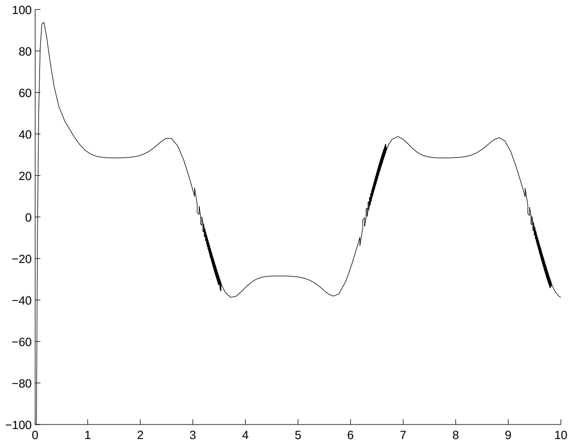

Fig. 3. Input signal

## 5. CONCLUSIONS

In this paper we have presented a practical approach to construct output regulators for nonlinear systems. In our method, we combine regulation theory, Takagi-Sugeno fuzzy models, sliding modes control and LMIs techniques. Based on the existence of local regulators, we developed a numerical algorithm to reduce systematically, or even to eliminate the overall tracking error by means of a discontinuous term. The simulations carried out suggest its validity.

This approach can be applied to the original nonlinear system. In that case the result will depend on the grade of approximation of the TS fuzzy model.

## REFERENCES


Boyd, S., L. El Ghaoui, E. Feron and V. Balakrishnan (1994). *Linear Matrix Inequalities in Systems and Control Theory.* SIAM. Philadelphia, PA.

Byrnes, C. I., F. Delli Priscoli and A. Isidori (1997). *Output Regulation of Uncertain Nonlinear Systems.* Birkhäuser, Boston.

Castillo-Toledo, B., J. A. Meda-Campaña and A. Titli (2003). A fuzzy output regulator for takagi-sugeno fuzzy models. In: *Proceedings of the 2003 IEEE International Symposium on Intelligent Control.* Vol. 2. Houston, Texas. pp. 310 – 315.

Francis, B. A. (1977). The linear multivariable regulator problem. *SIAM J. Cont. Optimiz.* **15**, 486–505.

Isidori, A. (1995). *Nonlinear Control Systems.* Springer. Berlin.

Isidori, A. and C. I. Byrnes (1990). Output regulation of nonlinear systems. *IEEE Transactions on Automatic Control* **35**, 131–140.

Johanson, M. and A. Rantzer (1997). On the computation of piecewise quadratic lyapunov functions. In: *Proceedings oof the 36th Conference on Decision and Control).* San Diego, California. pp. 3515–3520.

Knobloch, H. W., A. Isidori and D. Flockerzi (1993). *Topics in Control Theory.* Birkhäuser Verlag. Basel-Boston-Berlin.

Lee, Ho Jae, Jin Bae Park and Young Hoon Joo (2003). Comments on Output tracking and regulation on nonlinear system based on Takagi-Sugeno fuzzy model. *IEEE Transactions on Systems, Man, and Cybertics-Part B:Cybernetics* **33**(3), 521–523.

Meda-Campańa, J.A. and Bernardino. Castillo-Toledo (2005). On the output regulation for ts fuzzy models using sliding modes. *Submitted to the 24th American Control Conference (ACC).*

Tanaka, K. and H. O. Wang (2001). *Fuzzy control systems design and analysis. A linear matrix inequality approach.* John Wiley and Sons, Inc.

Tanaka, K. and M. Sugeno (1992). Stability analysis and design of fuzzy control systems. *Fuzzy sets Syst.* **45**(2), 135–156.

Utkin, V., J. Guldner and J. Shi (1999). *Sliding Mode Control in Electromechanical Systems.* Taylor & Francis, Philadelphia, PA.

Wang, Li-Xin (1997). *A Course in Fuzzy Systems and Control.* Prentice Hall PTR. Upper Saddle River, NJ.

Xiao-Jun, M. and S. Zeng-Qi (2000). Output tracking and regulation on nonlinear system based on Takagi-Sugeno fuzzy model. *IEEE Transactions on Systems, Man, and Cybertics-Part B:Cybernetics* **30**(1), 47–59.