DETERMINISTIC LEARNING AND RAPID DYNAMICAL PATTERN RECOGNITION

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Abstract: Recognition of temporal/dynamical patterns is among the most difficult pattern recognition tasks. In this paper, based on a recent result on deterministic learning theory, a unified, deterministic approach is proposed for effective representation and rapid recognition of dynamical patterns. Firstly, it is shown that time-varying dynamical patterns can be effectively represented in a time-invariant and spatially-distributed manner through deterministic learning. Then, by characterizing the similarity of dynamical patterns based on the system dynamics inherently within them, a dynamical recognition mechanism is proposed. Rapid recognition of dynamical patterns can be implemented when state synchronization is achieved according to a kind of indirect and dynamical matching on system dynamics. The synchronization errors can be taken as the measure of similarity between the test and training patterns. The significance of the paper is that the problem of dynamical pattern recognition is turned into a problem of stability and convergence of a closed-loop recognition system, so that a completely dynamical approach is presented for rapid recognition of dynamical patterns.

Keywords: Dynamical pattern recognition, deterministic learning, persistence of excitation (PE) condition, recognition by synchronization.

1. INTRODUCTION

Humans generally excel in dealing with temporal patterns. Human recognition of temporal patterns is an integrated process, in which patterns of information distributed over time can be effectively identified, represented, and classified (Covey *et al.*, 1993). These recognition mechanisms, although not fully understood, are quite different from the existing neural network and statistical approaches for pattern recognition (Bishop, 1995; Jain *et al.*, 2000; Webb, 2002). So far, only limited success has been reported in the literature for temporal pattern recognition.

In this paper, we investigate the recognition of a class of temporal patterns generated from nonlinear dynamical systems. Specifically, we consider a general nonlinear dynamical system:

$$\dot{x} = F(x; p), \ x(t_0) = x_0 \tag{1}$$

where $x = [x_1, \cdots, x_n]^T \in R^n$ is the state of the system, p is a system parameter vector, $F(x; p) = [f_1(x; p), \cdots, f_n(x; p)]^T$ is a smooth but unknown nonlinear vector field. Assume that (i) the system state $x(t)$ is uniformly bounded, and the system trajectory starting from x_0, denoted as $\varphi_\zeta(x_0; p)$, is in either periodic or periodic-like (recurrent) motion. This kind of periodic or recurrent motion is defined in this study as a *dynamical pattern*, and is denoted as φ_ζ for concise presentation.

It has been reported that nonlinear dynamical systems are capable of exhibiting various types of dynamical patterns (Rabinovich, *et al.*, 2000). The recognition process of such dynamical patterns con-

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sists of two phases: the identification phase and the recognition phase. Here, "identification" involves working out the essential features of the pattern one does not recognize, while "recognition" means looking at a pattern and realizing that it is the same or a similar pattern to one seen earlier. For identification of dynamical patterns, we recently proposed a theory of deterministic learning of nonlinear dynamical systems (Wang *et al.*, 2003), in which localized RBF neural networks are employed, and locally accurate NN approximation of the system dynamics can be achieved. A brief review of the deterministic learning theory will be given in Section 2.

In temporal pattern recognition, a very difficult and fundamental problem is how to appropriately represent a time-varying pattern. Another important problem is the definition of similarity between two temporal (or dynamical) patterns. As dynamical patterns evolve with time, the existing similarity measures developed for static patterns do not seem appropriate for dynamical patterns. For this reason, there is no standard similarity definition for dynamical patterns in the current literature.

To solve the problems and to achieve dynamical pattern recognition, in this paper, we firstly propose that, by using the constant RBF networks obtained through deterministic learning, the time-varying dynamical patterns can be effectively represented by the locally accurate NN approximations of the underlying system dynamics. This representation is *time-invariant* and *spatially-distributed*, using a kind of complete information of both state and dynamics of dynamical patterns.

Secondly, we give two definitions for similarity of dynamical patterns based on system dynamics. From the qualitative analysis of nonlinear dynamical systems, it is understood that the similarity between two dynamical behaviors lies in the *topological equivalence* of two dynamical systems (Shilnikov *et al.*, 2001). Subsequently, it can be concluded that the similarity of dynamical patterns is determined by the *topological similarity* of the system dynamics inherently within these dynamical patterns. The issues of representation and similarity of dynamical patterns will be discussed in Section 3.

Finally, based on the time-invariant representation and the similarity definition, we propose a mechanism for rapid recognition of dynamical patterns. Using the constant RBF networks obtained in the identification phase, we construct a dynamical model for each training dynamical pattern. When a test pattern similar to one of the training patterns is presented, the closed-loop recognition system, consisting of the system generating the test pattern and the dynamical model corresponding to the training pattern, will achieve a form of exponential convergence (or state synchronization). The synchronization is yielded by *indirect* and *dynamical* matching of system dynamics, and the test dynamical pattern is thus being recognized as similar to a training pattern. The synchronization errors will be proven to be proportional to the differences of system dynamics, and thus can be taken as similarity measures between the test and the training dynamical patterns.

The significance of the paper is that a completely dynamical approach is presented in the sense that the problem of dynamical pattern recognition is turned into a problem of stability and convergence of a closed-loop recognition system. The proposed approach can distinguish and classify dynamical patterns with qualitatively different behaviors, and can assign dynamical patterns based on the similarity of system dynamics to predefined classes.

2. REVIEW OF DETERMINISTIC LEARNING

In this section, we present a brief review of the deterministic learning theory (Wang et al, 2003), which is essential for identification of dynamical patterns. Elements of deterministic learning include: (i) employment of the localized RBF neural network, (ii) satisfaction of a partial PE condition, (iii) guaranteed exponential stability of a closed-loop identification system, and (iv) partial parameter convergence and locally-accurate NN identification of the dynamics $F(x;p)$ of system (1).

2.1 *Dynamical Localized RBF Networks*

The following dynamical RBF network is employed:

$$\dot{\hat{x}}_i = -a_i(\hat{x}_i - x_i) + \widehat{W}_i^T S_i(x), i = 1, \cdots, n \quad (2)$$

where $\hat{x}_i$ is the state of the dynamical RBF network, x_i is the state of system (1), $a_i > 0$ is a design constant, and $\widehat{W}_i^T S_i(x)$ is a *localized* RBF network described by

$$\widehat{W}_i^T S_i(Z) = \sum_{j=1}^{N_i} \hat{w}_{ij} s_{ij}(Z) \quad (3)$$

where $Z \in \Omega_Z \subset R^q$ is the input vector, $\widehat{W}_i = [\hat{w}_{i1}, \cdots, \hat{w}_{iN_i}]^T \in R^{N_i}$ is the weight vector, $N_i > 1$ is the NN node number, and $S_i(Z) = [s_{i1}(Z), \cdots, s_{iN_i}(Z)]^T$, with $s_{ij}(\cdot)$ being the radial basis functions. Commonly used RBFs include the Gaussian function and the inverse Hardy's multiquadric function, both of which are localized basis functions in the sense that $s_{ij}(Z) \to 0$ as $\|Z\| \to \infty$ (Powell, 1992).

It has been shown (e.g. (Powell, 1992)) that for any continuous function $f(Z) : \Omega_Z \to R$, where $\Omega_Z \subset R^q$ is a compact set, and for the NN approximator (3) (the node number N is sufficiently large), there exists an ideal constant weight vector W^* such that for each $\epsilon^* > 0$

$$f(Z) = W^{*T} S(Z) + \epsilon(Z), \ \forall Z \in \Omega_Z \quad (4)$$

where $|\epsilon(Z)| < \epsilon^*$ ($\epsilon(Z)$ is denoted as ϵ hereafter to simplify the notation).

For localized RBF networks, the spatially localized learning capability implies that for any point Z_ζ, or any bounded trajectory $Z_\zeta(t)$ within the compact set Ω_Z, $f(Z)$ can be approximated by using a limited number of neurons located at the neighborhood of the point, or in a local region along the trajectory:

$$f(Z) = W_\zeta^{*T} S_\zeta(Z) + \epsilon_\zeta \tag{5}$$

where $S_\zeta(Z) = [s_{j_1}(Z), \cdots, s_{j_\zeta}(Z)]^T \in R^{N_\zeta}$ is a subvector of $S(Z)$ with $N_\zeta < N$, $|s_{j_k}| > \iota$ $(j_k = j_1, \cdots, j_\zeta)$, with ι being a small positive constant, and ϵ_ζ is the approximation error, with $|\epsilon_\zeta| - |\epsilon|$ being small.

It is well know that the concept of the PE condition is of great importance in adaptive systems, however, it is very difficult for the *a priori* verification (Narendra and Annaswamy, 1989). Based on some recent results on the PE condition (Kurdila *et al.*, 1995, Lu & Basar, 1998), we indicated explicitly in (Wang *et al.*, 2003) that any periodic or recurrent trajectory $Z(t)$ can lead to PE of a regression subvector $S_\zeta(Z)$ consisting of RBFs with centers located in a small neighborhood of $Z(t)$.

2.2 *Exponential Stability and Accurate Identification*

The NN weight adaptation law is given by:

$$\dot{\widehat{W}}_i = \dot{\widetilde{W}}_i = -\Gamma_i S_i(x)\tilde{x}_i - \sigma_i \Gamma_i \widehat{W}_i, \; i = 1, \cdots, n \tag{6}$$

where $\widetilde{W}_i = \widehat{W}_i - W_i^*$, $\widehat{W}_i$ is the estimate of W_i^*, $\Gamma_i = \Gamma_i^T > 0$, and $\sigma_i > 0$ is a small value.

By using the localization property of RBF networks, along the orbit $\varphi_\zeta(x_0)$ the closed-loop identification system, consisting of the nonlinear dynamical system (1), the dynamical RBF network (2), and the NN weight adaptation law (6), is described by:

$$\begin{bmatrix} \dot{\tilde{x}}_i \\ \dot{\widetilde{W}}_{\zeta i} \end{bmatrix} = \begin{bmatrix} -a_i & S_{\zeta i}(\varphi_\zeta)^T \\ -\Gamma_{\zeta i} S_{\zeta i}(\varphi_\zeta) & 0 \end{bmatrix} \begin{bmatrix} \tilde{x}_i \\ \widetilde{W}_{\zeta i} \end{bmatrix} + \begin{bmatrix} -\epsilon_{\zeta i} \\ -\sigma_i \Gamma_{\zeta i} \widehat{W}_{\zeta i} \end{bmatrix} \tag{7}$$

and

$$\dot{\widehat{W}}_{\bar{\zeta} i} = \dot{\widetilde{W}}_{\bar{\zeta} i} = -\Gamma_{\bar{\zeta} i} S_{\bar{\zeta} i}(\varphi_\zeta)\tilde{x}_i - \sigma_i \Gamma_{\bar{\zeta} i} \widehat{W}_{\bar{\zeta} i} \tag{8}$$

in which subscript $(\cdot)_{\zeta i}$ is defined in (5) and $(\cdot)_{\bar{\zeta} i}$ represents complement to $(\cdot)_{\zeta i}$, and $|\epsilon_{\zeta i}|$ is close to $|\epsilon_i|$.

The following theorem is useful in identifying the system dynamics $F(x;p)$ in system (1):

Theorem 1. (Wang *et al.*, 2003) Consider the closed-loop adaptive system, consisting of the nonlinear dynamical system (1), the dynamical RBF network (2), and the NN weight updating law (6). For any periodic or recurrent trajectory $\varphi_\zeta(x_0)$, starting from an initial condition $x_0 = x(0) \in \Omega$, and with initial values $\widehat{W}_i(0) = 0$, we have: (i) the neural-weight estimates $\widehat{W}_{\zeta i}$ (as given in (7)) converge to small neighborhoods of their optimal values $W_{\zeta i}^*$, and the neural weights $\widehat{W}_{\bar{\zeta} i}$ will remain small and constant; (ii) the RBF network $\widehat{W}_i^T S_i(x)$ can approximate the dynamics $f_i(x;p)$ along the trajectory $\varphi_\zeta(x_0)$ as:

$$f_i(\varphi_\zeta; p) = \widehat{W}_i^T S_i(\varphi_\zeta) + \epsilon_{i_1} \tag{9}$$

where $|\epsilon_{i_1}|$ is close to $|\epsilon_i|$; and (iii) the system dynamics $f_i(x;p)$ along the orbit $\varphi_\zeta(x_0)$ can be described using constant RBF networks as

$$f_i(\varphi_\zeta; p) = \overline{W}_i^T S_i(x) + \epsilon_{i_2} \tag{10}$$

where $|\epsilon_{i_2}|$ is small and is close to $|\epsilon_{i_1}|$, and $\overline{W}_i$ is obtained from

$$\overline{W}_i = \text{mean}_{t \in [t_a, t_b]} \widehat{W}_i(t) \tag{11}$$

where $t_b > t_a > 0$ represent a time segment after the transient process.

Remark 1. The NN weights updating law (6) is similar to the Lyapunov-based learning laws developed in the literature of adaptive neural control, e.g. (Ge and Wang, 2001). The novelty of deterministic learning lies in the satisfaction of a partial PE condition, which is generally not achieved in the literature. With the satisfaction of the partial PE condition of $S_{\zeta i}(\varphi_\zeta)$, locally-accurate NN approximation of $F(x;p)$ is achieved.

3. REPRESENTATION AND SIMILARITY

In conventional static pattern recognition, a pattern is usually a set of time-invariant measurements or observations represented in vector or matrix notation (Bishop, 1995; Jain *et al.*, 2000). For example, in statistical pattern recognition, a pattern is represented by a set of d features, or a d-dimensional feature vector which yields a d-dimensional feature space. Subsequently, the task of recognition or classification is accomplished when the d-dimensional feature space is partitioned into compact and disjoint regions, and decision boundaries are constructed in the feature space which separate patterns from different classes into different regions (Jain *et al.*, 2000; Webb, 2002). For dynamical patterns, since the measurements are mostly time-varying in nature, the above framework for static patterns may not be suitable for representation of dynamical patterns.

3.1 *A Time Invariant and Spatially Distributed Representation*

As introduced in Section 2, the system dynamics $F(x;p) = [f_1(x;p), \cdots, f_n(x;p)]^T$ of a dynamical pattern φ_ζ can be accurately approximated by

$\overline{W}_i^T S_i(x)$ $(i = 1, \cdots, n)$ in a local region along the periodic or recurrent orbit of the dynamical pattern φ_ζ. The *constant* RBF network $\overline{W}_i^T S(x)$ consists of two types of neural weights: (i) For neurons whose centers are close to the orbit $\varphi_\zeta(x_0)$, their neural weights, $\widehat{W}_{\zeta i}$, converge exponentially to a small neighborhood of their optimal values, $W_{\zeta i}^*$; and (ii) for the neurons with centers far away from the orbit $\varphi_\zeta(x_0)$, the neural weights, $\widehat{W}_{\bar{\zeta} i}$, will remain almost constant. Thus, constant neural weights are obtained for all neurons of the entire RBF network $\overline{W}_i^T S_i(x)$. Accordingly, from Theorem 1 and equations (10), we have the following statements concerning the representation of dynamical patterns.

(i) A dynamical pattern φ_ζ can be represented by using the constant RBF network $\overline{W}_i^T S_i(x)$, which provides a locally accurate NN approximation of the *time-invariant* system dynamics $f_i(\varphi_\zeta; p)$. This representation, based on the fundamental information extracted from the dynamical pattern φ_ζ, is independent of the time attribute. Therefore, we provide an effective solution to the problem of representation of time-varying dynamical patterns.

(ii) The representation by $\overline{W}_i^T S_i(x)$ is *spatially-distributed* in the sense that relevant information is stored in a large number of neurons distributed along the orbit of the dynamical pattern. Intuitively, this spatially-distributed information implies that a representation using a limited number of extracted features (as in statistical pattern recognition) is probably incomplete for representation of dynamical patterns.

(iii) Since the NN approximation represented in $\overline{W}_i^T S(x)$ is only accurate in a local region along the orbit $\varphi_\zeta(x_0)$, this local region (denoted by Ω_{φ_ζ}) can be described by:

$$\Omega_{\varphi_\zeta} := \Big\{ x \;\Big|\; \mathrm{dist}(x, \varphi_\zeta) < d \Rightarrow |\overline{W}_i^T S(x) - f_i(\varphi_\zeta; p)| < \xi_i^* \Big\} \quad (12)$$

where $d, \xi_i^* > 0$ are constants, ξ_i^* is the approximation error that is close to ϵ_i^* within Ω_{φ_ζ}. This knowledge stored in $\overline{W}_i^T S_i(x)$ can be recalled in a way that whenever the NN input $Z (= x)$ enters the region Ω_{φ_ζ}, the RBF network $\overline{W}_i^T S_i(x)$ will provide accurate approximation to the previously learned dynamics $f_i(\varphi_\zeta; p)$.

Note that the representation by $\overline{W}_i^T S_i(x)$ will not be used directly for recognition, i.e., recognition by direct comparison of the corresponding neural weights. Instead, for a training dynamical pattern φ_ζ, we construct a dynamical model using the constant $\overline{W}^T S(x) = [\overline{W}_1^T S_1(x), \cdots, \overline{W}_n^T S_n(x)]^T$ as:

$$\dot{\bar{x}} = -B(\bar{x} - x) + \overline{W}^T S(x) \quad (13)$$

where $\bar{x} = [\bar{x}_1, \cdots, \bar{x}_n]^T$ is the state of the dynamical model, x is the state of an input pattern generated from system (1), $B = diag\{b_1, \cdots, b_n\}$ is a diagonal matrix, with $b_i > 0$ normally smaller than a_i (a_i is given in (2)). As will be detailed in the following sections, this dynamical model will be used as a representative of the training dynamical pattern φ_ζ in the construction of a recognition system for rapid recognition of test dynamical patterns.

3.2 *A Fundamental Similarity Measure*

In the literature of pattern recognition, there are many definitions for similarity of static patterns, e.g., Euclidean distance, Manhattan distance, and cosine distance (Webb, 2002). To define the similarity of two dynamical patterns, the existing similarity measures developed for static patterns might become inappropriate. The dynamical patterns evolve with time and the effects of different initial conditions or system parameters influence the occurrence of dynamical patterns.

To be specific, consider the dynamical pattern φ_ζ (as given by (1)), and another dynamical pattern (denoted as φ_ς) generated from the following nonlinear dynamical system:

$$\dot{x} = F'(x; p'), \;\; x(t_0) = x_{\varsigma 0} \quad (14)$$

where the initial condition $x_{\varsigma 0}$, the system parameter vector p', and subsequently the nonlinear vector field $F'(x; p') = [f_1'(x; p'), \cdots, f_n'(x; p')]^T$, are possibly different with those for dynamical pattern φ_ζ. Since small changes in $x(t_0)$ and p' (or p in (1)) may lead to large change of $x(t)$, it is clear that the similarity of dynamical patterns φ_ζ and φ_ς cannot be established by using only the time-varying states $x(t)$ of the patterns, or by some non-fundamental features extracted from $x(t)$.

In the qualitative analysis of nonlinear dynamical systems (e.g., Shilnikov *et al.*, 2001), the contemporary understanding of the similarity between two dynamical behaviors lies in the *topological equivalence* of two dynamical systems. It is understood from the studies on nonlinear dynamical systems, that the similarity of dynamical patterns is determined by the *topological similarity* of the system dynamics inherently within these dynamical patterns. Accordingly in this paper, we propose the following definition of similarity for dynamical patterns based on information from both system dynamics and system states.

Definition 1. Dynamical pattern φ_ς (given by (14)) is said to be similar with dynamical pattern φ_ζ (given by (1)), if the state of pattern φ_ς stays within a neighborhood region of pattern φ_ζ, and the difference between the corresponding system dynamics is small along the state of pattern φ_ς, i.e.,:

$$|f_i'(x;p') - f_i(x;p)| < \varepsilon_i^*, \forall x \in \varphi_\varsigma(x_{\varsigma 0};p') \quad (15)$$

where $\varepsilon_i^* > 0$ is the similarity measure between the two dynamical patterns.

Remark 2. It is seen that the above similarity definition is related to both the states and system dynamics of the two dynamical patterns. It is based on the fundamental information of system dynamics of the two patterns, i.e., $f_i(x;p)$ and $f_i'(x;p')$, which are by definition time-invariant. The state information of the two patterns is also involved; however, it is not required that states of the two patterns match (exactly) in phase space or occur identically.

According to the definition, pattern φ_ς being similar to pattern φ_ζ does not necessarily imply that the reverse is true. On the other hand, when the dynamics of pattern φ_ζ has been accurately identified within a local region, Ω_{φ_ζ} (as described by (12)), and effectively represented by constant RBF network $\overline{W}^T S(x)$, we can further investigate how pattern φ_ς *is recognized* to be similar to pattern φ_ζ.

Combining (12) with (15), when the state x of the pattern φ_ς stays within the local region Ω_{φ_ζ}, we have

$$\text{dist}(x, \varphi_\zeta) < d \Rightarrow$$
$$\max_{x \in \varphi_\varsigma(x_{\varsigma 0};p')} \left| f_i'(x;p') - \overline{W}_i^T S_i(x) \right| < \varepsilon_i^* + \xi_i^* \quad (16)$$

which shows that the difference of system dynamics of patterns φ_ς and φ_ζ is expressed in terms of $f_i(x;p')$ and $\overline{W}_i^T S_(x)$. Thus, from Definition 1 we have

Definition 2. Dynamical pattern φ_ς (given by (14)) is recognized to be similar with dynamical pattern φ_ζ (based on the identification of φ_ζ), if the state of pattern φ_ς stays within the local region Ω_{φ_ζ} (as described by (12)), and the difference between the corresponding system dynamics, as expressed in (16), is small along the state of pattern φ_ς.

This definition will be useful for the purpose of rapid recognition of φ_ς in Section 4.

4. DYNAMICAL PATTERN RECOGNITION

4.1 *Problem Formulation*

Consider a training set containing dynamical patterns φ_ζ^k, $k = 1, \cdots, M$, with the kth training pattern φ_ζ^k generated from

$$\dot{x} = F^k(x),\ x(t_0) = x_{\zeta 0}^k \quad (17)$$

where p^k is the system parameter vector. As shown in Section 2, the system dynamics $F^k(x) = [f_1^k(x),\ \cdots, f_n^k(x)]^T$ can be accurately identified and stored in constant RBF networks $\overline{W}^{k^T} S(x) = [\overline{W}_i^{k^T} S(x), \cdots, \overline{W}_n^{k^T} S(x)]^T$.

Consider dynamical pattern φ_ς (given by (14)) as a test pattern. Without identifying the system dynamics of the test pattern φ_ς, the recognition problem is to search *rapidly* from the training dynamical patterns φ_ζ^k ($k = 1, \cdots, M$) for those *similar* to the given test pattern φ_ς in the sense of Definition 2.

4.2 *Rapid Recognition via Synchronization*

In this subsection, we present how rapid recognition of dynamical patterns can be implemented by synchronization. Specifically, for the kth training pattern, a dynamical model is constructed based on the time-invariant representation $\overline{W}^{k^T} S(x)$ as:

$$\dot{\bar{x}}^k = -B(\bar{x}^k - x) + \overline{W}^{k^T} S(x) \quad (18)$$

where $\bar{x}^k = [\bar{x}_1^k, \cdots, \bar{x}_n^k]^T$ is the state of the dynamical (template) model, x is the state of an input test pattern generated from (14), and $B = diag\{b_1, \cdots, b_n\}$ is a diagonal matrix which is kept the same for all training patterns. Note that b_i ($1 \le i \le n$) is not chosen as a large value. Then, corresponding to the test pattern φ_ς and the dynamical model (18) for the training pattern φ_ζ^k, we obtain the following closed-loop recognition system:

$$\dot{\tilde{x}}_i^k = -b_i \tilde{x}_i^k + \overline{W}_i^{k^T} S_i(x) - f_i(x, p')$$
$$i = 1, \cdots, n \quad (19)$$

where $\tilde{x}_i^k = \bar{x}_i^k - x_i$ is the state tracking error.

Note that without identifying the system dynamics of the test pattern φ_ς, the difference on system dynamics of the test and training patterns, i.e., $|\overline{W}_i^{k^T} S_i(x) - f_i(x, p')|$, is not available from direct computation. Nevertheless, it will be shown that the difference between system dynamics can be explicitly measured by $|\tilde{x}_i^k|$. Thus, if the state $\bar{x}_i^k$ of the dynamical model (18) tracks closely to (or synchronize with) the state x of dynamical pattern φ_ς, i.e., $|\tilde{x}_i^k|$ is small, then the test pattern φ_ς can be recognized as similar to the training pattern φ_ζ^k in the sense of Definition 2. Note that the synchronization is not achieved between the states of dynamical patterns φ_ς and φ_ζ^k.

The following theorem describes how a test dynamical pattern is rapidly recognized in a dynamical process by synchronization.

Theorem 2. Consider the closed-loop recognition system (19) corresponding to test pattern φ_ς and the dynamical model (18) for training pattern φ_ζ^k. Then, the synchronization error $\tilde{x}^k$ is proportional to the difference of system dynamics of the test and training patterns. Further, the test pattern φ_ς is recognized as similar to the training pattern φ_ζ^k if the state $\bar{x}^k$ of the dynamical model (18) synchronizes with the state x of test pattern φ_ς.

Proof: See (Wang and Hill, 2005). $\diamondsuit$

Remark 3. Recognition of a test dynamical pattern is turned into a problem of stability and convergence of closed-loop recognition system (19). Without identifying the system dynamics $F'(x; p')$ of the test pattern φ_ς, and so without comparing directly the system dynamics, the recognition of a test pattern is achieved according to a kind of *indirect* matching of the system dynamics. The synchronization error $|\tilde{x}_i|$ can be taken as the measure of similarity on system dynamics, and subsequently, the measure of similarity between the test and training patterns.

Remark 4. The recognition of test pattern φ_ς is also achieved rapidly, since the recognition process takes place from the beginning of measuring the state x of test pattern φ_ς. No feature extraction in conventional pattern recognition is required. Complicated computations for comparisons of the states of dynamical patterns, or for matching directly the system dynamics of dynamical patterns are also avoided. The rapid recognition is naturally implemented when the closed-loop recognition system (19) achieves exponential stability such that exponential synchronization is obtained.

5. CONCLUDING REMARKS

In this paper, we have proposed an effective approach for representation and rapid recognition of dynamical patterns. The elements of the recognition approach include: (i) a time-invariant and spatially-distributed representation for dynamical patterns; (ii) a similarity measure based on system dynamics; and (iii) a mechanism in which rapid recognition is achieved by state synchronization.

The proposed recognition approach will facilitate further construction of recognition systems for temporal/dynamical patterns. Specifically, the recognition system can be constructed using many dynamical (template) models (as described in (18)). Each of the dynamical models represents one training dynamical pattern. Since the similarity between the test and training dynamical patterns can be measured using the synchronization errors, the recognition system can be built up by using the nearest neighbor classification — a commonly used classification algorithm in pattern recognition (Jain *et al.*, 2000).

The constructed recognition system promises to be able to classify different classes of dynamical patterns, and distinguish a set of dynamical patterns generated from the same class. It can also be designed to identify bifurcation points, which is an important task for many industrial applications, such as in power systems. Extensions of the current work will explore these aspects and employ the recognition system in a human-like control strategy.

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