# CONTROL-ORIENTED PROPERTIES PRESERVATION IN LINEAR SYSTEMS WHEN APPLYING PR0 SUBSTITUTIONS

**G. Fernández-Anaya** *,3 **J. C. Martínez-García** **,1,2
**V. Kučera** ***,4 **J. J. Flores-Godoy** *,5

* *Departamento Física y Matemáticas, Universidad Iberoamericana, Paseo de la Reforma #880, Lomas de Santa Fe, México, D. F. 01210, MEXICO*
** *Departamento de Control Automático, CINVESTAV-IPN, A. P. 14-740, México D.F. 07300, MEXICO*
*** *Faculty of Electrical Engineering, Czech Technical University in Prague, Technicka 2, 16627 Prague 6, CZECH REPUBLIC*

Abstract: We present in this paper some results concerning the preservation of control-oriented properties in Linear Time-Invariant systems (LTI systems) when applying substitutions of the Laplace variable s by a particular class of positive real functions, the socalled Positive Real functions of zero relative degree (PR0 functions). In particular, we show that the families of Bounded Real (BR), Strictly Bounded Real (SBR), and Positive Real (PR), rational functions are closed under compositions with the specified class of positive real functions. We also give some results concerning the preservation of stability in proper real rational transfer functions (in fact this is our main result), as well as the preservation of the $\mathcal{H}^\infty$-norm bound for this class of systems. Our study is restricted to Single-Input Single Output (SISO) systems. 

Keywords: Linear time-invariant systems, robustness, positive real rational functions, stabilization, properties preservation.

## 1. INTRODUCTION

Passivity is a classic concept in both circuits theory and mathematical control theory. In fact, passivity, as a property of the socalled passive systems, has been extensively studied by the systems theory community. As is pointed out in (Narendra and Annaswamy, 1989), the concept of positive realness of a transfer function plays a central role in Stability Theory. The definition of rational Positive Real functions (PR functions) arose in the context of Circuit Theory and Network Synthesis (Anderson and Vongpanitlerd, 1972), (Narendra and Annaswamy, 1989), (Guillemin, 1959), (Weinberg, 1962), (Piekarski and Uruski, 1990). In fact, the driving point impedance or admittance of a passive network is rational and positive real. If the network is dissipative (due to the presence of resistors), the driving point impedance of the network is a Strictly Positive Real transfer function (SPR function). Thus,

[1] Corresponding author
[2] Email: jcmg1964@prodigy.net.mx
[3] Email: guillermo.fernandez@uia.mx
[4] Email: kucera@fel.cvut.cz
[5] Email: job.flores@uia.mx

positive real systems, also called passive systems, are systems that do not generate energy. These systems are relevant to the analysis of absolute stability of nonlinear Lure systems and the design of adaptive schemes (see (Arcak *et al.*, 2003), (Khalil, 1996), (Lozano *et al.*, 2000), (Haddad and Bernstein, 1993), (Anderson *et al.*, 1982), (Dorato *et al.*, 1989)). Also a well-known fact is that a sufficient condition for the convergence of several recursive algorithms of adaptive schemes is the SPR property of a suitable family of transfer functions (Goodwin and Sin, 1984), (Anderson *et al.*, 1990), (Youla and Saito, 1967). In controller design (Abdallah *et al.*, 1995) and design of strictly positive real and positive real systems (Geromel and Gapsky, 1997), (Huang *et al.*, 1999), (Safonov *et al.*, 1987), (A. Tesi and Zappa, 1992). More recently it has been proposed in (Choi and Swaminathan, 2000) a method to synthesize lumped-element circuits of embedded RF components directly from rational functions, based in the theory of networks synthesis and analysis (Anderson and Vongpanitlerd, 1972), (Guillemin, 1959), (Weinberg, 1962).

Concerning the substitution of the Laplace variable s (in transfer functions describing LTI systems in frequency terms) by rational functions, they arise from the study of the socalled uniform systems (see (Polyak and Tsypkin, 1996) and references therein). More recently, some results have been published in (Fernández-Anaya, 1999), (Fernández-Anaya *et al.*, 2003) and (Fernández-Anaya *et al.*, 2004), mainly focused on the preservation of robustness properties in closed-loop control schemes.

We present in this paper some results concerning the preservation of control-oriented properties in Linear Time-Invariant systems (LTI systems) when applying substitutions of the Laplace variable s by a particular class of positive real functions, the socalled Positive Real functions of zero relative degree (PR0 functions). In particular, we show that the families of Bounded Real (BR), Strictly Bounded Real (SBR), and Positive Real (PR), are closed under compositions with the specified class of positive real functions. We also give some results concerning the preservation of stability in proper real rational transfer functions (in fact this is our main result), as well as the preservation of the $\mathcal{H}^\infty$-norm bound for this class of systems. Our study is based on some classic results concerning networks synthesis and analysis(see (Guillemin, 1959), (Weinberg, 1962)), it is restricted to Single-Input Single Output (SISO) systems, and extends some results presented in (Fernández-Anaya, 1999), (Fernández-Anaya *et al.*, 2003), (Fernández-Anaya *et al.*, 2004) and (Fernández-Anaya and Torres-Muños, 2002), which are limited to substitutions by Strictly Positive Real rational functions of zero relative degree (SPR0 functions).

The paper is organized as follows: We present in Section 2 the notation, as well as some basic definitions and concepts concerning positive real functions. Moreover, we recall a seminal result presented in (Weinberg, 1962), which shows that a rational function resulting from a Positive Real substitution (of zero relative degree) in a Positive Real rational function is also Positive Real. Our study is mainly based on this result. We provide our main results in Section 3, i. e., the preservation of stability and $\mathcal{H}^\infty$-norm bound properties, when applying positive real substitutions of zero relative degree in proper and stable LTI systems. We conclude with some final comments in Section 4.

## 2. PRELIMINARIES

We present in this section the notation, as well as some basic definitions and concepts concerning positive real functions. Moreover, we recall a seminal result coming from (Weinberg, 1962), which shows that a rational function resulting from a Positive Real substitution (of zero relative degree) in a Positive Real rational function is also Positive Real. Our study is mainly based on this result.

Notation :

Let $\mathbb{R}$ be the field of real numbers, $\mathbb{C}$ the complex plane, $\operatorname{Im}\mathbb{C}$ the imaginary axis, $\mathbb{C}^+$ the open right-half-plane of the complex plane, $\mathbb{C}_e^+ \equiv \mathbb{C}^+ \cup \{\infty\}$, $\overline{\mathbb{C}}^+ \equiv \mathbb{C}^+ \cup \operatorname{Im}\mathbb{C}$ and $\overline{\mathbb{C}}_e^+ \equiv \mathbb{C}^+ \cup \{\infty\} \cup \operatorname{Im}\mathbb{C}$.

First of all we recall some definitions concerning proper and stable systems, as well as positive real rational functions.

*Definition 1.* Let $\mathcal{RH}^\infty$ be the Euclidean domain of the proper, stable and rational real functions, and $R(s)$ is the field of real rational functions and $R_p(s)$ is the ring of real rational and proper functions.

*Definition 2.* (Narendra and Annaswamy, 1989) A rational function $q(s) \in R(s)$ of complex variable $s = \sigma + j\omega$ is positive real (PR function) if

(1) $q(s)$ is real for $s$ real,
(2) $q(s)$ is analytic in $\operatorname{Re}[s] > 0$,
(3) $\operatorname{Re}[q(s)] \geq 0$ for all $\operatorname{Re}[s] > 0$.

*Definition 3.* (Narendra and Annaswamy, 1989) A rational function is strictly positive real (SPR) if $q(s - \varepsilon)$ is PR for some $\varepsilon > 0$.

This definition is equivalent to:

*Definition 4.* (Narendra and Annaswamy, 1989), (Ioannou and Tao, 1987) A rational function $q(s)$ of zero relative degree is SPR (SPR0 function) if and only if

(1) $q(s)$ is analytic in $\mathrm{Re}[s] \geq 0$,
(2) $\mathrm{Re}[q(j\omega)] > 0$ for all $\omega \in R$.

*Remark 5.* We shall denote in the sequel the set of Positive Real functions by PR and the set of strictly positive real functions by SPR. Moreover, we shall denote PR0 the set of Positive Real functions of zero relative degree.

*Definition 6.* (Lozano *et al.*, 2000), (Haddad and Bernstein, 1993) A rational function $q(s)$ is bounded real (BR) if

(1) $q(s)$ is analytic in $\mathrm{Re}[s] > 0$,
(2) $\|q(s)\|_\infty \leq 1$.

*Definition 7.* (Lozano *et al.*, 2000), (Haddad and Bernstein, 1993) A rational function $q(s)$ is strictly bounded real (SBR) if

(1) $q(s)$ is analytic in $\mathrm{Re}[s] \geq 0$,
(2) $\|q(s)\|_\infty < 1$.

The following lemma characterizes the socalled Positive Real rational functions (PR functions).

*Lemma 8.* (Lozano *et al.*, 2000) A rational function $q(s)$ is PR if and only if

(1) $q(s)$ is analytic in $\mathrm{Re}[s] > 0$,
(2) $\mathrm{Re}[q(j\omega)] \geq 0$ for all $\omega$ such that $j\omega$ is not a pole in $p(s)$,
(3) If $s = j\omega_0$ is a pole in $q(s)$, then it is a simple pole, and $\omega_0$ is finite, then the residual
$$\mathrm{Res}_{s=j\omega_0}\, q(s) = \lim_{s\to j\omega_0}(s - j\omega_0)q(s)$$
is real and positive (note that $\omega_0$ can be zero). If $\omega_0$ is infinite, then the limit
$$R_\infty \equiv \lim_{\omega\to\infty}\frac{q(s)}{j\omega}$$
is real and positive.

We recall now a basic result which is seminal for our current study.

*Lemma 9.* (Weinberg, 1962) The composition of PR functions is a PR function i.e., if $p(s)$ and $q(s)$ are PR then $p(q(s))$ is PR.

*Corollary 10.* If $p(s)$ is PR and $q(s)$ is SPR, then $p(q(s))$ is SPR. In particular, composition of SPR functions is a SPR function.

**PROOF.** [Corollary 10] By Lemma 9, $p(q(s)) \in$ PR because SPR $\subset$ PR. By Definition 3, $q(s-\varepsilon) \in$ PR and then $p(q(s-\varepsilon)) \in$ PR for some $\varepsilon > 0$. Therefore by Definition 3 again, $p(q(s)) \in$ SPR. $\square$

*Remark 11.* This corollary generalizes the Theorem 2.2 in (Fernández-Anaya, 1999), because in this corollary is not necessary assume that the SPR functions are of zero relative degree.

We conclude this section with the following properties of PR, which will be useful in the sequel:

(1) If $p(s)$ is PR function, then $1/p(s)$ is also PR function.
(2) If $p_1(s)$ and $p_2(s)$ are PR functions, then $\alpha p_1(s)+\beta p_2(s)$ is a PR function for $\alpha,\ \beta \geq 0$.

## 3. PRESERVATION OF STABILITY AND $\mathcal{H}_\infty$-NORM BOUNDS PROPERTIES

We provide in this section our main result, i. e., it establishes the preservation of the stability property in proper and stable LTI systems when applying positive real substitutions of zero relative degree. Moreover, we show that a $\mathcal{H}_\infty$-norm bound associated to a proper and stable LTI system is preserved for the considered class of substitutions.

We can present at this moment our main result.

*Theorem 12.* Consider a rational function $q(s) = \frac{N_q(s)}{D_q(s)}$ with $N_q(s)$ and $D_q(s)$ given coprime polynomials, then

(1) If $p(s) = \frac{N_p(s)}{D_p(s)} \in \mathcal{RH}^\infty$ and $q(s) \in$ PR0, then $p(q(s)) \in \mathcal{RH}^\infty$.
(2) If $p(q(s)) \in \mathcal{RH}^\infty$ and $p(1/q(s)) \in \mathcal{RH}^\infty$ for all $p(s) \in \mathcal{RH}^\infty$, then $q(s) \in$ PR0.

**PROOF.** [Theorem 12]

(1) Suppose that $q(s)$ has poles in $j\omega_0, \ldots, j\omega_l$. Now we prove that $p(q(s)) \in \mathcal{RH}^\infty$. By item 3) in Definition 2 2, we have that
$$q\left(\overline{\mathbb{C}}^+\right) \subset \overline{\mathbb{C}}^+$$
except in the poles $j\omega_1, \ldots, j\omega_l$, and since composition of analytic functions is an analytic function, then $p(q(s))$ is analytic in $\overline{\mathbb{C}}^+$. The poles $j\omega_1, \ldots, j\omega_l$ have no effect at all. In order to prove this last statement consider the polynomials
$$N_p(s) = a_n s^n + \cdots + a_0$$
$$D_p(s) = b_m s^m + \cdots + b_0$$
and making the substitution, we obtain

$$
\begin{aligned}
p\left(q(s)\right) &= \frac{N_p(q(s))}{D_p(q(s))} = \frac{N_p\left(\frac{N_q(s)}{D_q(s)}\right)}{D_p\left(\frac{N_q(s)}{D_q(s)}\right)} \\
&= \frac{a_n\left[q(s)\right]^n + \cdots + a_1\left[q(s)\right] + a_0}{b_n\left[q(s)\right]^m + \cdots + b_1\left[q(s)\right] + b_0} \\
&= \frac{\sum_{i=0}^{n} a_i\left[D_q(s)\right]^{m-i}\left[N_q(s)\right]^i}{\sum_{i=0}^{m} b_i\left[D_p(s)\right]^{m-i}\left[N_q(s)\right]^i}
\end{aligned}
$$

Now note that in the poles $j\omega_1, \ldots, j\omega_l$ i. e., the roots of $D_q(s)$, we have that:

$$
p\left(q\left(j\omega_i\right)\right) = \begin{cases} \dfrac{a_n}{b_m}, & \text{if } m = n \\ 0, & \text{if } m > n \end{cases}
$$

for $i = 1, ..., l$, because $N_q(s)$ and $D_q(s)$ are coprime polynomials. Also, observe that if $q(s) \in$ PR0, then $p(q(s))$ is proper rational function for all $p(s) \in \mathcal{RH}^\infty$. Therefore $p(q(s)) \in \mathcal{RH}^\infty$.

(2) We just need to show that $p(q(s))$ satisfies Definition 2 for $q(s)$.

Definition 2 item 1) is obvious.

Definition 2 item 2), if there exists $s_0 \in \mathbb{C}$ with $\mathrm{Re}[s_0] > 0$ and such that $q(s)$ is not an analytic function at $s_0$ (i.e., $s_0$ is a pole of $q(s)$), then $p(1/q(s))$ is not an analytic function in $s_0$ for some $p(s) \in \mathcal{RH}^\infty$, because $1/q(s) = 0$ at $s_0$ and there exist $p(s) \in \mathcal{RH}^\infty$ that are not analytic at $s_0$. But by hypothesis $p(1/q(s)) \in \mathcal{RH}^\infty$ for all $p(s) \in \mathcal{RH}^\infty$, hence by a contradiction argument, item ii) is proved. Notice that by property Theorem 12 item 1) $1/q(s) \in$ PR0.

Definition 2 item 3), observe that $\mathrm{Re}[q(s)] \geq 0$ for all $\mathrm{Re}[s] > 0$ is equivalent to $q\left(\mathbb{C}^+\right) \subseteq \overline{\mathbb{C}}^+$. Again, suppose that there exists $s_0 \in \mathbb{C}$ with $\mathrm{Re}[s_0] > 0$ such that $q(s_0) \in \mathbb{C}^-$, then there exists $p_0(s) \in \mathcal{RH}^\infty$ so that $p_0(s)$ is not analytic at $q(s_0)$. But by hypothesis, $p(q(s)) \in \mathcal{RH}^\infty$ for all $p(s) \in \mathcal{RH}^\infty$. Therefore, by a contradiction argument, Definition 2 item 3) is proved. Finally, by arguments about the relative degree, we can conclude that $q(s) \in$ PR0.

□

*Remark 13.* Since $p(q(s)) \in \mathcal{RH}^\infty$ for all $p(s) \in \mathcal{RH}^\infty$ and taking $q(s) \in$ PR0 fixed, we can define the function $F : \mathcal{RH}^\infty \to \mathcal{RH}^\infty$ where $F(p(s)) \triangleq p(q(s))$. This function is an homomorphism of the Euclidean domain $\mathcal{RH}^\infty$ in itself (i.e., this function preserves sums, products, constants, units and inverses in $\mathcal{RH}^\infty$).

*Remark 14.* A direct consequence of Theorem 12 is that, the set PR0 is the greatest subset in $R(s)$ of rational functions $q(s)$, which satisfies the property $p(q(s)) \in \mathcal{RH}^\infty$ and $p(1/q(s)) \in \mathcal{RH}^\infty$ (for all $p(s) \in \mathcal{RH}^\infty$ with $q(s) \in R(s)$), which is to say it does not exists a set $X \subset R(s)$ such that $PR0 \in X$ and $PR0 \neq X$ and such that $p(q(s)) \in \mathrm{PR0} \subset \mathrm{X}$ and $\mathrm{PR0} \neq \mathrm{X}$ and such that $p(q(s)) \in \mathcal{RH}^\infty$ and $p(1/q(s)) \in \mathcal{RH}^\infty$ for all $p(s) \in \mathcal{RH}^\infty$ with $q(s) \in \mathrm{X}$.

*Remark 15.* Since SPR0 $\subset$ PR0, Theorem 12 item 1, generalizes Theorem 2.1 in (Fernández-Anaya, 1999).

*Remark 16.* On the other hand, the results in (Fernández-Anaya and Torres-Muños, 2002) can be generalized, changing the substitution $q(s) \in$ SPR0 by the substitution $q(s) \in$ PR0, also.

The following proposition establishes the preservation of the $\mathcal{H}^\infty$-norm, bound in proper and stable LTI systems, when applying the substitution of the Laplace variable $s$ by PR0 functions.

*Proposition 17.* Consider a given positive bound $\gamma_P$. If $P(s) \in \mathcal{RH}^\infty$ and $P(s)$ is non-constant, where $P(s) \neq 1$ for all $\mathrm{Re}[s] \geq 0$ with $\|P(s)\|_\infty \leq \gamma_P$, then $\|P(q(s))\|_\infty \leq \gamma_P$ for each $q(s) \in$ PR0.

**PROOF.** [Proposition 17] Define $P_\gamma(s) \equiv \gamma_P^{-1} P(s)$. By Theorem 2.9 in (Lozano *et al.*, 2000), if $P_\gamma(s) \neq 1$ for all $\mathrm{Re}[s] \geq 0$, then

$$
H_\gamma(s) = \frac{1 + P_\gamma(s)}{1 - P_\gamma(s)}
$$

is PR0 if and only if $P_\gamma(s)$ is BR of zero relative degree. Now by Lemma 9

$$
H_\gamma(q(s)) = \frac{1 + P_\gamma(q(s))}{1 - P_\gamma(q(s))}
$$

is PR0 for each $q(s) \in$ PR0. Then, again by Theorem 2.9 in (Lozano *et al.*, 2000), $P_\gamma(q(s))$ is BR of zero relative degree, for each $q(s) \in$ PR0. Therefore, using Definition 6 we have that $P_\gamma(q(s))$ is analytic in $\mathrm{Re}[s] \geq 0$, and $\|P_\gamma(q(s))\|_\infty \leq 1$ for each $q(s) \in$ PR0. Now as

$$
\|P_\gamma(q(s))\|_\infty = \gamma_P^{-1}\|P(q(s))\|_\infty \leq 1,
$$

then $\|P(q(s))\|_\infty \leq \gamma_P$ for each $q(s) \in$ PR0. □

*Remark 18.* This Proposition generalizes the Lemma 3 in (Fernández-Anaya *et al.*, 2003).

We conclude this section with the he following corollary, which concerns the closedness of both the family of BR functions and the family of SBR under PR0 substitutions.

*Corollary 19.* (1) If $P(s)$ is BR and $q(s) \in$ PR0, then $P(q(s))$ is BR of zero relative degree.

(2) If $P(s)$ is SBR and $q(s) \in$ PR0, then $P(q(s))$ is SBR of zero relative degree.

**PROOF.** [Corollary 19] The proof is consequence of the Proposition 17 and the Definitions 6 and 7.

## 4. FINAL COMMENTS

In this paper we mainly prove that:

(1) The families of proper and stable, positive real, bounded real, and strictly bounded real, rational functions are closed under compositions by positive real rational functions of zero relative degree.
(2) The $\mathcal{H}_\infty$-norm bound of a proper and stable real rational transfer function is preserved when applying positive real rational substitutions of zero relative degree.

Based on the results presented here, we conjecture that all the results given in (Fernández-Anaya *et al.*, 2004) (which concern strictly PRO substitutions) can be generalized, i.e., just changing SPR0 functions in (Fernández-Anaya *et al.*, 2004) by PR0 functions. Moreover, we think that almost all the results given in (Fernández-Anaya *et al.*, 2003) can also be extended to the PR0 case. In fact, it is easy to prove that coprime factorizations of real rational matrix transfer functions are preserved when applying PR0 substitutions.

## REFERENCES


A. Tesi, A. Vicino and G. Zappa (1992). New results for the design of robust strict positive real systems. In: *Proc. IEEE Symposium on Circuits and Systems*. San Diego, CA, USA. pp. 2729–2732.

Abdallah, C., P. Dorato, F. Pérez and D. Docampo (1995). Controller synthesis for a class of interval plants. *Automatica* **31**(2), 341–343.

Anderson, B. D. O. and S. Vongpanitlerd (1972). *Network Analysis and Synthesis. A modern Systems Approach*. Prentice Hall. Englewood Cliffs, USA.

Anderson, B. D. O., R. Bitmead, C. Johnson, P. Kokotovic, R. Kosut, I. Mareels, L. Praly and B. Riedle (1982). *Stability of Adaptive Systems - Passivity and Averaging Analysis*. MIT Press. Cambridge, USA.

Anderson, B. D. O., S. Dasgupta, P. Khargonekar, K. J. Krauss and M. Mansour (1990). Robust strict positive realness: characterization and construction. *IEEE Transactions on circuits and Systems 1* **37**, 869–876.

Arcak, M., M. Larsen and P. Kokotović (2003). Circle and popov criteria as tools for nonlinear feedback design. *Automatica* **39**, 643–650.

Choi, K. L. and M. Swaminathan (2000). Development of model libraries for embedded passives using network synthesis. *IEEE Transactions on Circuits and Systems II: Analog and Digital Signal Processing* **47**(4), 249–260.

Dorato, P., H.-B. Parks and Y. Li (1989). An algorithm for interpolation with units in $H_\infty$, with applications to feedback stabilization. *Automatica* **25**(3), 427–430.

Fernández-Anaya, G. (1999). Preservation of SPR functions and stabilization by substitutions in SISO plants. *IEEE Trans. Automat. Contr.* **44**, 2161–2164.

Fernández-Anaya, G. and J. Torres-Muños (2002). Preservation of stability in multidimensional systems using SPR0 functions. *IEEE Transaction on Circuits and Systems: Fundamental Theory and Application* **49**, 1654–1658.

Fernández-Anaya, G., J. C. Martínez-García and V. Kučera (2003). $H_\infty$-robustness properties preservation in SISO systems when applying SPR substitutions. *International Journal of Control* **76**(7), 728–740.

Fernández-Anaya, G., J. C. Martínez-García, V. Kučera and D. Aguilar-George (2004). MIMO systems properties preservation under SPR substitutions. *IEEE Transactions on Circuits and Systems II: Express Briefs* **51**(5), 222–227.

Geromel, J. C. and P. B. Gapsky (1997). Synthesis of positive real $H_2$ controllers. *IEEE Transactions on Automatic Control* **42**, 988–992.

Goodwin, G. C. and K. S. Sin (1984). *Adaptive Filtering, Prediction and Control*. Prentice-Hall International Editions. Englewood-Cliffs, NJ, USA.

Guillemin, E. A. (1959). *Synthesis of Passive Network*. John Wiley & Sons, Inc.. USA.

Haddad, W. M. and D. S. Bernstein (1993). Explicit construction of quadratic lyapunov functions for the small gain, positivity, circle, and popov theorems and their application to robust stability part i: Continuous-time theory. *Int. J. Robust and Nonlinear Control* **3**, 313–339.

Huang, C. H., P. A. Ioannou, J. Maroulas and M. G. Safonov (1999). Design of strictly positive real systems using constant output feedback. *IEEE Transactions on Automatic Control* **44**(3), 569–572.

Ioannou, P. A. and G. Tao (1987). Frequency domain conditions for strictly positive real functions. *IEEE Transactions on Automatic Control* **32**, 53–54.

Khalil, H. K. (1996). *Nonlinear Systems*. Prentice-Hall International Editions. Englewood-Cliffs, NJ, USA.

Lozano, R., B. Brogliato, . Maschke B and O. Egeland (2000). *Dissipative Systems Analysis and*

*Control: Theory and Applications*. Springer-Verlag. London, UK.

Narendra, K. S. and A. M. Annaswamy (1989). *Stable Adaptive Systems*. Prentice-Hall. Englewood Cliffs, NJ, USA.

Piekarski, M. S. and M. Uruski (1990). A network approach to interpolation with symmetric positive-real matrices. *Journal of the Franklin Institute* **327**(6), 841–866.

Polyak, B. T. and Ya. Z. Tsypkin (1996). Stability and robust stability of uniform systems. *Automation and Remote Contr.* **57**, 1606–1617.

Safonov, M. G., E. A. Jonckheere, M. Verma and D. J. Limebeer (1987). Synthesis of positive real multivariable feedback systems. *Int. J. Contr.* **45**(3), 817–842.

Weinberg, L. (1962). *Network Analysis and Synthesis*. McGraw-Hill Book Company, Inc.. USA.

Youla, D. C. and M. Saito (1967). Interpolation with positive-real functions. *Journal of the Franklin Institute* **284**(2), 77–108.