PASS SCHEDULE OPTIMIZATION FOR A TANDEM COLD MILL

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Abstract: Pass schedules for a tandem cold mill affect not only the uniformity quality and productivity of rolled strips but also the operation safety. This paper describes pass schedule optimization, which consists of two stages. One is robust pass schedule optimization (RPSO) before rolling, which optimizes pass schedules under distributions in strips' properties and rolling conditions. The other is mill balance control (MBC) during rolling, which slowly changes the pass schedules to cope with remaining distributions and unpredictable during-rolling variations. Results with an actual mill showed an 8% decrease in off-gage length and a 2.4% increase in maximum rolling speed without deterioration in gage accuracy.

Keywords: steel manufacture, control applications, optimization problems, robustness, multivariable control, linear quadratic regulators

1. INTRODUCTION

A tandem cold mill (TCM) is one of the plants in the steel works, which rolls a strip from initial gage to final gage in order to produce high surface-quality strips. Specifically, the mill is composed of several stands as shown in Fig.1, and the strip is reduced in gage by the stands' rolling forces and interstand tensions which are dependent on a pass schedule. The pass schedule, which designates the combination of reductions in gage at all stands and all interstand tensions in this paper, is crucial for proper rolling. There have been a number of attempts to improve the pass schedules (Okado, et al., 1969), but there has been no attempt to optimize the pass schedules taking account of the distributions and variations in strips' properties and rolling conditions.

In this paper, pass schedule optimization with two stages is considered. One is robust pass schedule optimization (RPSO) before rolling, and the other is mill balance control (MBC) during rolling at constant rolling speeds. The outline of pass schedule optimization is described in chapter 2. The RPSO and the MBC are illustrated in chapter 3 and chapter 4 respectively.

2. OUTLINE OF PASS SCHEDULE OPTIMIZATION

Figure 1 shows a 5-stand tandem cold mill. In cold rolling, the rolling forces of the first and last stands as well as motor currents are important to achieve safe

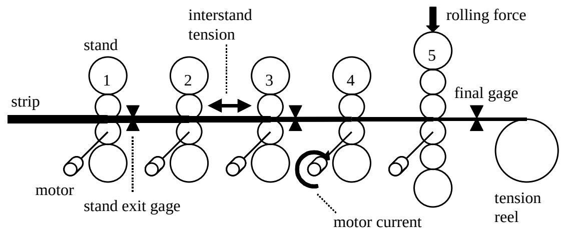

Fig. 1. Tandem cold mill

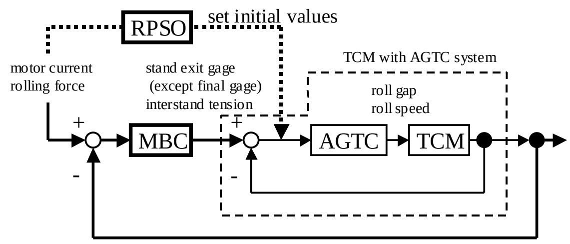

Fig. 2. Pass schedule optimization which consists of robust pass schedule optimization and mill balance control

operation and good strip quality.
Here, a type of automatic gage control system with interstand tension control, which has recently become a trend, is assumed, and it is referred to as automatic gage and tension control (AGTC) hereafter.

Figure 2 shows the outline of pass schedule optimization. First, prior to the rolling, the RPSO calculates each pass schedule for its corresponding group of strips and sets them as initial values of stand exit gages and interstand tensions. Then, during the constant-speed rolling, the MBC operates the stand exit gages and interstand tensions to regulate rolling forces and motor currents. The MBC is constructed as a cascade control system by adding a new control loop to the AGTC loop.

3. ROBUST PASS SCHEDULE OPTIMIZATION

3.1 Outline of robust pass schedule optimization

Firstly, the RPSO is considered which is the first stage of pass schedule optimization. Usually, the strips to be rolled are classified into groups depending on the initial gages, the final gages, the widths and the hardnesses of the strips, and each group has its own pass schedule in a table. Here, the values of a pass schedule, i.e. the values of reductions and interstand tensions, can be arbitrarily determined because only the values of the initial and the final gages are fixed.

In general, the determination of pass schedules is done in consideration of the uniformity quality and productivity of the rolled strips and safety operation. The reason is that the reductions and interstand tensions affect the rolling forces and motor currents at the stands and these values affect the quality, productivity, and operation safety. For example, inappropriate rolling forces can cause poor shape quality and snaking of the rolled strip, and inappropriate motor currents can cause motor damage and mill operation stoppage in the worst case. Moreover, manual adjustments to change the inappropriate reductions and interstand tensions can increase the off-gage length of the rolled strips. These are the facts that are taken into account when designing a pass schedule.

Conventionally, some pass schedules are designed based on prior good pass schedules which were arrived at empirically by mill operators, and others are designed based on numerical optimization methods (Okado, et al., 1969). However, in recent years, the required quality of the steel strips is becoming higher, demand for productivity is becoming stronger, and safety operation is becoming more important than ever. And, the distributions in strips' properties and rolling conditions are becoming not negligible. Therefore it is becoming more difficult for conventional methods to determine appropriate pass schedules.

Accordingly, a new pass schedule design method has been developed as a robust optimization problem which utilizes a rolling model. The utilization of the model enables not only quantitative evaluation of interactions and tradeoffs of the rolling variables but also quick redesign of pass schedules when rolling conditions such as the mill entry tension are changed. Moreover, the robustness for distributions of strip properties and rolling conditions is taken into consideration since the strips of the same group are not exactly the same in size, hardness and rolling condition. This is achieved by identifying the distribution from past rolling data and introducing reliability based constraints (Thanedar, 1991) with the sigmoid function.

In this way, the pass schedules are optimized to minimize the effect of the rolling variables' distributions, and each strip starts to be rolled based on the pass schedule of its group.

3.2 Model

The rolling force and the motor current at No.i stand (hereafter referred to as No.i motor current ditto for other variables) are modelled as follows (Hirano, et al., 1984):

$$P_i = P(H_1, H_i, h_i, q_{fi}, q_{bi}, k_i, \mu_i, w) \quad (i = 1, \cdots, 5) \quad (1)$$

$$G_i = G(H_1, H_i, h_i, q_{fi}, q_{bi}, k_i, \mu_i, w) \quad (i = 1, \cdots, 5) \quad (2)$$

where
P_i : rolling force, G_i : motor current, H_i : stand entry gage, h_i : stand exit gage, q_{fi} : front tension, q_{bi} : back tension, k_i : deformation resistance, μ_i : friction coefficient, w : strip width.

3.3 Optimization using Sequential Quadratic Programming method considering robustness

The performance index and constraints are determined as follows:

$$\text{minimize} \sum_{j=1}^{Nall} J_j(\mathbf{x}) \quad (3)$$

such that

$$Prob(g_k(\mathbf{x}) \geq 0) \equiv \frac{N(g_k(\mathbf{x}) \geq 0)}{N_{all}} \approx \frac{\sum_{j=1}^{Nall} sigmf(g_k(\mathbf{x}), a_k)}{N_{all}} \geq R_k \quad (k = 1, \cdots, M) \quad (4)$$

where
N_{all} : the number of previously-rolled strips of a group under concern, $J_j(\mathbf{x})$: part of performance index corresponding to j-th strip, $\mathbf{x}$: an (8×1) vector of the pass schedule variables, $Prob()$: probability, $g_k(\mathbf{x})$: k-th constraint function, $N()$: the number of strips which satisfy the k-th constraint, $sigmf()$: the sigmoid function, a_k : a parameter of the sigmoid function, R_k : an approximate reliability, M : the number of constraints.

Here, since the constraint functions must be differentiable so that the problem is solvable by the Sequential Quadratic Programming (SQP) method (Ibaraki, et al., 1991), the sigmoid function is used. Because of this, a reliability R_k is treated as an approximate value and the optimization problem is solved in an approximate manner.

The details of the variables and functions are determined mostly from the operators' empirical knowledge as follows:

$$\mathbf{x} = \left[h_1, h_2, h_3, h_4, q_{f1}, q_{f2}, q_{f3}, q_{f4}\right]^T \tag{5}$$

$$r_i = (H_i - h_i)/H_i \tag{6}$$

$$J_j(\mathbf{x}) = \sum_{i=1}^{5} \alpha_i \times (r_{j,i} - r_{j,di})^2 + \sum_{i=1}^{5} \beta_i \times (p_{j,i} - p_{j,di})^2 + \sum_{j=1}^{4} \gamma_i \times (G_{j,i} - G_{j,i+1})^2 \tag{7}$$

$$g_1(\mathbf{x}) = p_{j,1} - \underline{p_{j,1}} \quad (8), \qquad g_2(\mathbf{x}) = \overline{p_{j,1}} - p_{j,1} \quad (9)$$

$$g_3(\mathbf{x}) = p_{j,1} - \underline{\underline{p_{j,1}}} \quad (10), \qquad g_4(\mathbf{x}) = \overline{\overline{p_{j,1}}} - p_{j,1} \quad (11)$$

$$g_5(\mathbf{x}) = p_{j,5} - \underline{p_{j,5}} \quad (12), \qquad g_6(\mathbf{x}) = \overline{p_{j,5}} - p_{j,5} \quad (13)$$

$$g_{i+6}(\mathbf{x}) = G_{j,i} - \underline{G_{j,i}} \quad (i = 1,\cdots,5) \tag{14}$$

$$g_{12}(\mathbf{x}) = p_{j,1} - p_{j,5} \tag{15}$$

$$g_{i+12}(\mathbf{x}) = \overline{G_{j,i}} - G_{j,i} \quad (i = 1,\cdots,5) \tag{16}$$

$$g_{18}(\mathbf{x}) = r_{j,2} + \Delta r_{23} - r_{j,3} \tag{17}$$

$$g_{19}(\mathbf{x}) = r_{j,3} + \Delta r_{34} - r_{j,4} \tag{18}$$

$$g_{20}(\mathbf{x}) = r_{j,4} + \Delta r_{45} - r_{j,5} \quad (19), \qquad g_{21}(\mathbf{x}) = r_{j,5} - \underline{r_{j,5}} \quad (20)$$

$$sigmf(b,a) = \frac{1}{1 + exp(-ab)} \tag{21}$$

where
r : reduction in gage, p : rolling force per unit width (P/w), $*_{j,i}$: No.i rolling value of j-th strip, $*_{j,di}$: No.i desired rolling value of j-th strip, $\alpha_i, \beta_i, \gamma_i$: weights, $\overline{*}, \underline{*}, \overline{\overline{*}}, \underline{\underline{*}}$: desired upper limits and lower limits, $\Delta r_{i,i+1}$: desired difference between No.i and No.i+1 stand reductions.

Eqs. (3), (7) mean that a new pass schedule is obtained near the desired one. Eqs. (4), (8) - (21) mean that the new pass schedule satisfies the constraints on a probabilistic basis. The approximate reliabilities R_k 's are determined by decreasing them until the optimal solution is obtained.

3.4 Application of robust pass schedule optimization

Figure 3 shows actual rolling data before and after a robust optimal pass schedule is applied to one group of strips. Firstly, subfigures (1-1) and (1-2) show that the root mean square errors (RMSE) of No.1 rolling forces and No.5 rolling forces were decreased from 1.74 to 0.73 and from 0.84 to 0.75 [kN/mm] respectively so that strips can be rolled in good shape without snaking.

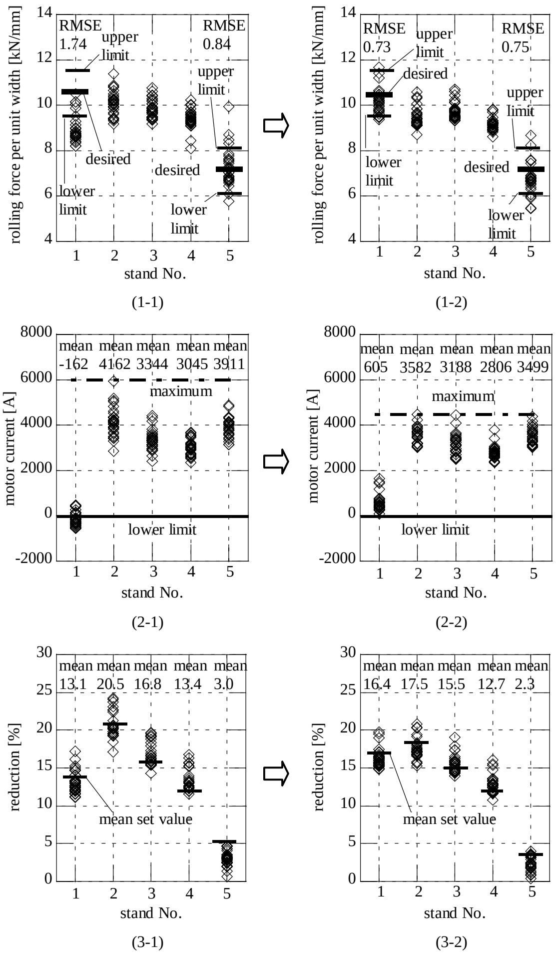

Fig. 3. Rolling data before (*-1) and after (*-2) a robust optimal pass schedule is applied

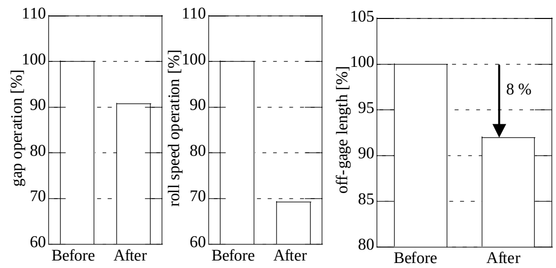

Fig. 4. Effects of robust optimal pass schedules

Here, the thick solid lines indicate desired rolling forces, and the thin solid lines indicate desired upper limits and lower limits. As is shown, the number of strips whose No.1 rolling forces were between the upper and the lower limits increased and the pass schedule is improved. Secondly, subfigures (2-1) and (2-2) show that No.1 currents are increased to be greater than 0 [A] to achieve safe operation. And the No.2, No.3 and No.4 motor currents are changed to be close to each other so that the maximum motor current becomes low and allows a margin to increase the rolling speed. As is shown in subfigures (3-1) and (3-2), the above-mentioned changes are achieved by increasing No.1 reductions and decreasing No.2 and No.5 reductions.

Figure 4 shows that gap operation and roll speed operation by operators and other automatic control compensations have been reduced. As a result, the off-gage length was decreased by 8 [%].

4. MILL BALANCE CONTROL

4.1 Outline of mill balance control

Secondly, the MBC is considered which is the second stage of pass schedule optimization. In principle, if the pass schedule is appropriate and there is no disturbance, changing it is not necessary. However, in reality, there exist the above-mentioned distributions, so rolling forces and motor currents can be inappropriate. Furthermore, there exist disturbances such as hardness variation and friction coefficient variation, so rolling forces and motor currents can change for the worse during the rolling.

Conventionally, an inappropriate pass schedule is modified by operators' manual adjustments or by a simple static single-shot compensation (Okamura, et al., 1999). However, due to the interaction among the rolling variables, it is difficult to determine how to change the pass schedule, i.e. which variables of reductions and interstand tensions to operate and how much to operate them. Moreover, if the pass-schedule change is quick and large, the gage accuracy can be deteriorated.

Accordingly, a new on-line pass schedule changing method to regulate the rolling forces and motor currents is developed as multivariable control. Hereafter, all the rolling forces and motor currents are referred to as "mill balance", and the control of the mill balance by changing a pass schedule is referred to as "mill balance control (MBC)". The control law is designed based on an ILQ design method (Fujii, 1987, 1994), which is a design method of LQ regulators and was developed by reverse application of pertinent results on the inverse regulator problem. An ILQ control law requires no Riccati solutions, and it is obtained in an explicit form unlike a conventional LQ control law. Besides, it has fewer tuning parameters, so the ILQ design method has a big advantage in the practical application of multivariable control in terms of implementation and tuning.

Moreover, ramp feedforward compensation is added in order to change an inappropriate pass schedule to an appropriate one more quickly than the feedback compensation.

In this way, the pass schedules are changed during the rolling to reduce the effect of the remaining distributions and the unpredictable variations. Consequently, strips are rolled maintaining an appropriate mill balance.

4.2 Model of TCM with AGTC system

The MBC controls a plant which consists of the mill and the AGTC system. The model of the plant is derived as simply as possible with a view to applying the MBC. Specifically, firstly, a static model of the plant is derived from the rolling theories, and secondly, a dynamic model of the plant is obtained by combining the static model with the dynamic property of the plant.

The static model is obtained using influence coefficients. Equations of the volumetric velocity, the exit strip speeds, the exit strip gages, the AGTC-controlled variables, the non-controlled variables, the rolling forces and the motor currents are derived based on the rolling theories(Hirano, et al., 1984), and they are linearized around a steady rolling state. As a result, the static model is described as follows:

$$\mathbf{x}_{m\&c} = \mathbf{A}_{u\_to\_x} \cdot \mathbf{u} \tag{22}$$

where

$$\mathbf{x}_{m\&c} = \left[\frac{\Delta P_1}{P_1}, \frac{\Delta P_2}{P_2}, \frac{\Delta P_3}{P_3}, \frac{\Delta P_4}{P_4}, \frac{\Delta P_5}{P_5}, \frac{\Delta G_1}{G_1}, \frac{\Delta G_2}{G_2}, \frac{\Delta G_3}{G_3}, \frac{\Delta G_4}{G_4}, \frac{\Delta G_5}{G_5} \right]^T \tag{23}$$

$$\mathbf{u} = \left[\frac{\Delta h_1}{h_1}, \frac{\Delta h_2}{h_2}, \frac{\Delta h_3}{h_3}, \frac{\Delta h_4}{h_4}, \frac{\Delta q_{f1}}{q_{f1}}, \frac{\Delta q_{f2}}{q_{f2}}, \frac{\Delta q_{f3}}{q_{f3}}, \frac{\Delta q_{f4}}{q_{f4}} \right]^T \tag{24}$$

and $\mathbf{A}_{u\_to\_x}$: influence coefficient matrix, ΔP_i : rolling force deviation, ΔG_i : motor current deviation.

Next, by combining the static model with the dynamic property of the plant, the dynamic model is constructed. Considering the strip travel delay and the AGTC response, the dynamics of the plant are approximated to a first-order lag which corresponds to the slowest dominant response of the plant. $T_{m\&c}$ denotes the time constant of the dominant response. The dynamic model is described by the following state equations:

$$\dot{\mathbf{x}}_{m\&c} = \mathbf{A} \cdot \mathbf{x}_{m\&c} + \mathbf{B} \cdot \mathbf{u} \tag{25}$$

$$\mathbf{y}_{m\&c} = \mathbf{C} \cdot \mathbf{x}_{m\&c} \tag{26}$$

where

$$\mathbf{A} = -\frac{1}{T_{m\&c}} \cdot \mathbf{I} \tag{27}$$

$$\mathbf{B} = \frac{1}{T_{m\&c}} \cdot \mathbf{A}_{u\_to\_x} \tag{28}$$

and $\mathbf{I}$: a unit matrix, $\mathbf{y}_{m\&c}$: an (8×1) output vector, $\mathbf{C}$: an arbitrary (8×10) matrix.

The matrix $\mathbf{C}$ is determined so that the variations of the important rolling variables at the final stand are dispersed to the upstream stands.

$$\mathbf{C} = \begin{bmatrix} 0.2 & -0.2 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.2 & -0.2 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.2 & -0.2 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.2 & -1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.2 & -0.2 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.2 & -0.2 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.2 & -0.2 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.2 & -1.0 \end{bmatrix} \tag{29}$$

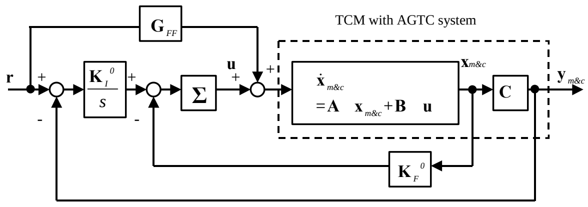

Fig. 5. Block diagram of a mill balance control system based on the ILQ design method

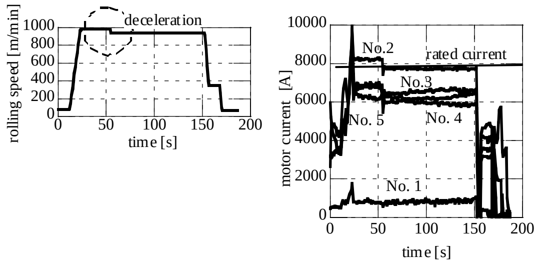

Fig. 6. Deceleration to avoid excess motor current when mill balance control is not applied

4.3 Design of MBC

Feedback control based on the ILQ design method Figure 5 shows the block diagram of the MBC system based on the ILQ design method. $\mathbf{K}_F{}^0$ and $\mathbf{K}_I{}^0$ are gain matrices, and $\mathbf{\Sigma}$ is a diagonal matrix whose elements σ's adjust the norms of $\mathbf{K}_F{}^0$ and $\mathbf{K}_I{}^0$. The σ's are tuning parameters.

For the case where the initial gage is 4.2 mm, the final gage is 1.01 [mm], and the time constant $T_{m\&c}$ is 1.0[s], the coefficient matrix is

$$\mathbf{A}_{u\_to\_x} = \begin{bmatrix} -1.73 & 0.00 & 0.00 & 0.01 & -0.10 & 0.00 & 0.00 & 0.01 \\ 0.50 & -0.99 & 0.00 & 0.01 & -0.21 & -0.11 & 0.00 & 0.00 \\ 0.00 & 0.74 & -1.04 & 0.00 & 0.00 & -0.20 & -0.11 & 0.00 \\ 0.00 & 0.00 & 0.80 & -1.02 & 0.00 & 0.00 & -0.19 & -0.10 \\ 0.00 & 0.00 & 0.00 & 4.18 & 0.00 & 0.00 & 0.00 & -0.24 \\ -9.59 & 0.00 & 0.00 & 0.18 & -1.58 & 0.00 & 0.00 & -0.01 \\ 2.67 & -3.30 & 0.00 & 0.16 & 0.59 & -0.57 & 0.00 & -0.01 \\ 0.00 & 3.19 & -3.67 & 0.16 & 0.00 & 0.60 & -0.59 & -0.01 \\ 0.00 & 0.00 & 3.39 & -3.64 & 0.00 & 0.00 & 0.54 & -0.55 \\ 0.00 & 0.00 & 0.00 & 7.12 & 0.00 & 0.00 & 0.00 & 0.89 \end{bmatrix}. \tag{30}$$

And the gain matrices are obtained in explicit form as follows:

$$\mathbf{K}_F{}^0 = T_{m\&c} \cdot \left[\mathbf{C} \cdot \mathbf{A}_{u\_to\_x}\right]^{-1} \cdot \mathbf{C} \tag{31}$$

$$\mathbf{K}_I{}^0 = T_{m\&c} \cdot \left[\mathbf{C} \cdot \mathbf{A}_{u\_to\_x}\right]^{-1} \cdot \mathbf{Diag}\left(\frac{1}{T_{mbci}}\right) \tag{32}$$

where $\sigma_i \ (i = 1, \cdots, 8)$ and another tuning parameter $T_{mbci} \ (i = 1, \cdots, 8)$ were tuned to be 0.05 and 10.0[s] respectively. These values are determined so that the MBC response is slow and do not deteriorate final gage accuracy.

Addition of feedforward compensation The above-mentioned feedback control of the MBC is suitable for suppression of slowly changing rolling forces and motor currents. However, when the variations of rolling variables such as motor currents exist from the beginning, they should be changed more quickly as long as final gage accuracy is maintained. For this reason, feedforward compensation in ramp form is added, as shown in Fig.5.

$$\mathbf{G}_{FF} = \left[\mathbf{C} \cdot \mathbf{A}_{u\_to\_x}\right]^{-1} \tag{33}$$

4.4 Application of MBC

Motor current regulation The MBC has been applied to an actual process. Without the MBC, as Fig.6 shows, the rolling speed is decreased manually when a motor current exceeds its rated value 8000 [A]. At first, the No.2 motor current is greater than its rated value due to the remaining distributions and during-rolling variations. In order to decrease the No.2 motor current for safety, the rolling speed is reduced by a mill operator.

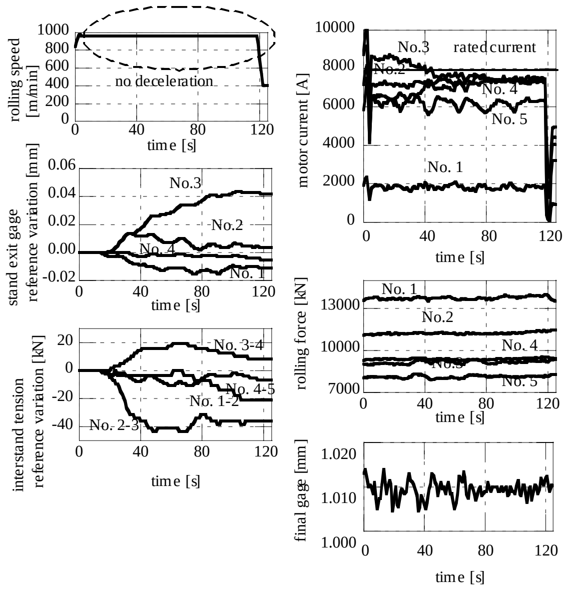

Fig. 7. Motor current regulation by mill balance control

In Fig.7, with the MBC mentioned in 4.3, the desired motor currents of No.2, No.3 and No.4 stands are properly changed automatically into the average of the three motor currents by the MBC as an example. At first, the No.3 motor current is about 8300 [A] and exceeds its rated value. Then the references of the stand exit gages except the final gage and interstand tensions are operated by the MBC. The No.3 stand exit gage reference is increased by almost 40 [μ m] and the No.2-3 interstand tension reference is decreased by almost 40 [kN]. As a result, No.3 motor current is controlled under the rated value.

Furthermore, the rolling forces are kept almost constant. Besides, since the response of the MBC is slower than that of the AGTC so as to avoid interference between the two control loops, the strip gage accuracy under the MBC is maintained, as is shown in Fig.7.

In this way, the MBC compensates the remaining distributions and during-rolling variations, and it is possible to avoid decreasing the rolling speed while maintaining gage uniformity.

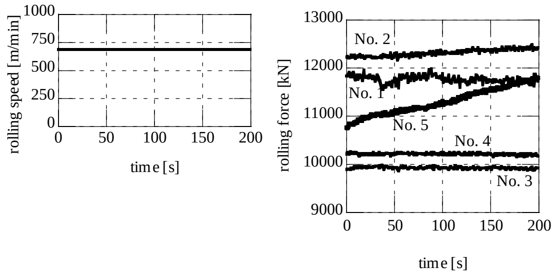

Fig.8. Rolling force variation when mill balance control is not applied

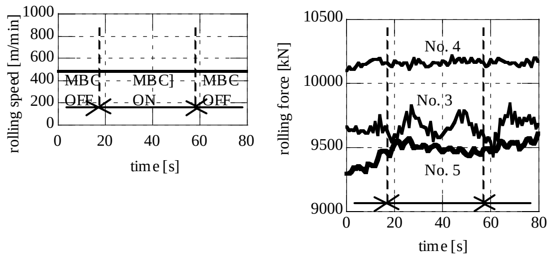

Fig. 9. Rolling force regulation by mill balance control

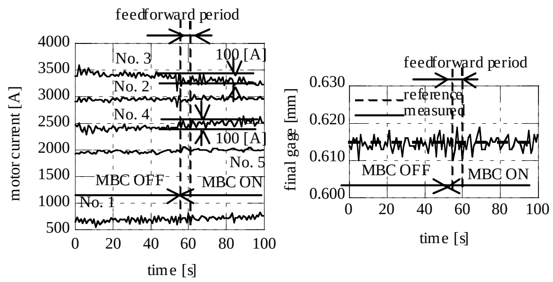

Fig.10 Motor current regulation by mill-balance-control feedforward compensation

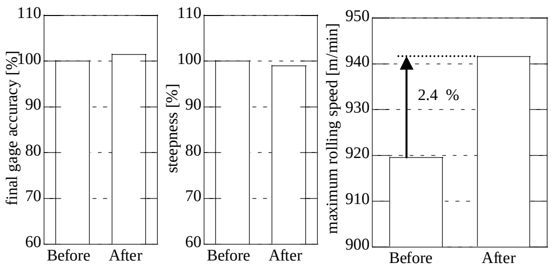

Fig.11 Effects of mill balance control

Rolling force regulation Without the MBC, as Fig.8 shows, there were cases where the No.5 rolling force increased by during-rolling disturbances such as hardness variations. In contrast, with the MBC, Fig.9 shows that the No.5 rolling force stops increasing. The influence of the disturbance is dispersed to all stands, and the rolling forces are maintained almost constant. In the experiments, the steepness of the rolled strip was 0.6[%] and there was no strip shape defect.

In this way, with the MBC, it is possible to obtain strips with desired shape thorough the rolling without manual adjustments.

Feedforward compensation The feedforward compensation is added in ramp form for 5[s] after the MBC begins to work. Figure 10 shows the result where the MBC was turned on in the middle of a constant speed rolling. The No.3 motor current decreased by almost 100 [A], and the No.4 motor current increased almost 100[A] while maintaining the final gage accuracy. When putting the MBC to practical use, the changes of motor currents are limited to 300 [A]. Figure 11 shows that the mean maximum rolling speed increased by 2.4 [%] while maintaining strip quality.

5. CONCLUSIONS

A new pass schedule optimization method, which consists of robust pass schedule optimization and mill balance control, was developed. The pass schedules are robustly optimized prior to the rolling in consideration of distributions, and they are refined during the rolling by the ILQ-design-method based mill balance control. The rolling forces and the motor currents were maintained in optimal condition through the rolling. The results with an actual mill showed an 8% decrease in off gage length and a 2.4% increase in maximum rolling speed without deterioration in gage accuracy.

REFERENCES

Fujii, T. (1987). A New Approach to the LQ Design from the Viewpoint of the Inverse Regulator Problem. *IEEE Transactions on Automatic Control*, **Vol.AC-32**, **No.11**, pp.995-1004.

Fujii, T. (1994). Robustness of Optimal Regulators. In: *Robust Control* (in Japanese), pp.109-156, Corona Publishing, Tokyo.

Hirano, T. et al. (1984). *Theory and Practice of Flat Rolling* (in Japanese), the Iron and Steel Institute of Japan, Tokyo.

Ibaraki, T. and M. Fukushima (1991). *Fortran 77 Optimization Programming* (in Japanese), Iwanami Shoten, Tokyo

Okado, M.and H. Suzuki (1969), Optimising Theory of Pass Reduction Schedules and Some Criterion Functions. (in Japanese) *Journal of the Japan society for technology of plasticity*, **Vol.10**, **No.106**, pp.808-816

Okamura, Y. and I. Hoshino (1999). Robust Gauge Control for Hot Finishing Mills. (in Japanese) *Transactions of the ISCIE*, **Vol.12**, **No.8**, pp.480-488

Thanedar, P.B. and S. Kodiyalam (1991). Structural Optimization Using Probabilistic Constraints. , *Proceedings of AIAA/ASME/ACE/AHS/ASC Structures, Structural Dynamics and Materials Conference*, pp.205-212