DECENTRALIZED CONTROL VIA DISTURBANCE ATTENUATION AND EIGENSTRUCTURE ASSIGNMENT

Batool Labibi

K.N.Toosi University of Technology
Tehran, Iran
labibi@kntu.ac.ir

Abstract: In this paper a simple approach is proposed for decentralized control of linear large-scale systems. Sufficient conditions for diagonal dominance of closed-loop large-scale systems are derived. Based on these conditions, the interactions between the subsystems can be considered as external disturbances for each isolated subsystem. Then a previously proposed approach is used to attenuate disturbances via dynamic output compensators based on complete parametric eigenstructure assignment. Through attenuation of the disturbances, the closed-loop poles of the overall system are assigned to the desirable region, by assigning the eigenstructure of each isolated subsystem appropriately. An example is given to show the effectiveness of the proposed method.

Keywords: Decentralised control, Diagonal dominance, Disturbance rejection, Eigenstructure assignment.

1. INTRODUCTION

Decentralised control with Nyquist-like methods can be very effective, if one can obtain the required degree of diagonal dominance fairly easily (Nwokah, *et al.*, 1993). In (Labibi, *et al*., 2003), it is shown that in order to achieve the proper conditions for overall closed-loop diagonal dominance, the interactions between the subsystems are taken as external disturbances for each isolated subsystem. Then it is tried to attenuate the effect of the disturbances by solving properly defined local H_∞ problems. In this paper, the interactions between the subsystems are taken as external disturbances for each isolated subsystem like the proposed method in (Labibi, *et al*., 2003). But, in order to attenuate the disturbances, the method proposed in (Duan, *et al.*, 2000) is used. The proposed methodology can be applied to non-square, non-minimum phase and open-loop unstable systems, which will thereby guarantee a closed-loop diagonally dominant system and this is achieved by using a decentralized controller.

This paper is organized as follows: In section 2, the problem of finding suitable decentralized controllers for the subsystems of a linear large-scale system is presented. In section 3, the eignstructure assignment methodology proposed in (Duan, *et al.*, 2000). is examined. In section 4 the new method for decentralized control of large scale systems is proposed, and it is shown that by assigning the closed-loop eigenstructure of each isolated subsystem appropriately, the interactions between the subsystems are attenuated, while the closed loop poles of the system are tried to be assigned to the desirable region, through proper assignment of eigenstructiure of the isolated subsystems. In section

5 an example is given to show the effectiveness of the proposed method.

2. PROBLEM FORMULATION

Consider a large-scale system $G(s)$, with the following state-space equations

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where $x \in R^n$, $u \in R^m$, $y \in R^l$, $A \in R^{n\times n}$, $B \in R^{n\times m}$, and $C \in R^{l\times n}$, composed of N linear time-invariant subsystems $G_i(s)$, described by

$$\begin{aligned} \dot{x}_i &= A_{ii}x_i + B_{ii}u_i + \sum_{\substack{j=1 \\ j\neq i}}^{N} A_{ij}x_j \\ y_i &= C_{ii}x_i \end{aligned} \tag{2}$$

where
$x_i \in R^{n_i}$, $u_i \in R^{m_i}$, $y_i \in R^{l_i}$, $A_{ii} \in R^{n_i\times n_i}$, $B_{ii} \in R^{n_i\times m_i}$, $C_{ii} \in R^{l_i\times n_i}$, $\sum_{i=1}^{N} n_i = n$, $\sum_{i=1}^{N} m_i = m$, and $\sum_{i=1}^{N} l_i = l$.

It is assumed that all (A_{ii}, B_{ii}) and (A_{ii}, C_{ii}) are controllable and are observable respectively and all of B_{ii}'s and C_{ii}'s are full rank. The term $\sum_{\substack{j=1 \\ j\neq i}}^{N} A_{ij}x_j$ is associated to the interactions of the other subsystems.

The objective of this paper is to design a local output feedback dynamic controller

$$U_i(s) = K_i(s)(Y_i(s) - R_i(s)) \tag{3}$$

for each isolated subsystem

$$G_{di} = \begin{cases} \dot{x}_i(t) = A_{ii}x_i(t) + B_{ii}u_i(t) \\ y_i(t) = C_{ii}x_i(t) \end{cases} \tag{4}$$

where R_i is the i-th reference input vector, such that the eigenstructure of the subsystem is assigned appropriately. Therefore, the decentralised controller

$$K(s) = \quad diag\{K_i(s)\} \tag{5}$$

assigns the overall closed loop poles in the desirable region, if some sufficient conditions are satisfied.

3. EIGENSTRUCTURE ASSIGNMENT

In this section the method for eigenstructure assignment proposed in (Duan, *et al.*, 2000) is investigated.

Consider the isolated i-th subsystem given by equations (4). Let (A_{ii}, B_{ii}) be controllable and (A_{ii}, C_{ii}) observable and the matrices B_{ii} and C_{ii} are full rank. A general output dynamic compensator, of order p_i, for the subsystem can be written in the following form

$$\begin{aligned} \dot{z}_i(t) &= K_{i22}z_i(t) + K_{i21}(r_i + y_i) \\ u_i(t) &= K_{i12}z_i(t) + K_{i11}(r_i + y_i) \end{aligned} \tag{6}$$

where $z_i \in R^{p_i}$ is the compensator state vector and K_{ijl}, $j, l = 1,2$ are four controller coefficient matrices of appropriate dimensions. Applying the dynamic compensator to the i-th subsystem, gives following closed-loop system

$$\begin{aligned} \dot{\zeta}_i(t) &= A_{ci}\zeta_i(t) + M_i r_i(t) \\ y_i(t) &= C_{ci}\zeta_i(t) \end{aligned} \tag{7}$$

where $\zeta_i^T = \begin{bmatrix} x_i^T & z_i^T \end{bmatrix}^T$, $\quad M_i = \begin{bmatrix} -B_{ii}K_{i11} \\ -K_{i21} \end{bmatrix}$,

$A_{ci} = \begin{bmatrix} A_{ii} + B_{ii}K_{i11}C_{ii} & B_{ii}K_{i12} \\ K_{i21}C_{ii} & K_{i22} \end{bmatrix}$, and $C_{ci} = \begin{bmatrix} C_{ii} & 0 \end{bmatrix}$.

Since, eigenvalues of a non-defective matrix are less sensitive to parameter perturbations in the matrix, the closed loop system matrix, A_{ci} is assumed to be non-defective, where its Jordan form is a diagonal matrix.

Subsystem (4) is controllable and observable and the matrices B_{ii} and C_{ii} are full rank, therefore there hold the following right co-prime factorisations

$$(sI - A_{ii})^{-1}B_{ii} = N_i(s)D_i^{-1}(s) \tag{8}$$

$$(sI - A_{ii}^T)^{-1}C_{ii}^T = H_i(s)L_i^{-1}(s) \tag{9}$$

where $N_i(s) \in R^{n_i\times m_i}$, $D_i(s) \in R^{m_i\times m_i}$, $H_i(s) \in R^{n_i\times l_i}$ and $L_i(s) \in R^{l_i\times l_i}$ are all polynomial matrices, and $N_i(s)$ and $D_i(s)$, $H_i(s)$ and $L_i(s)$ are both right co-prime.

Lemma 3.1 (see (Duan, *et al.*, 2000) for proof). For the i-th isolated subsystem, let (A_{ii}, B_{ii}) be controllable and (A_{ii}, C_{ii}) be observable, and

s_{ij}, $j=1,...,n_i+p_i$ be a group of self-conjugate complex numbers, then it follows that:
a) matrices K_{ijl} $j,l=1,2$, T_i, and $V_i \in R^{(n_i+p_i)\times(n_i+p_i)}$ do exist, such that

$$A_{ci}=\begin{bmatrix} A_{ii}+B_{ii}K_{i11}C_{ii} & B_{ii}K_{i12} \\ K_{i21}C_{ii} & K_{i22} \end{bmatrix}=V_i\Lambda_i T_i^T \quad (10)$$

$$T_i^T V_i = I \quad (11)$$

$$\Lambda_i = diag\{s_{ij}\} \qquad j=1,2,...,n_i+p_i \quad (12)$$

hold for a set of self conjugate complex numbers s_{ij} $j=1,2,...,n_i+p_i$, if and only if there were vectors $f_{ijl}\in C^{m_i}$ and $g_{ijl}\in C^{l_i}$ $j=0,1,\ l=1,2,...,n_i+p_i$, satisfying below equations:

$$C1:\ f_{ijl}=\bar{f}_{ijk} \quad g_{ijl}=\bar{g}_{ijk} \quad if \quad s_j=\bar{s}_k$$
$$j=0,1 \quad l,k=1,..,n_i+p_i$$
$$C2:\ f_{i0j}N_i^T(s_{ij})H_i(s_{il})g_{i0l}+f_{i1j}g_{i1l}=\delta_{jl}$$
$$j,l=1,2,...,n_i+p_i$$

where δ_{jl} is the Kronecker function.
b) when constraints C1 and C2 are met, the matrix V_i is given by

$$V=\begin{bmatrix} V_{i0} \\ V_{i1} \end{bmatrix} \quad (13)$$

where
$V_{i0}=\left[N_i(s_{i1})f_{i01} \quad N_i(s_{i2})f_{i02} \quad \cdots \quad N_i(s_{i(n_i+p_i)})f_{i0(n_i+p_i)}\right]$, $V_{i1}=\left[f_{i11} \quad f_{i12} \quad \cdots \quad f_{i1(n_i+p_i)}\right]$

and the matrix T_i is given by

$$T_i=\begin{bmatrix} T_{i0} \\ T_{i1} \end{bmatrix} \quad (14)$$

with
$T_{i0}=$
$\left[H_i(s_{i1})g_{i01} \quad H_i(s_{i2})g_{i02} \quad \cdots \quad H_i(s_{i(n_i+p_i)})g_{i0(n_i+p_i)}\right]$
$T_{i1}=\left[g_{i11} \quad g_{i12} \quad \cdots \quad g_{i1(n_i+p_i)}\right]$

the corresponding matrices K_{ijl} are either given by

$$K_{i11}=W_{i0}\Phi_i\,, \qquad K_{i12}=W_{i0}\Psi_i-K_{i11}C_{ii}V_{i0}\Psi_i\,,$$
$$K_{i21}=W_{i1}\Phi_i\,,\ K_{i22}=W_{i1}\Psi_i-K_{i21}C_{ii}V_{i0}\Psi_i \quad (15)$$

with
$W_{i0}=$
$\left[D_i(s_{i1})f_{i01} \quad D_i(s_{i2})f_{i02} \quad \cdots \quad D_i(s_{i(n_i+p_i)})f_{i0(n_i+p_i)}\right]$
, $W_{i1}=\left[s_{i1}f_{i11} \quad s_{i2}f_{i12} \quad \cdots \quad s_{i(n_i+p_i)}f_{i1(n_i+p_i)}\right]$

$$\Psi_i=V_{i1}^T(V_{i1}V_{i1}^T)^{-1}, \qquad \Phi_i=\Gamma_i(C_{ii}V_{i0}\Gamma_i)^{-1},$$
$$\Gamma_i=(I-\Psi_i V_{i1})(C_{ii}V_{i0})^T$$

or by

$$K_{i11}=\hat{\Phi}_i Z_{i0}^T\,, \qquad K_{i21}=\hat{\Psi}_i(Z_{i0}^T-B_{ii}^T T_{i0}K_{i11})\,,$$
$$K_{i12}=\hat{\Phi}_i Z_{i1}^T\,,\ K_{i22}=\hat{\Psi}_i(Z_{i1}^T-B_{ii}^T T_{i0}K_{i21}) \quad (16)$$

with
$Z_{i0}=$
$\left[L_i(s_{i1})g_{i01} \quad L_i(s_{i2})g_{i02} \quad \cdots \quad L_i(s_{i(n_i+p_i)})g_{i0(n_i+p_i)}\right]$,
$Z_{i1}=\left[s_{i1}g_{i11} \quad s_{i2}g_{i12} \quad \cdots \quad s_{i(n_i+p_i)}g_{i1(n_i+p_i)}\right]$,

$$\hat{\Psi}_i=(T_{i1}T_{i1}^T)^{-1}T_{i1}\,, \qquad \hat{\Phi}_i=(\hat{\Gamma}_i T_{i0}^T B_{ii})^{-1}\hat{\Gamma}_i\,,$$
$$\hat{\Gamma}_i=B_{ii}^T T_{i0}(I-T_{i1}\hat{\Psi}_i)$$

The parameters f_{ijl} and g_{ijl} $j=0,1;\ l=1,2,...,n_i+p_i$ represent the degrees of freedom available in the compensator design for the i-th isolated subsystem.

4. DECENTRALISED CONTROL VIA DISTURBANCE ATTENUATION

Consider the i-th subsystem given by equations (4). In general the controller designed for each isolated subsystem is a dynamic controller. Assuming the i-th controller $K_i(s)$ has the state-space equations given by (6). It is simple to show that designing dynamic output feedback controller for the subsystem can be reduced to designing a static controller for the augmented subsystem with the state space equations given by (7).

Applying the designed controller to the i-th system, the closed-loop subsystem has the following equation

$$\begin{bmatrix}0 & \cdots & C_{ci} & \cdots & 0\end{bmatrix}\zeta(s)-C_{ci}P_iH_i\zeta(s)=C_{ci}P_iM_iR_i \quad (17)$$
$$Y_i(s)=C_{ci}P_iH_i\zeta(s)+C_{ci}P_iM_iR_i \quad i=1,...,N \quad (18)$$

where

$$H_i=\begin{bmatrix} A_{i1} & 0 & \cdots & A_{ii-1} & 0 & 0 & 0 & A_{ii+1} & 0 & \cdots & A_{iN} & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

$\zeta^T=\left[\zeta_1^T \quad \cdots \quad \zeta_{i-1}^T \quad \zeta_i^T \quad \zeta_{i+1}^T \quad \cdots \quad \zeta_N^T\right]^T$, and $P_i=(sI-A_{ci})^{-1}$.

From equation (17), it is clear if $\|C_{ci}P_iH_i\| < \alpha_i\|C_{ci}\| \quad i = 1,...,N$, where $\alpha_i < 1$ is a positive scalar, then

$$Y_i(s) = C_{ci}\zeta_i(s) \cong C_{ci}P_iM_iR_i \quad i = 1,...,N \tag{19}$$

This means that $[0 \ \cdots \ 0 \ C_{ci} \ 0 \ \cdots \ 0]$ is an approximation to $[0 \ \cdots \ 0 \ C_{ci} \ 0 \ \cdots \ 0] - C_{ci}P_iH_i$. The residual of $[0 \ \cdots \ 0 \ C_{ci} \ 0 \ \cdots \ 0]$ is the matrix $C_{ci}P_iH_i$. The relative error in $[0 \ \cdots \ 0 \ C_{ci} \ 0 \ \cdots \ 0] - C_{ci}P_iH_i$ is the number $\frac{\|C_{ci}P_iH_i\|}{\|C_{ci}\|} < \alpha_i$ (Stewart, 1973).

Therefore by having small values for $\alpha_i \quad i = 1,...,N$, the overall closed loop system is diagonal dominant.

Considering equations (18), it can be seen that minimizing the term $C_{ci}P_iH_i$, minimizes the interactions between the subsystems. It means, the states of the other subsystems may be considered as external disturbance for each isolated subsystem. Thus, to attenuate the effects of other subsystems on the i-th output, the following index can be minimized.

$$J_i = \sum_{\substack{j=1 \\ j \neq i}}^{N} \left\| C_{ci}(sI - A_{ci})^{-1}H_{ij} \right\|_2^2 \tag{20}$$

where $H_{ij} = \begin{bmatrix} A_{ij} \\ 0 \end{bmatrix}$.

Since the i-th isolated subsystem is stabilized, the following Lyapunov matrix equation

$$A_{ci}P_i + P_iA_{ci}^T = -H_{ij}H_{ij}^T \tag{21}$$

has a unique solution with respect to P_i, and that this solution is also symmetric semi-positive definite. Further, it follows that the following equation holds:

$$\left\| C_{ci}(sI - A_{ci})^{-1}H_{ij} \right\|_2^2 = C_{ci}P_iC_{ci}^T \tag{22}$$

(Duan, *et al.* 2000). Using equations (10)-(11), and (21)

$$(V_i\Lambda_iV_i^{-1})P_i + P_i(V_i\Lambda_iV_i^{-1})^T = -H_{ij}H_{ij}^T \tag{23}$$

Assuming

$$Q_i = V_i^{-1}P_iV_i^{-T} \tag{24}$$

or

$$P_i = V_iQ_iV_i^T \tag{25}$$

where the matrix Q_i is also symmetric semi-positive definite,

$$\left\| C_{ci}(sI - A_{ci})^{-1}H_{ij} \right\|_2^2 = C_{ci}V_iQ_iV_i^TC_{ci}^T \tag{26}$$

and

$$\Lambda_iQ_i + Q_i\Lambda_i = -T_i^TH_{ij}H_{ij}^TT_i \tag{27}$$

Considering the structure of C_{ci}, equation (20) can be expressed as

$$\left\| C_{ci}(sI - A_{ci})^{-1}H_{ij} \right\|_2^2 = C_{ii}V_{i0}Q_iV_{i0}^TC_{ii}^T \tag{28}$$

Denoting $Q_i = [\, q_{ijl} \,]_{(n_i+p_i)\times(n_i+p_i)}$ equation (24) can then be decomposed as

$$(s_{ij} + s_{il})q_{ijl} = -t_{ij}^TH_{ij}H_{il}^Tt_{il} \quad j,l = 1,2,...n_i + p_i \tag{29}$$

which gives

$$q_{ijl} = \frac{-t_{ij}^TH_{ij}H_{il}^Tt_{il}}{(s_{ij} + s_{il})} \quad j,l = 1,2,...,n_i + p_i \quad j \neq l \tag{30}$$

An algorithm example to solve a decentralized control problem is given below:

4.1 Algorithm:

(a) Select $p_i = 0$ degree of controller for the i-th subsystem.
b) For the i-th subsystem, solve for the polynomial matrices $N_i(s)$, $H_i(s)$, $D_i(s)$, $L_i(s)$, satisfying the right co-prime factorisation equations (8) and (9).
(c) Solve for the expression of constraint C1, C2 and the parametric expression for matrix V_{i0} according to the equations (13).
(d) Solve for the expression of index J_i according to the equations (28), and (30) .
(e) Specify the desired closed-loop eigenvalue location regions, according to the closed-loop stability and performance requirements.
(f) Solve the optimization problem
$\min J_i$
s.t constraints C1, C2, hold and $s_{ij} \quad j = 1,...,n_i + p_i$
belong to the desired region in left of the complex plane.

with some numerical optimization algorithm. If α_i is small enough, go to step g), otherwise $p_i = p_i + 1$ and go to step d).

(g) Solve K_{ijl} according to equations (15) or (16).

Remark - In realistic large-scale systems, the interactions of the subsystems are usually not known. In this case, conditions and cost functions can satisfactorily be modified as

$$\|C_{ci}P_i\| < \frac{\|C_{ci}\|}{\|H_i\|} \quad i = 1,...,N \tag{31}$$

and

$$J_i = \sum_{j=1}^{N} \left\| C_{ci}(sI - A_{ci})^{-1} e_j \right\|_2^2 \tag{32}$$

where e_j is the j-th column of the identity matrix with appropriate dimensions.

5. An EXAMPLE

Consider a system whose dynamics are described by:

$$A = \begin{bmatrix} -2 & 1 & 1 & 1 \\ 3 & 0 & 0 & 2 \\ -1 & 0 & -2 & -3 \\ -2 & -1 & 2 & -1 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix},$$
$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

(Veillette, *et al.* 1992). The system is unstable and highly interacted. Assuming the desirable dynamic characteristic is a minimum decay rate $\alpha = -1$. The system is consisted of two isolated subsystems

$$A_{11} = \begin{bmatrix} -2 & 1 \\ 3 & 0 \end{bmatrix}, B_{11} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C_{11} = \begin{bmatrix} 1 & 0 \end{bmatrix}, D_{11} = 0$$
$$A_{22} = \begin{bmatrix} -2 & -3 \\ 2 & -1 \end{bmatrix}, B_{22} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C_{22} = \begin{bmatrix} 1 & 0 \end{bmatrix}, D_{22} = 0 \ .$$

$A_{12} = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$, $A_{21} = \begin{bmatrix} -1 & 0 \\ -2 & -1 \end{bmatrix}$ are associated to the interactions of the other subsystems. For this system, $\|C_{ii}\|_2 = 1, i = 1,2$. Minimizing the appropriately defined cost functions for the isolated subsystems, $J_1 = 0.0047$, $J_2 = 0.0469$ are achieved. Hence, $\alpha_{1,2}$ are small enough and the overall closed loop system is diagonal dominant. The designed decentralized controller has the following state space matrices.

$$A_k = \begin{bmatrix} -39.0873 & 0 \\ 0 & -3.9644 \end{bmatrix}, B_k = \begin{bmatrix} 146.4679 & 0 \\ 0 & -7.4262 \end{bmatrix},$$
$$C_k = \begin{bmatrix} 101.5331 & 0 \\ 0 & 0.0823 \end{bmatrix}, D_k = \begin{bmatrix} -420.4589 & 0 \\ 0 & -1.6864 \end{bmatrix}$$

This controller assigns the closed loop poles of the overall system at

$$e = \{-1.0083 + 1.6025j, -1.0083 - 1.6025j, -3.8882, -4.3276, -18.9096 + 4.4808j, -18.9096 - 4.4808j\}$$

which are close to eigenvalues of the isolated subsystems given by

$$e1 = \{-19.1298, -17.6827, -4.2748\},$$
$$e2 = \{-3.6072, -1.9846, -1.3726\}.$$

6. CONCLUSION

This paper introduces a new approach for designing a decentralised controller for large-scale systems. Sufficient conditions for diagonal dominance of overall closed loop system are derived. Based on these conditions, the interactions between the subsystems can be considered as external disturbances for each isolated subsystem. Then the proposed approach in (Duan, *et al.*, 2000) is used for disturbance attenuation via dynamic output compensators based on complete parametric eigenstructure assignment. By attenuating the disturbances, the closed loop poles are assigned to the desirable region by assigning the eigenstructure of isolated subsystems appropriately.

REFERENCES

Duan, G. R., G. W. Irwin, and G. P. Liu. (2000). Disturbance attenuation in linear systems via dynamical compensators: A parametric eigenstructure assignment approach. *IEE Proceedings Part D, Control Theory Applications*, 146, 2, 129–136.

Labibi, B., B. Lohmann, A. Khaki Sedigh, and P. Jabedar Maralani. (2003) Output feedback decentralized control of large-scale systems using weighted sensitivity functions minimization Sys. Contr. Letters,. 47, 3, 191-198.

Nwokah, O. D. & Yau, C. H. (1993). Quantitative feedback design of decentralized control systems, Journal of Dynamic Systems, Measurment, and Control, 115, 3, 452-464.

Stewart, G. W. (1973). *Introduction to matrix computations*, Academic Press.

Veillette, R. J., J. V. Medanic, and W. R. Perkins.

(1992). Design of reliable control system. IEEE Transactions on Automatic Control**, 37, 3**, 290-304.