ADAPTIVE CONTROL DESIGN FOR NONSMOOTH SYSTEMS WITH UNCERTAINTY [1]

Takashi Nakakuki * **Tielong Shen** *
Katsutoshi Tamura *

* *Department of Mechanical Engineering, Graduate Division of Science and Technology, Sophia University, 7-1, Kioicho, Chiyoda-ku, Tokyo, 1028554*
{*t-nakaku, tetu-sin, k-tamura*}*@sophia.ac.jp*

Abstract: The paper presents an adaptive controller design for a class of nonsmooth systems with uncertainty. The design method is based on the concept of the Filippov solution since a classical approach can not be applied to establish the stability of the adaptive control system. It is shown by means of a solid nonsmooth analysis that the adaptive control system is globally strongly stable and the state of the controlled system converges to the origin while the uniqueness of the solution of the closed loop system is not necessarily guaranteed. *Copyright©2005 IFAC.*

Keywords: Discontinuities, Discontinuous control, Adaptive control, Stability analysis, Lyapunov stability

1. INTRODUCTION

Control and stability analysis of nonsmooth systems have attracted considerable research interests in both mathematical issues and practical applications. Especially, a characterization of solutions of nonsmooth systems has been proposed by Filippov, and the existence and uniqueness of the solution have been also explored (Filippov, 1964). Based on the concept of the Filippov solution, the existence and uniqueness of the solution of nonsmooth adaptive control systems including switching adaptive schemes have been investigated by Polycarpou *et al.* (Polycarpou and Ioannou, 1993). Recently, the existence of Carathéodory solution of general nonlinear systems with discontinuous controller has been also pursued by Kim *et al.* (Kim and Ha, 2004). These papers imply the importance of the solid analysis of the nonsmooth or discontinuous closed loop system.

On the other hand, controller designs for nonlinear systems with nonsmoothness such as stick-slip motion, backlash, switching control and switching structure of hybrid systems have been considered in Filippov's framework (Paden and Sastry, 1987; Polycarpou and Ioannou, 1993; Orlov *et al.*, 2003; Sekhavat *et al.*, 2004; Alvarez *et al.*, 2000). Since these results require the preknowledge of the upper and lower bounds of nonsmooth function, the challenge to adaptive control is one of future topics. Also a controller design for hybrid systems such as switched piecewise continuous affine systems has been proposed by utilizing a piecewise quadratic Lyapunov functions while the preknowledge of the system matrices is required to solve a linear matrix inequalities (Johansson and Rantzer, 1998; Rantzer and Johansson, 2000). For adaptive control of nons-

[1] This research is supported by the fund of Open Research Center Project from MEXT of Japanese Government (2002-2006).

mooth systems, there is a remarkable book by Tao *et al.* (Tao and Lewis, 2001). In the book various mechanical systems with nonsmoothness are considered whereas the stability analysis for each system is individually made by each specialized methodology.

In this paper, adaptive control of nonsmooth systems with uncertainty is considered. The system, whose uncertainty is linearly parameterized in an unknown vector and a known discontinuous function, is addressed, which is more general and comprehensive system than that of (Tao and Lewis, 2001). For such a nonsmooth system, generally speaking, the classical stability analysis (Krstic *et al.*, 1995) can not be applied due to the presence of the discontinuous function which may create a sliding motion while our approach assures the stability during the sliding motion as well as the non-sliding motion. Besides the stability property, the nonsmooth adaptive system has an interesting nature in comparison to a smooth adaptive system: Although the uniqueness of the Filippov solution of the closed loop system is not necessarily guaranteed, global strong stability and the convergence of the state of the controlled system to the origin is achieved. Examples include a hybrid system with uncertain parameters and a mechanical system with uncertain discontinuity. As an application of our design, adaptive control version of a hybrid system introduced by Branicky (Branicky, 1998) is considered.

2. PROBLEM STATEMENT

Consider the nonlinear system

$$\dot{x} = f(x) + g(x)\big(u + \phi(x)\theta\big) \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, and $f : \mathbb{R}^n \to \mathbb{R}^n$ and $g : \mathbb{R}^n \to \mathbb{R}^{n\times m}$ are locally Lipschitz on $\mathbb{R}^n$ such that $f(0) = 0$ and $g(x) \neq 0$ for all $x \in \mathbb{R}^n$. $\theta \in \mathbb{R}^p$ is an unknown vector, and $\phi : \mathbb{R}^n \to \mathbb{R}^{m\times p}$ is a known discontinuous function satisfying *Condition B* (See Appendix) and is given by

$$\phi(x) = \begin{cases} \phi^+(x) & \text{if } S(x) > 0 \\ \phi^-(x) & \text{if } S(x) < 0 \end{cases} \tag{2}$$

where $\phi^+,\ \phi^- : \mathbb{R}^n \to \mathbb{R}^{m\times p}$ are locally Lipschitz, and $S(x)$ is a smooth function.

The objective of this paper is to show a method of an adaptive controller design for system (1) in despite of the discontinuity ϕ. The adaptive control problem for system (1) with smooth function ϕ has long been studied (Krstic *et al.*, 1995). Notice that even if we employ a smooth control law $u = \alpha(x)$ for system (1), the closed loop system should be described by the differential inclusion form

$$\dot{x} \in K[\mathcal{F}](x) = K[f + g(\alpha + \phi\theta)](x) \tag{3}$$

due to the presence of the discontinuity ϕ, where $K[\cdot]$ is the Filippov set (See Appendix).

Some examples of the system (1) include a hybrid system with unknown parameters (Example 2.1) and a mechanical system with discontinuous physical property (Example 3.1).

Example 2.1. Consider a hybrid system (Branicky, 1998) described by

$$\begin{cases} \dot{x} = A_1 x + u & \text{if } x \in \text{quadrant I or III} \\ \dot{x} = A_2 x + u & \text{if } x \in \text{quadrant II or IV} \end{cases} \tag{4}$$

where stable matrices A_1 and A_2 are given by

$$A_1 = \begin{bmatrix} -1 & 100 \\ -10 & -1 \end{bmatrix},\ A_2 = \begin{bmatrix} -1 & 10 \\ -100 & -1 \end{bmatrix}. \tag{5}$$

Although the system matrices A_1 and A_2 are stable, the response of the free system with $u \equiv 0$ becomes unstable (Branicky, 1998). Hence, to assure the stability of the system a stabilizing control law $u = \alpha(x)$ must be applied. If both A_1 and A_2 are known, we can construct a quadratic Lyapunov function by means of solving a certain type of linear matrix inequalities (Johansson and Rantzer, 1998; Rantzer and Johansson, 2000). On the other hand, in case that A_1 and A_2 are unknown, system (4) can be rewritten as

$$\dot{x} = \phi(x)\theta + u \tag{6}$$

where

$$\phi(x) = \begin{bmatrix} px_1 & px_2 & 0 & 0 & qx_1 & qx_2 & 0 & 0 \\ 0 & 0 & px_1 & px_2 & 0 & 0 & qx_1 & qx_2 \end{bmatrix} \tag{7}$$

$$p = \frac{1 + \mathrm{sgn}(x_1x_2)}{2},\quad q = \frac{1 - \mathrm{sgn}(x_1x_2)}{2} \tag{8}$$

$$\theta^T = [a_1^{(11)}\ a_1^{(12)}\ a_1^{(21)}\ a_1^{(22)}\ a_2^{(11)}\ a_2^{(12)}\ a_2^{(21)}\ a_2^{(22)}]$$
$$a_i^{(jk)} :\ (jk) \text{ element of } A_i. \tag{9}$$

In this case the discontinuous surface is given by $S(x) = x_1x_2 = 0$. ∎

3. ADAPTIVE CONTROLLER DESIGN

In this section, we consider the adaptive controller design for system (1). We now begin to discuss with the following assumption.

Assumption 1. For system $\dot{x} = f(x) + g(x)u$, there exists a Lipschitz continuous control law $u = \alpha_0(x)$ such that the origin $x = 0$ is globally asymptotically stable, that is, $f(0) + g(0)\alpha_0(0) = 0$ and there exists a continuously differentiable function $V_0(x) : \mathbb{R}^n \to \mathbb{R}$ such that

$$V_0(0) = 0 \text{ and } V_0(x) > 0,\ \forall x \neq 0 \tag{10}$$

$$\|x\| \to \infty \Rightarrow V_0(x) \to \infty \tag{11}$$

$$\dot{V}_0(x) = \frac{\partial V_0}{\partial x}\big(f + g\alpha_0\big) \le -W(x) < 0,\ \forall x \ne 0 \tag{12}$$

where $W(x)$ is a continuously differentiable and positive definite function. ∎

Employing an estimation $\hat{\theta}(t)$ and the update law $\Phi(x)$, the control law is given by

$$\begin{aligned} u &= \alpha_0(x) - \phi(x)\hat{\theta} \\ \dot{\hat{\theta}} &= \Phi(x). \end{aligned} \tag{13}$$

Then, the closed loop system is given by

$$\begin{bmatrix} \dot{x} \\ \dot{\tilde{\theta}} \end{bmatrix} = F(x,\tilde{\theta}) = \begin{bmatrix} f + g\alpha_0 + g\phi\theta - g\phi\hat{\theta} \\ -\Phi \end{bmatrix} \tag{14}$$

where $\tilde{\theta} = \theta - \hat{\theta}$ is the parameter error. As a result we have the following differential inclusion [2]

$$\begin{bmatrix} \dot{x} \\ \dot{\tilde{\theta}} \end{bmatrix} \in K[F](x,\tilde{\theta}) \subset \begin{bmatrix} \{f+g\alpha_0\} + K[g\phi\theta] - K[g\phi\hat{\theta}] \\ -K[\Phi] \end{bmatrix}. \tag{15}$$

It is very important to notice that in (15) we can not describe $K[g\phi\theta] - K[g\phi\hat{\theta}]$ as $K[g\phi\tilde{\theta}]$ since ϕ in (1) is different from ϕ in (13) in physical terms. The former ϕ is of a nature of the system while the latter ϕ is of an implementation of an actuator (See (Yakubovich *et al.*, 2004), pp.12).

Thus, we can not proceed the adaptive controller design in a classical manner (e.g. (Krstic *et al.*, 1995)). In order to overcome the difficulty, we split the stability analysis into two regions

$$\begin{aligned} \Omega &= \{(x,\tilde{\theta}) \in \mathbb{R}^{(n+p)} \mid S(x) = 0\} \\ \overline{\Omega} &= \{(x,\tilde{\theta}) \in \mathbb{R}^{(n+p)} \mid S(x) \ne 0\}. \end{aligned} \tag{16}$$

Theorem 3.1. Consider system (1). Let the adaptive control law be given by

$$\begin{aligned} u &= \alpha_0(x) - \phi(x)\hat{\theta} \\ \dot{\hat{\theta}} &= \gamma\left(L_g V_0 \phi(x)\right)^T \end{aligned} \tag{17}$$

where $L_gV_0 = (\partial V_0/\partial x)g(x)$. Then, the closed loop system is globally strongly stable. Moreover for every solution $(x(t),\hat{\theta}(t)) \in \Xi$, where Ξ is a set of solutions starting from $(x(0),\hat{\theta}(0)) = (x_0,\hat{\theta}_0)$, $x(t)$ converges to 0 as t tends to ∞.

proof: The closed loop system is given by

$$\begin{bmatrix} \dot{x} \\ \dot{\tilde{\theta}} \end{bmatrix} \in K[F](x,\tilde{\theta}) \subset \begin{bmatrix} \{f+g\alpha_0\} + K[g\phi\theta] - K[g\phi\hat{\theta}] \\ -K\left[\gamma\left(L_gV_0\phi\right)^T\right] \end{bmatrix}. \tag{18}$$

[2] Throughout the paper, the addition of two sets $A \subseteq \mathbb{R}^n$ and $B \subseteq \mathbb{R}^n$ is defined by $A + B = \{a + b \in \mathbb{R}^n \mid a \in A,\ b \in B\}$.

Let $V : \mathbb{R}^{n+p} \to \mathbb{R}$ be a Lipschitz, regular and positive definite function given by

$$V(x,\tilde{\theta}) = V_0(x) + \frac{1}{2\gamma}\tilde{\theta}^T\tilde{\theta},\quad \gamma > 0. \tag{19}$$

Note that $V(x(t),\tilde{\theta}(t))$ is absolutely continuous in t by Theorem 2.2 in (Shevitz and Parden, 1994). In what follows we will evaluate the behavior of V in two regions Ω and $\overline{\Omega}$, respectively.

(I) The evaluation of $\dot{V}$ in $(x,\tilde{\theta}) \in \overline{\Omega}$

Since the discontinuity of the system occurs on Ω, we treat system (18) as the following usual differential equation

$$\begin{bmatrix} \dot{x} \\ \dot{\tilde{\theta}} \end{bmatrix} = F(x,\tilde{\theta}) = \begin{bmatrix} f + g\alpha_0 + g\phi\tilde{\theta} \\ -\gamma\left(L_gV_0\phi\right)^T \end{bmatrix}. \tag{20}$$

Thus, calculating the time derivative of V along the trajectories of system (20), we obtain

$$\begin{aligned} \dot{V} &= \begin{bmatrix} \frac{\partial V_0}{\partial x} & \frac{1}{\gamma}\tilde{\theta}^T \end{bmatrix} \cdot \begin{bmatrix} f+g\alpha_0+g\phi\tilde{\theta} \\ -\gamma\left(L_gV_0\phi\right)^T \end{bmatrix} \\ &\le -W(x) + L_gV_0\phi\tilde{\theta} - \tilde{\theta}^T\left(L_gV_0\phi\right)^T \\ &= -W(x),\quad \forall (x,\tilde{\theta}) \in \overline{\Omega} \end{aligned} \tag{21}$$

(II) The evaluation of $\dot{V}$ in $(x,\tilde{\theta}) \in \Omega$

In this case we should pay attention to two situations, that is, one is that a trajectory $(x(t),\tilde{\theta}(t))$ intersects the surface Ω with Lebesgue measure 0 with respect to t, and another is that a trajectory $(x(t),\tilde{\theta}(t))$ has a sliding motion on Ω. In the former case the time derivative of V on a point of measure 0 does not change and is ignored to evaluate. Thus, we focus on the latter case here.

$S(x) = 0$ is a smooth surface which divides the state space into two domains $\Omega^+ = \{(x,\tilde{\theta}) \mid S(x) > 0\}$ and $\Omega^- = \{(x,\tilde{\theta}) \mid S(x) < 0\}$. Now let $N(x,\tilde{\theta}) = [n(x)^T\ 0]^T$ be the normal vector to the surface $S(x)$ at a point $(x,\tilde{\theta})$ directed from Ω^- to Ω^+. Let $(x,\tilde{\theta}) \in \Omega$, and $F^-(x,\tilde{\theta})$ and $F^+(x,\tilde{\theta})$ be limiting values where $(x,\tilde{\theta})$ is approached from Ω^- and Ω^+ respectively:

$$\begin{aligned} F^+(x,\tilde{\theta}) &= \begin{bmatrix} f + g\alpha_0 + g\phi^+\tilde{\theta} \\ -\gamma(L_gV_0\phi^+)^T \end{bmatrix} \\ F^+(x,\tilde{\theta}) &= \begin{bmatrix} f + g\alpha_0 + g\phi^-\tilde{\theta} \\ -\gamma(L_gV_0\phi^-)^T \end{bmatrix}. \end{aligned} \tag{22}$$

The projection of $F^-(x,\tilde{\theta})$ and $F^+(x,\tilde{\theta})$ to $N(x,\tilde{\theta})$ is defined as

$$\begin{aligned} F_N^+(x,\tilde{\theta}) &= \langle N(x,\tilde{\theta}),\ F^+(x,\tilde{\theta})\rangle \\ &= n(x)^T(f + g\alpha_0) + n(x)^Tg\phi^+\tilde{\theta} \\ F_N^-(x,\tilde{\theta}) &= \langle N(x,\tilde{\theta}),\ F^-(x,\tilde{\theta})\rangle \\ &= n(x)^T(f + g\alpha_0) + n(x)^Tg\phi^-\tilde{\theta}. \end{aligned} \tag{23}$$

According to Lemma 3 in (Filippov, 1964), a trajectory during a sliding motion is dominated by the following differential equation

$$\begin{bmatrix} \dot{x} \\ \dot{\tilde{\theta}} \end{bmatrix} = \alpha F^{+}(x,\tilde{\theta}) + (1-\alpha)F^{-}(x,\tilde{\theta}), \quad \alpha = \frac{F_N^-}{F_N^- - F_N^+} \tag{24}$$

subject to the constraint $F_N^+ \leq 0$, $F_N^- \geq 0$ and $F_N^- - F_N^+ > 0$. Thus, by substituting (22) and (23) into (24),

$$\begin{aligned} \dot{x} = & \frac{1}{n^T g(\phi^- - \phi^+)\tilde{\theta}} \Big\{ n^T g(\phi^- - \phi^+)\tilde{\theta}(f + g\alpha_0) \\ & - n^T (f + g\alpha_0) g(\phi^- - \phi^+)\tilde{\theta} + n^T g\phi^- \tilde{\theta} g\phi^+ \tilde{\theta} \\ & - n^T g\phi^+ \tilde{\theta} g\phi^- \tilde{\theta} \Big\} \end{aligned} \tag{25}$$

$$\begin{aligned} \dot{\tilde{\theta}} = & \frac{-\gamma}{n^T g(\phi^- - \phi^+)\tilde{\theta}} \cdot \\ & \Big\{ \left(n^T (f + g\alpha_0) + n^T g\phi^- \tilde{\theta} \right) (L_g V_0 \phi^+)^T \\ & - \left(n^T (f + g\alpha_0) + n^T g\phi^+ \tilde{\theta} \right) (L_g V_0 \phi^-)^T \Big\}. \end{aligned} \tag{26}$$

Now we examine the time derivative of V along the trajectories of system (25) and (26) during the sliding motion on Ω. By calculating the time derivative of V,

$$\begin{aligned} \dot{V} &= \begin{bmatrix} \frac{\partial V_0}{\partial x} & \frac{1}{\gamma}\tilde{\theta}^T \end{bmatrix} \cdot \begin{bmatrix} \dot{x} \\ \dot{\tilde{\theta}} \end{bmatrix} \\ &= \frac{1}{n^T g(\phi^- - \phi^+)\tilde{\theta}} \cdot \\ & \quad \frac{\partial V_0}{\partial x} \Big\{ n^T g(\phi^- - \phi^+)\tilde{\theta}(f + g\alpha_0) \\ & \quad - n^T (f + g\alpha_0) g(\phi^- - \phi^+)\tilde{\theta} + n^T g\phi^- \tilde{\theta} g\phi^+ \tilde{\theta} \\ & \quad - n^T g\phi^+ \tilde{\theta} g\phi^- \tilde{\theta} \Big\} \\ & \quad - \frac{1}{n^T g(\phi^- - \phi^+)\tilde{\theta}} \cdot \\ & \quad \tilde{\theta}^T \Big\{ \left(n^T (f + g\alpha_0) + n^T g\phi^- \tilde{\theta} \right) (L_g V_0 \phi^+)^T \\ & \quad - \left(n^T (f + g\alpha_0) + n^T g\phi^+ \tilde{\theta} \right) (L_g V_0 \phi^-)^T \Big\} \\ &= \frac{n^T g(\phi^- - \phi^+)\tilde{\theta}}{n^T g(\phi^- - \phi^+)\tilde{\theta}} \cdot \frac{\partial V_0}{\partial x}(f + g\alpha_0) \\ &= \frac{\partial V_0}{\partial x}(f + g\alpha_0) \\ &\leq -W(x), \quad \forall (x, \tilde{\theta}) \in \Omega \end{aligned} \tag{27}$$

where $n^T g(\phi^- - \phi^+)\tilde{\theta} \neq 0$ for all t on Ω due to the constraint $F_N^- - F_N^+ > 0$.

(III) The stability of the adaptive system

From (21) and (27) we can conclude that for almost all t

$$\dot{V} \leq -W(x) < 0, \quad \text{a.e. } t, \ \forall x \neq 0, \ \forall \tilde{\theta}. \tag{28}$$

which implies that the origin of the closed loop system is globally strongly stable by directly applying Theorem 1 in (Bacciotti and Ceragioli, 1999). In order to show the attractiveness of $x(t)$, consider a trajectory $(x(t), \hat{\theta}(t)) \in \Xi$. From (28), it follows that

$$\sup_{(x,\tilde{\theta}) \in \Xi} V(x(t), \tilde{\theta}(t)) \leq V(x_0, \tilde{\theta}_0) \leq M, \quad \forall t \geq 0 \tag{29}$$

where M is a sufficiently large positive constant. Thus, Ξ is uniformly bounded. Moreover, $\dot{x}(t)$ is also essentially uniformly bounded since $\dot{x} \in K[f + g\alpha_0 + g\phi\theta - g\phi\hat{\theta}](x, \tilde{\theta})$, and f, g, α_0, ϕ^+ and ϕ^- are continuous functions. Thus, Ξ is equicontinuous. From (28) and (29),

$$\int_0^\infty W(x(t))dt \leq M. \tag{30}$$

Since $W(x)$ is continuously differentiable positive definite function, $W(x(t))$ is uniformly continuous. Hence, Barbalat's lemma follows that $W(x(t))$ converges to 0 as $t \to \infty$ which yields that $x(t)$ converges to 0. ∎

Remark 3.1. In Theorem 3.1, the convergence of $x(t)$ to 0 is shown. But, the uniqueness of the solution of the closed loop system is not guaranteed. The following example shows a non-uniqueness of the solution.

Example 3.1. Consider a second order mechanical system with Coulomb friction

$$\dot{x} = Ax + b(u - \theta \text{sgn}(x_2)) \tag{31}$$

where $x = [x_1 \ x_2]^T$, $\theta > 0$ is an unknown parameter, and

$$A = \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad a_0 > 0, \ a_1 > 0. \tag{32}$$

Then, from (17) the adaptive control law is given by

$$\begin{aligned} u &= kx - \hat{\theta}\text{sgn}(x_2) \\ \dot{\hat{\theta}} &= \gamma x^T P b \ \text{sgn}(x_2) \end{aligned} \tag{33}$$

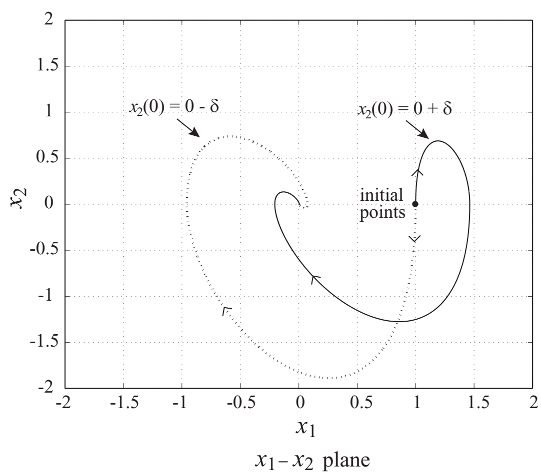

Fig. 1. Simulation result : perturbation $\delta = 10^{-5}$ setting : $\theta = 2$, $-a_0 + k_1 = -2$, $-a_1 + k_2 = -3$ initial condition : $x_1(0) = 1$, $x_2(0) = 0 \pm \delta$, $\hat{\theta}(0) = -8$

where $k = [k_1\ k_2]$ is an adequate state feedback gain, and P is a positive definite solution of Lyapunov equation satisfying

$$P(A+bk)+(A+bk)^T P = -Q, \quad Q = Q^T > 0. \tag{34}$$

Thus, the closed loop system is given by

$$\begin{bmatrix} \dot{x} \\ \dot{\tilde{\theta}} \end{bmatrix} = F(x,\tilde{\theta}) = \begin{bmatrix} A_s x - b\theta\ \mathrm{sgn}(x_2) - b\hat{\theta}\mathrm{sgn}(x_2) \\ \gamma x^T P b\ \mathrm{sgn}(x_2) \end{bmatrix} \tag{35}$$

where $A_s = A + bk$, and $\tilde{\theta} = -\theta - \hat{\theta}$. Since the discontinuous surface is $\Omega = \{(x,\tilde{\theta}) \in \mathbb{R}^3 \mid S(x) = x_2 = 0\}$, the state vector space $\mathbb{R}^3$ is divided into the domains

$$\begin{aligned} \Omega^+ &= \{(x,\tilde{\theta}) \in \mathbb{R}^3 \mid S(x) > 0\} \\ \Omega^- &= \{(x,\tilde{\theta}) \in \mathbb{R}^3 \mid S(x) < 0\}. \end{aligned} \tag{36}$$

Thus, by utilizing Theorem 14 in (Filippov, 1964), we can show that the uniqueness and continuous dependence of the solution starting from an initial condition satisfying $\hat{\theta}(0) < 0$ and $|\hat{\theta}(0)| > \theta + |(-a_0 + k_1)x_1(0)|$ can not be assured.

Fig.1 shows two trajectories starting from some points on the neighborhood of Ω. The trajectories starting from $x_2(0) = 0 \pm \delta$, where $\delta > 0$ is a perturbation, indicate two critically different behaviors, which implies that the uniqueness and continuous dependence of the solution of the closed loop system fail. ∎

4. SIMULATION

Consider the stabilization problem of hybrid system (4) in Example 2.1 again. Now assume that system matrices A_1 and A_2 are unknown.

Since system (4) can be represented into system (6), from Theorem 3.1 the adaptive control law is given by

$$\begin{aligned} u &= Kx - \phi(x)\hat{\theta} \\ \dot{\hat{\theta}} &= \gamma\big(x^T P \phi(x)\big)^T \end{aligned} \tag{37}$$

where

$$K = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, \tag{38}$$

and P is a positive definite solution of Lyapunov equation satisfying

$$PK + K^T P = -Q, \quad Q = Q^T > 0. \tag{39}$$

As shown in Fig.2, the closed loop system is globally strongly stable, and $x(t)$ converges to 0 as t tends to ∞. Notice that $\hat{\theta}$ does not necessarily converge to the nominal values.

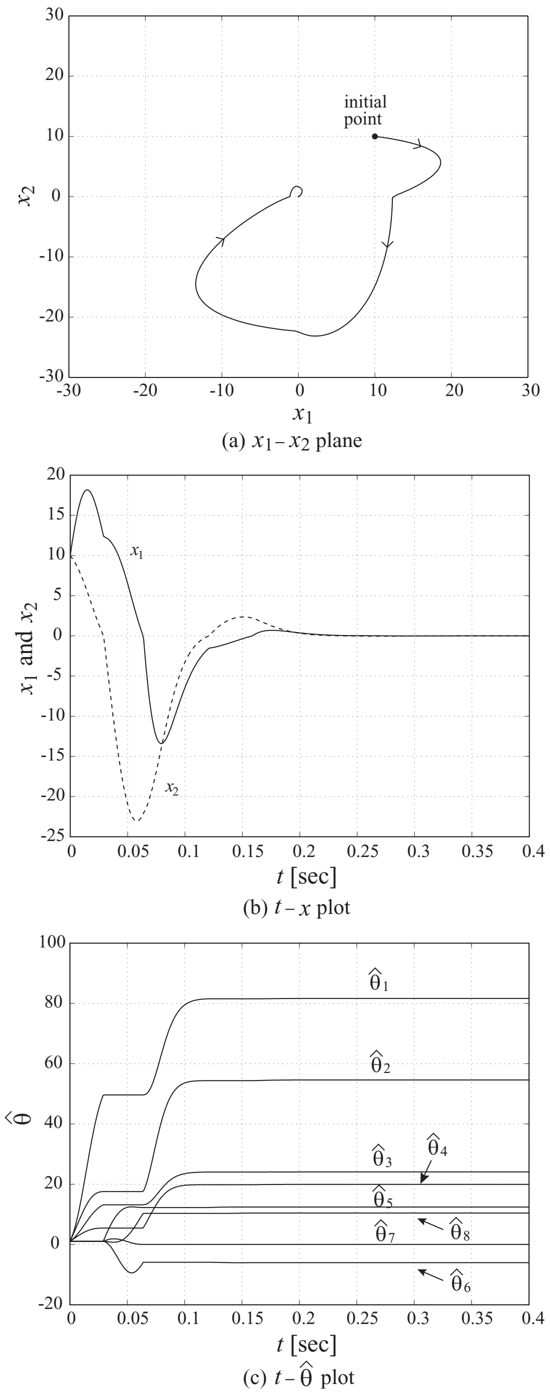

Fig. 2. Simulation result of adaptive control ($\gamma = 5$)

5. CONCLUSIONS

We have presented an adaptive controller design for nonsmooth systems with uncertainty. The characteristics of the adaptive control system is also investigated. It is shown that the closed loop system is globally strongly stable and the state of the controlled system converges to 0 although the uniqueness of the solution is not necessarily guaranteed. We also give an example of nonuniqueness of the solution of the proposed adaptive system. As an application, our controller is successfully applied to a hybrid system with unknown parameters.

REFERENCES

Alvarez, J., I. Orlov and L. Acho (2000). An invariance principle for discontinuous dynamic systems with application to a coulomb friction oscillator. *Journal of Dynamic Systems, Measurement, and Control* **122**, 687–690.

Bacciotti, A. and F. Ceragioli (1999). Stability and stabilization of discontinuous systems and nonsmooth lyapunov functions. *ESAIM Control, Optimisation and Calculus of Variations* **4**, 361–376.

Branicky, M. S. (1998). Multiple lyapunov functions and other analysis tools for switched and hybrid systems. *IEEE Transactions on Automatic Control* **43**, 475/482.

F.Clarke (1983). *Optimization and Nonsmooth Analysis*. Addison-Wesley. New York.

Filippov, A. (1964). Differential equations with discontinuous right-hand side. *American Mathematical Society Translations* **42**, 199–231.

Johansson, M. and A. Rantzer (1998). Computation of piecewise quadratic lyapunov functions for hybrid systems. *IEEE Transaction on Automatic Control* **43**, 555/559.

Kim, S-J. and I-J. Ha (2004). Existence of carathéodory solutions in nonlinear systems with discontinuous switching feedback controllers. *IEEE Transactions on Automatic Control* **49**, 1167–1171.

Krstic, M., I. Kanellakopoulos and P. Kokotovic (1995). *Nonlinear and Adaptive Control Design*. John Wiley & Sons. New York.

Orlov, Y., L. Aguilar and J. Cadiou (2003). Switched chattering control vs. backlash friction phenomena in electrical servo-motors. *International Journal of Control* **76**, 959–967.

Paden, B. and S. S. Sastry (1987). A calculus for computing filippov's differential inclusion with application to the variable structure control of robot manipulators. *IEEE Transactions on Circuits and Systems* **34**, 73/82.

Polycarpou, M. M. and P. A. Ioannou (1993). On the existence and uniqueness of solutions in adaptive control systems. *IEEE Transactions on Automatic Control* **38**, 474–479.

Rantzer, A. and M. Johansson (2000). Piecewise linear quadratic optimal control. *IEEE Trans. on Automatic Control* **45**, 629/637.

Sekhavat, P., Q. Wu and N. Sepehri (2004). Lyapunov-based friction compensation for accurate positioning of a hydraulic actuator. In: *Proceeding of the 2004 American Control Conference*. Boston, June 30- July 23. pp. 418–423.

Shevitz, D. and B. Parden (1994). Lyapunov stability theory of nonsmooth systems. *IEEE Transactions on Automatic Control* **39**, 1910–1914.

Tao, G. and F. L. Lewis (2001). *Adaptive Control of Nonsmooth Dynamic Systems*. Springer-Verlag. London.

Yakubovich, V., G. Leonov and A. Gelig (2004). *Stability of Stationary Sets in Control Systems with Discontinuous Nonlinearities*. World Scientific Pub Co Inc. Singapore.

Appendix A

Consider the vector differential equation

$$\dot{x} = f(x,t), \;\; x(t_0) = x_0 \tag{A.1}$$

where $x \in \mathbb{R}^n$ and $f : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ satisfies the following condition.

Condition B: f is defined almost everywhere and is measurable in an open region $D \subset R^{n+1}$. Further, for every compact set $Q \subset D$, there exists an integrable $B(t)$ such that $\|f(x,t)\| \leq B(t)$ a.e. in Q.

Definition A.1. (Filippov solution (Filippov, 1964)). A vector function $x(t)$ is called a solution of (A.1) on $[t_0, t_1]$ if $x(t)$ is absolutely continuous on $[t_0, t_1]$, and for almost all $t \in [t_0, t_1]$

$$\dot{x} \in K[f](x,t) \tag{A.2}$$

where $K[\cdot]$ is the Filippov set defined by

$$K[f](x,t) = \bigcap_{\delta>0} \bigcap_{\mu N=0} \overline{\text{co}} f(B(x,\delta) - N, t). \tag{A.3}$$

$\bigcap_{\mu N=0}$ denotes the intersection over all sets N of Lebesgue measure zero, $B(x,\delta) = \{y \in R^n | \; \|y - x\| < \delta\}$ and $\overline{\text{co}}$ denotes the convex closure. □

Theorem A.1. (Chain rule (Shevitz and Parden, 1994)). Let $x(t)$ be a Filippov solution of (A.1) on an interval containing t and $V : R^n \to R$ be a Lipschitz and regular function (See (F.Clarke, 1983) for definition).

Then, $V(x(t))$ is absolutely continuous, $(d/dt)V(x(t))$ exists almost everywhere and

$$\frac{d}{dt}V(x(t)) \in^{\text{a.e.}} \dot{\tilde{V}}(x) \tag{A.4}$$

where

$$\dot{\tilde{V}}(x) = \bigcap_{\xi \in \partial V(x(t))} \xi^T K[f](x(t)). \tag{A.5}$$