

TRACK FOLLOWING CONTROLLER FOR OPTICAL DISK DRIVES BASED ON ADAPTIVE OUTPUT REGULATION

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Abstract: This paper presents an adaptive output regulator. It is an add-on type output regulator with frequency adaptation algorithm. A frequency adaptation algorithm based on *internal model principle* is used to identify frequency of sinusoidal disturbance. It is shown that the proposed method is stable for frequency within a connected compact set. Its stability is proven under the assumption that the frequency of disturbance varies slowly in time. The performance of the proposed method is verified with simulation and experiment results. There is a good agreement between simulation and experiment results.
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1. INTRODUCTION

Disturbance rejection is a large and important topic in control theory. Cancellation of sinusoidal disturbance is special interest because they commonly appear in practice (Sacks *et al.*, 1996). In (Francis, 1977), Internal Model Principle (IMP) to reject periodic disturbance was proposed. For perfect disturbance cancellation, Francis *et al.* proposed that controller must have a pair poles of disturbance. Various approaches based on IMP have been studied (Hara *et al.*, 1988; Kempf *et al.*, 1993; Bodson and Douglas, 1997; Knobloch *et al.*, 1993; Francis, 1977; Shim *et al.*, 2004).

Repetitive control has been shown to be very effective for rejecting repetitive disturbance (Hara *et al.*, 1988; Kempf *et al.*, 1993). Its advantages and disadvantages are well summarized in (Kempf *et al.*, 1993) based on four different algorithms used for cancellation of periodic disturbance. Al-

though repetitive control enables perfect rejecting of periodic disturbances by employing the internal model principle with a periodic generator, it requires the exact knowledge of the period-time of the external signals. In literature several solutions have been proposed to resolve this problem, most of them use a supervisory adaptive scheme by estimating period-time from measuring closed-loop response (Tsao and Nemani, 1992; Dötch *et al.*, 1995; Manayathara *et al.*, 1996). In contrast to the literature, recently a new structure for repetitive control is introduced in (Steinbuch, 2002), which is robust for changes in period-time. However, this method requires increased number of memory location. Further it is hard to reject disturbance when the range of uncertain frequency is wide.

Adaptive feedforward cancellation (AFC) based on the phase-locked loop technique is also able to reject periodic disturbance (Bodson and Douglas, 1997). In (Bodson and Douglas, 1997), they proposed advanced method to reject sinusoidal

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disturbances with uncertain frequency. However, to reject sinusoidal disturbance by AFC, it is required to compute the gain of plant at all estimated frequencies. In practice, it is difficult to obtain the gain and thus this method may not be easy to apply if the plants are complicated. Furthermore, this method is not applicable even for a parametric model uncertainty which is acceptable to the output regulator.

In (Knobloch *et al.*, 1993; Francis, 1977), output regulator perfectly rejecting sinusoidal disturbance is proposed. It is shown that the full state feedback and error feedback regulator problems are solvable, under the standard assumptions of stabilizability and detectability, if and only if a pair of regulator equations is solvable. Their proposed method is known to be able to reject disturbance perfectly even under the parametric uncertainty of plant model. Recently applying this idea to the track following problem of optical disk drive, Shim *et al.* proposed an add-on type output regulator assuming that the frequency of disturbance is known (Shim *et al.*, 2004). In practice, the frequency of disturbance is, however, usually unknown and may even vary during operation of system.

In this paper, extending this result we propose an adaptive output regulator to reject periodic disturbance with unknown frequency. The adaptive output regulator consists of add-on type output regulator and adaptive algorithm proposed in (Brown and Zhang, 2001). The adaptive algorithm is based on *internal model principle* (IMP) which can track frequency of sinusoidal disturbance. By adding the adaptive algorithm to add-on output regulator, the output regulator is able to reject disturbance even though its frequency is unknown while preserving the advantages of the output regulator. It is shown that the proposed method is stable for frequency within a connected compact set. Its stability is proven under the assumption that the frequency of disturbance varies slowly in time. Its performance is verified with simulation and experiment results. Finally, we observed that there was a good agreement between simulation and experiment results.

2. ADAPTIVE OUTPUT REGULATOR

We construct an adaptive output regulator for generic linear systems and disturbance written by

$$\begin{aligned} \dot{x} &= Ax + Bu + Pw, \\ e &= Cx + Qw, \end{aligned} \quad (1)$$

$$\dot{w} = S_\sigma w, \quad S_\sigma = \begin{bmatrix} 0 & 1 \\ -\sigma^2 & 0 \end{bmatrix}, \quad \sigma \in \Sigma \subset \mathbb{R}_{>0} \quad (2)$$

where x is the state, u is the control input, w is the periodic disturbance and Σ is a connected compact set. We suppose that the error e can be measured while the state x is not measurable, disturbance frequency σ is unknown and the initial condition $w(0) \in W \subset \mathbb{R}^2$, which is connected and compact.

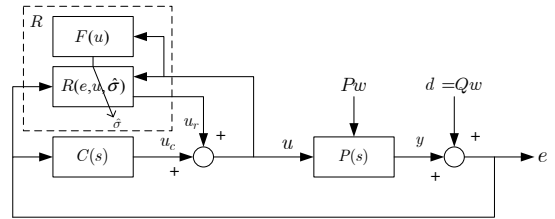


Fig. 1. Schematic of control system. $P(s)$: plant, $C(s)$: pre-designed stabilizing control, R : adaptive output regulator, $F(u)$: adaptive algorithm, $R(e, u, \hat{\sigma})$: output regulator.

Our control goal is to design an error feedback controller so that the closed-loop system in Fig. 1 is asymptotically stable and the error $e(t)$ goes to zero as time goes to infinity. In our approach, the goal of closed-loop stability is achieved by the controller $C(s)$ while the goal of asymptotic disturbance rejection is gained by the adaptive output regulator R . In particular, we propose a design method for R assuming that the controller $C(s)$ is pre-installed and that we do not know any information about $C(s)$ except that it stabilizes the plant $P(s)$ when there's no disturbance.

We adopt the add-on type output regulator proposed in (Shim *et al.*, 2004). It was shown that the controller achieves asymptotic disturbance rejection (i.e., perfect rejection) if the Assumption 1-2 proposed in (Shim *et al.*, 2004) are satisfied. The following assumptions are extension of the assumptions.

Assumption 1. For the plant (1) with $w \equiv 0$, there exists a dynamic controller $C(s)$, whose realization is given by

$$\begin{aligned} \dot{z} &= Fz + Ge, \\ u_c &= Hz + Je \end{aligned} \quad (3)$$

which stabilizes the closed-loop system. In other words, the matrix

$$\begin{bmatrix} A + BJC & BH \\ GC & F \end{bmatrix}$$

is Hurwitz. \diamond

Assumption 2. The following two conditions hold.

- (1) There exist matrices Π_σ and Γ_σ such that

$$\Pi_\sigma S_\sigma = A\Pi_\sigma + B\Gamma_\sigma + P, \quad (4)$$

$$0 = C\Pi_\sigma + Q \quad (5)$$

for all $\sigma \in \Sigma$.

- (2) The matrix pair

$$\left([C \ Q], \begin{bmatrix} A & P \\ 0 & S_\sigma \end{bmatrix} \right)$$

is detectable for all $\sigma \in \Sigma$.

- (3) The pair (A, B) is stabilizable. \diamond

The adaptive output regulator consists of a frequency adaptation algorithm $F(u)$ and output regulator $R(e, u, \hat{\sigma})$ as illustrated in Fig. 1.

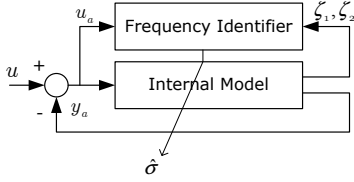


Fig. 2. Adaptive Algorithm $F(u)$

2.1 Design of Output Regulator $R(e, u, \hat{\sigma})$

The output regulator $R(e, u, \hat{\sigma})$ consists of a state observer and a state feedback gain (Shim *et al.*, 2004). Then, the controller can be rewritten by

$$\begin{aligned} \dot{\xi} &= \begin{pmatrix} A - K_1 C & P - K_1 Q \\ -K_2 C & S_{\hat{\sigma}} - K_2 Q \end{pmatrix} \xi + \begin{pmatrix} K_1 \\ K_2 \end{pmatrix} e + \begin{pmatrix} B \\ 0 \end{pmatrix} u, \\ u_r &= (0 \ \Gamma_{\hat{\sigma}}) \xi \end{aligned} \quad (6)$$

where $\hat{\sigma} \in \Sigma$ ($\hat{\sigma}$ is the estimated value of actual disturbance frequency σ) and $\Gamma_{\hat{\sigma}}$ is the solution to (4) and (5) given $S_{\hat{\sigma}}$. Now to design the output regulator (6), we need the following assumption.

Assumption 3. The observer gain K_1 and K_2 in (6) are designed such that

$$\left\{ \begin{bmatrix} A & P \\ 0 & S_{\hat{\sigma}} \end{bmatrix} - \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} [C \ Q] \right\} \quad (7)$$

is Hurwitz for all $\hat{\sigma} \in \Sigma$. \diamond

2.2 Design of Adaptive Algorithm $F(u)$

The adaptive algorithm $F(u)$ in Fig. 1 consists of an internal model for sinusoidal disturbance and a frequency identifier as shown in Fig. 2. We adopt the internal model proposed in (Brown and Zhang, 2001), but we modify it with additional feedback loop using y_a for stability. Then, the internal model with additional feedback loop y_a is

$$\begin{aligned} \begin{bmatrix} \dot{\zeta}_1 \\ \dot{\zeta}_2 \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ -\hat{\sigma}^2 & -K_f \end{bmatrix} \begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix} + \begin{bmatrix} 0 \\ K_f \end{bmatrix} u, \\ y_a &= [0 \ 1] \begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix}. \end{aligned} \quad (8)$$

Remark 1. The feedback loop y_a is new proposed structure in this paper. For the structure, the system matrix in (8)

$$\begin{bmatrix} 0 & 1 \\ -\hat{\sigma}^2 & -K_f \end{bmatrix}$$

is Hurwitz because $\hat{\sigma}$ and K_f is positive, and degree of the system matrix is second order. This feature is very important for stability analysis in section 3. \diamond

Under the Remark 1, our frequency identifier is given by

$$\dot{\hat{\sigma}} = K_e \epsilon = -K_e \frac{\hat{\sigma} K_f \zeta_1 (u - \zeta_2)}{(\hat{\sigma} \zeta_1)^2 + \zeta_2^2} \quad (9)$$

where K_e is a *small* positive constant to be chosen. This identifier is obtained with the following argument (which will be formally justified in the next section). We assume that the exosystem has the frequency σ so that the vector $w(t)$ is sinusoidal with the same frequency. If the closed-loop system is stable, then the input $u(t)$ will converge to its steady-state as $u_{ss}(t) = A_u \sin(\sigma t + \varphi_u)$. Then, by Remark 1 for the system (8), we have

$$\begin{aligned} \zeta_{1,ss}(t) &= A_{\zeta} \sin(\sigma t + \varphi_{\zeta}) \\ \zeta_{2,ss}(t) &= \sigma A_{\zeta} \cos(\sigma t + \varphi_{\zeta}) \end{aligned} \quad (10)$$

where²

$$\begin{aligned} \varphi_u &= \tan^{-1} \left(\frac{K_f \sigma}{\hat{\sigma}^2 - \sigma^2} \right) + \varphi_{\zeta} \\ A_u &= \frac{A_{\zeta} \sqrt{(\hat{\sigma}^2 - \sigma^2)^2 + (\sigma K_f)^2}}{K_f}. \end{aligned}$$

From this, we see that

$$\tan^{-1} \left(\frac{\sigma \zeta_{1,ss}}{\zeta_{2,ss}} \right) = \sigma t + \varphi_{\zeta}. \quad (11)$$

The derivative of (11) with respect to time is

$$\frac{d}{dt} \left(\tan^{-1} \left(\frac{\sigma \zeta_{1,ss}}{\zeta_{2,ss}} \right) \right) = \sigma. \quad (12)$$

Let the error be $\tilde{\sigma} = \hat{\sigma} - \sigma$. Then,

$$\tilde{\sigma} = \hat{\sigma} - \frac{\sigma (\dot{\zeta}_{1,ss} \zeta_{2,ss} - \zeta_{1,ss} \dot{\zeta}_{2,ss})}{(\sigma \zeta_{1,ss})^2 + \zeta_{2,ss}^2}.$$

In the case that $\tilde{\sigma} \neq 0$, we employ the parameter update law as

$$\dot{\tilde{\sigma}} = -K_e \tilde{\sigma} = -K_e \left(\hat{\sigma} - \frac{\sigma (\dot{\zeta}_{1,ss} \zeta_{2,ss} - \zeta_{1,ss} \dot{\zeta}_{2,ss})}{(\sigma \zeta_{1,ss})^2 + \zeta_{2,ss}^2} \right)$$

where $K_e > 0$ is a gain. However, because the value σ is unknown, we replace it by its estimate $\hat{\sigma}$, and then, we obtain by some calculation that

$$\dot{\tilde{\sigma}} = -K_e \frac{\hat{\sigma} K_f \zeta_{1,ss} (u_{ss} - \zeta_{2,ss})}{(\hat{\sigma} \zeta_{1,ss})^2 + \zeta_{2,ss}^2}.$$

Inspired by this form, we will use (9) as our parameter update law whose stability and convergence properties are analyzed in the next section.

3. ANALYSIS ON STABILITY AND CONVERGENCE

In this section, we show the stability and error convergence for the closed-loop system. In fact, the closed-loop system, consisting of the plant (1), the stabilizing controller (3), the output regulator (6), and the frequency identifier (8) and (9), has the structure of singularly perturbed system because the gain K_e is chosen sufficiently small. In other words, the update law (9) of frequency estimate yields the reduced system while the rest

² In this section, the function $\tan^{-1}(y/x)$ should be understood as the arctangent of y/x , using the signs of both arguments x and y to determine the quadrant of the return value.

constitutes the boundary-layer system in the standard singular perturbation theory³.

To show this, we begin by investigating the fast dynamics (i.e., the boundary-layer system) assuming that $\hat{\sigma}$ is frozen (i.e., a constant). With the control input

$$u = u_c + u_r = Hz + Je + (0 \ \Gamma_{\hat{\sigma}}) \xi,$$

the closed-loop consisting of the plant (1), the controller (3), and the output regulator (6) can be written as

$$\begin{aligned} \dot{\tilde{x}} &= (A + BJC)\tilde{x} + BH z + B\Gamma_{\sigma} e_w + B(\Gamma_{\hat{\sigma}} - \Gamma_{\sigma})w, \\ \dot{z} &= GC\tilde{x} + Fz, \\ \dot{e}_x &= (A - K_1C)e_x + (P - K_1Q)e_w, \\ \dot{e}_w &= (-K_2C)e_x + (S_{\hat{\sigma}} - K_2Q)e_w + (S_{\hat{\sigma}} - S_{\sigma})w, \end{aligned} \quad (13)$$

where $\tilde{x} := x - \Pi w$, $e_x := \xi_x - x$ and $e_w := \xi_w - w$ (where $\xi^T = [\xi_x^T, \xi_w^T]$), and the signal w comes from the exosystem

$$\dot{w} = \begin{bmatrix} 0 & 1 \\ -\sigma^2 & 0 \end{bmatrix} w, \quad \sigma \in \Sigma \subset \mathbb{R}_{>0}, \quad w(0) \in W \subset \mathbb{R}^2.$$

It follows that

$$\begin{aligned} w_1(t) &= \frac{A_w}{\sigma} \sin(\sigma t + \varphi_w) \\ w_2(t) &= A_w \cos(\sigma t + \varphi_w), \end{aligned}$$

with certain A_w and φ_w depending on the initial condition $w(0)$. By Assumptions 1 and 3, the matrices

$$\begin{bmatrix} A + BJC & BH \\ GC & F \end{bmatrix}, \quad \begin{bmatrix} A - K_1C & P - K_1Q \\ -K_2C & S_{\hat{\sigma}} - K_2Q \end{bmatrix}$$

are Hurwitz, and thus, the steady-state solutions of $\tilde{x}(t)$, $z(t)$, $e_x(t)$ and $e_w(t)$ in (13) are all sinusoidal with the frequency σ . This, in turn, implies that the steady-state input is also sinusoidal that can be expressed by

$$\begin{aligned} u_{ss}(t) &= Hz_{ss} + Je_{ss} + \Gamma_{\hat{\sigma}} \xi_{w,ss} \\ &= Hz_{ss} + JC\tilde{x}_{ss} + \Gamma_{\hat{\sigma}}(e_{w,ss} + w) \\ &= A_u \sin(\sigma t + \varphi_u) \end{aligned}$$

with certain A_u and φ_u .

On the other hand, a part of the frequency identifier (8) in the closed-loop system is given by

$$\begin{aligned} \dot{\zeta}_1 &= \zeta_2, \\ \dot{\zeta}_2 &= -\hat{\sigma}^2 \zeta_1 - K_f \zeta_2 + K_f u. \end{aligned} \quad (14)$$

Since $u(t) \rightarrow u_{ss}(t)$ and by Remark 1, we obtain the steady-state solution of ζ_1 and ζ_2 as (10), and the solution of (14) converges to its steady-state solution.

In summary, with $X := [\tilde{x}^T, z^T, e_x^T, e_w^T, \zeta_1^T, \zeta_2^T]^T$ and its steady-state signal $X_{ss}(t)$, we define $\tilde{X}(t) = [x_e^T, z_e^T, e_{xe}^T, e_{we}^T, \zeta_{1e}^T, \zeta_{2e}^T]^T := X(t) - X_{ss}(t)$. Then, we have

$$\dot{\tilde{X}} = \bar{g}(\tilde{X}, \hat{\sigma}) \quad (15)$$

where $\bar{g}(0, \hat{\sigma}) = 0$ and $\frac{\partial \bar{g}}{\partial \tilde{X}}(0, \hat{\sigma})$ is Hurwitz for each frozen $\hat{\sigma}$. Indeed, it is easily seen that

$$\begin{aligned} \begin{bmatrix} \dot{x}_e \\ \dot{z}_e \end{bmatrix} &= \begin{bmatrix} A + BJC & BH \\ GC & F \end{bmatrix} \begin{bmatrix} x_e \\ z_e \end{bmatrix} + \begin{bmatrix} B\Gamma_{\sigma} e_{we} \\ 0 \end{bmatrix} \\ \begin{bmatrix} \dot{e}_{xe} \\ \dot{e}_{we} \end{bmatrix} &= \begin{bmatrix} A - K_1C & P - K_1Q \\ -K_2C & S_{\hat{\sigma}} - K_2Q \end{bmatrix} \begin{bmatrix} e_{xe} \\ e_{we} \end{bmatrix} \\ \begin{bmatrix} \dot{\zeta}_{1e} \\ \dot{\zeta}_{2e} \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ -\hat{\sigma}^2 & -K_f \end{bmatrix} \begin{bmatrix} \zeta_{1e} \\ \zeta_{2e} \end{bmatrix} \\ &\quad + \begin{bmatrix} 0 \\ K_f(Hz_e + JCx_e + \Gamma_{\hat{\sigma}} e_{we}) \end{bmatrix}, \end{aligned}$$

which ensures that (15) is globally exponentially stable for every σ and every frozen $\hat{\sigma}$.

Now, consider the rest of the closed-loop system (i.e., the update law (9) for the unknown frequency);

$$\dot{\hat{\sigma}} = -K_e \frac{\hat{\sigma} K_f \zeta_1 (u - \zeta_2)}{(\hat{\sigma} \zeta_1)^2 + \zeta_2^2}. \quad (16)$$

With the state $X(t)$ being its steady-state $X_{ss}(t)$, this becomes

$$\begin{aligned} \dot{\hat{\sigma}} &= -K_e \frac{\hat{\sigma} K_f \zeta_{1,ss} (u_{ss} - \zeta_{2,ss})}{(\hat{\sigma} \zeta_{1,ss})^2 + \zeta_{2,ss}^2} \\ &= -K_e \frac{\hat{\sigma}^2 - \sigma^2}{\hat{\sigma}} \frac{\sin^2 \theta}{\sin^2 \theta + \frac{\sigma^2}{\hat{\sigma}^2} \cos^2 \theta} \end{aligned}$$

where $\theta(t) = \sigma t + \varphi_{\zeta}$. From this and (13), it is seen that $\hat{\sigma}_{ss}(t) = \sigma$ and $X_{ss}(t) = [0, 0, 0, 0, \zeta_{1,ss}(t), \zeta_{2,ss}(t)]^T$ is one of the steady-state solutions of the closed-loop system, which is desired because $e(t) = Cx(t) + Qw(t) = C\tilde{x}(t) = 0$ in the steady-state.

Now consider the system (16) equivalently written, with $\tilde{\sigma} := \hat{\sigma} - \sigma$, as

$$\begin{aligned} \dot{\tilde{\sigma}} &= -K_e \frac{(\tilde{\sigma} + \sigma) K_f (\zeta_{1e} + \zeta_{1,ss})}{(\tilde{\sigma} + \sigma)^2 (\zeta_{1e} + \zeta_{1,ss})^2 + (\zeta_{2e} + \zeta_{2,ss})^2} \\ &\quad \times [H(z_e + z_{ss}) + JC(x_e + \tilde{x}_{ss}) \\ &\quad + \Gamma_{\hat{\sigma}}(e_{we} + e_{w,ss}) + \Gamma_{\hat{\sigma}} w - \zeta_{2e} - \zeta_{2,ss}] \\ &=: K_e f(t, \tilde{X}, \tilde{\sigma}). \end{aligned}$$

Then, by defining a new time scale $\tau := K_e t$ and $\epsilon := K_e$, we have

$$\epsilon \frac{d\tilde{X}}{d\tau} = \bar{g}(\tilde{X}, \tilde{\sigma} + \sigma) =: g(\tilde{X}, \tilde{\sigma}) \quad (17)$$

$$\frac{d\tilde{\sigma}}{d\tau} = f(\tau/\epsilon, \tilde{X}, \tilde{\sigma}), \quad (18)$$

which is in the standard form of singularly perturbed systems.

We have already shown that the origin (i.e., $\tilde{X} = 0$) of the boundary-layer system (17) is globally exponentially stable with a frozen $\tilde{\sigma}$ (because $g(0, \tilde{\sigma}) = 0$ and $\frac{\partial g}{\partial \tilde{X}}(0, \tilde{\sigma})$ is Hurwitz). On the other hand, the reduced system in the slow manifold (i.e., $\tilde{X} = 0$) is described by $\dot{\tilde{\sigma}} = \epsilon f(t, 0, \tilde{\sigma})$ whose origin $\tilde{\sigma} = 0$ can be shown to be exponentially stable with sufficiently small $\epsilon = K_e$ as follows. The system under consideration is given by

³ We refer to (Khalil, 2002) for the singular perturbation theory, but the situation in this paper is more suitable to (Riedle and Kokotovic, 1986).

$$\begin{aligned}\dot{\tilde{\sigma}} &= \epsilon f(t, 0, \tilde{\sigma}) \\ &= -\epsilon \frac{(\tilde{\sigma} + \sigma)^2 - \sigma^2}{\tilde{\sigma} + \sigma} \frac{\sin^2 \theta(t)}{\sin^2 \theta(t) + \frac{\sigma^2}{(\tilde{\sigma} + \sigma)^2} \cos^2 \theta(t)}.\end{aligned}$$

The above system is in the standard form for the averaging theory (Khalil, 2002, Sec. 10.4) since ϵ is small and the system is periodic in t with a period $T = 2\pi/\sigma$. Then, the averaged system is obtained, with $\tilde{\sigma}$ small so that $(\tilde{\sigma} + \sigma)$ is positive,⁴ as

$$\begin{aligned}\dot{\tilde{\sigma}} &:= \frac{\epsilon}{T} \int_0^T f(t, 0, \tilde{\sigma}) dt \\ &= -\frac{\epsilon}{\pi} \frac{(\tilde{\sigma} + \sigma)^2 - \sigma^2}{\tilde{\sigma} + \sigma} \int_0^\pi \frac{\sin^2 \theta}{\sin^2 \theta + \frac{\sigma^2}{(\tilde{\sigma} + \sigma)^2} \cos^2 \theta} d\theta \\ &= -\epsilon \tilde{\sigma}.\end{aligned}$$

Therefore, by (Khalil, 2002, Theorem 10.4, or Example 10.8), we conclude that there exists an $\epsilon_1^* > 0$ such that, for each $\epsilon \in (0, \epsilon_1^*)$, the origin $\tilde{\sigma} = 0$ (i.e., $\hat{\sigma} = \sigma$) of the reduced system (18) is locally exponentially stable.

We finally prove that there exists an $0 < \epsilon^* \leq \epsilon_1^*$ such that, with $K_e = \epsilon \in (0, \epsilon^*)$, the closed-loop system is exponentially stable and any trajectory converges to the desired steady-state solution. (This in turn means that $e(t) = C\tilde{x}(t)$ converges to zero as time goes to infinity.) This is done by employing (Khalil, 2002, Theorem 11.3). Although (Khalil, 2002, Theorem 11.3) only handles the time-invariant case, it is not difficult to follow the proof with our time-varying system (18) under consideration. Note, in particular, that due to the exponential stability of (17) and (18), and their continuous differentiability, the *interconnection condition* discussed in (Khalil, 2002, p. 453) holds, which simplifies the proof.

4. TRACK FOLLOWING CONTROL FOR OPTICAL DISK DRIVE

We applied the designed controller to an optical disk drive (ODD). The overall configuration of the closed loop system is illustrated in Fig. 3. We have obtained the optical disk drive model of LG $\times 52$ CD-ROM drive experimentally using LDV (Laser Doppler Velocimeter), whose plant is

$$P(s) = \frac{818.22}{s^2 + 64.73s + 166800} (m/V). \quad (19)$$

The plant can be represented in state-space by

$$\dot{x} = Ax + Bu = \begin{bmatrix} 0 & 1 \\ a_1 & a_2 \end{bmatrix} x + \begin{bmatrix} 0 \\ b \end{bmatrix} u, \quad (20)$$

$$y = Cx = [1 \ 0] x \quad (21)$$

where u is the force and y is the position. The tracking error of ODD system is $e(t) = y + d$ where y is the plant output and d is the

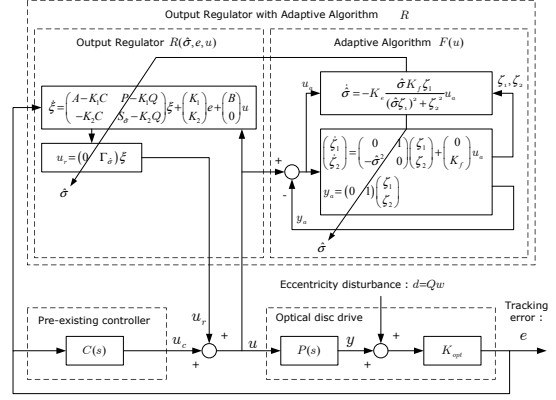


Fig. 3. Actual system configuration with adaptive output regulator

disturbance. The disturbance is in the form of sinusoidal whose frequency is the unknown. Thus, it can be expressed by

$$e = Cx + Qw = [1 \ 0] x + [-1 \ 0] w, \quad (22)$$

$$\dot{w} = S\sigma w = \begin{bmatrix} 0 & 1 \\ -\sigma^2 & 0 \end{bmatrix} w \quad (23)$$

for the control goal that $e(t) = x_1(t) - w_1(t) \rightarrow 0$. Note that the state x and w are not measurable but e and u are measurable. Here, the equations (20) and (22) are now in the form of (1) (with $P = 0$).

Remark 2. There is, in fact, sensor gain K_{opt} in the feedback loop (see Fig. 3). The gain converts the position displacement into voltage. Our experiment shows $K_{opt} \approx 1.25 \times 10^6 V/m$, but we regarded this value as 1 for simple discussion in the above. \diamond

The state feedback gain $\Gamma_{\hat{\sigma}}$ of output regulator $R(e, u, \hat{\sigma})$ in Fig. 3 is obtained in (4) and (5). In this case, we have

$$\Gamma_{\hat{\sigma}} = \frac{1}{b} (a_1 Q + a_2 Q S \hat{\sigma} - Q S \hat{\sigma}^2). \quad (24)$$

We also assume that the $C(s)$ has been designed (Assumption 1). Here, we simply assume that the following lead-lag compensator has been designed:

$$C(s) = -\frac{2.4364s^2 + 17420s + 12558500}{s^2 + 97515s + 7309900}. \quad (25)$$

Remark 3. In our ODD system, disturbance frequency is 68Hz on ID(inner track of disk) and 57Hz on OD(outer track of disk). The observer gain K_1 and K_2 are selected to satisfy assumption 3 for $2\pi \cdot 55 \leq \hat{\sigma} \leq 2\pi \cdot 70$ by *Kharitonov Theorem* (Kharitonov, 1978) as

$$K_1 = \begin{bmatrix} -6479.4 \\ 42932.0 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -7914.6 \\ -302314.7 \end{bmatrix}. \quad (26)$$

A simulation results is shown in Fig. 4. The initial value of $\hat{\sigma}$ is 60Hz. In Fig. 4 (d), the

⁴ We use the fact that

$$\int \frac{\sin^2 \theta}{\sin^2 \theta + a^2 \cos^2 \theta} d\theta = \frac{1}{1-a^2} \left(\theta - a \tan^{-1} \left(\frac{\tan \theta}{a} \right) \right).$$

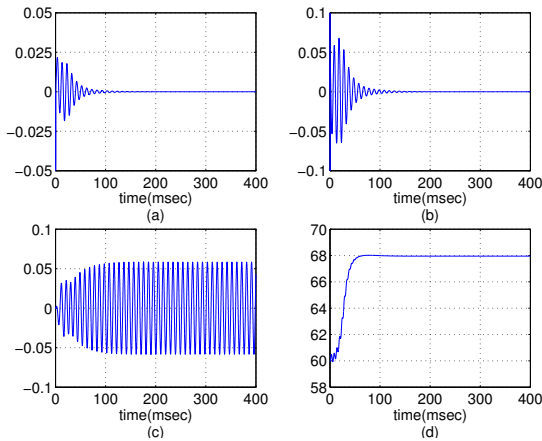


Fig. 4. Simulation result. (a) Tracking error e . (b) Output of the pre-installed controller $C(s)$. (c) Output of the adaptive output regulator R . (d) The adaptive frequency ρ whose initial values is 60Hz.

frequency $\hat{\sigma}$ estimated by the adaptive algorithm $F(u)$ converges to the real frequency 68Hz. The tracking error is perfectly cancelled, as shown in Fig. 4 (a).

The adaptive output regulator is implemented using TMS320VC33 DSP (manufactured by TI Co.) and is applied to LG $\times 52$ CD-ROM disk drive. Configuration for the experiment is the same as Fig. 3 except that we have added a low-pass filter in front of A/D converter because the measured signal was too noisy. The experiment result in Fig. 5 shows the convergence of actual frequency and the tracking error reduction as the simulation result of Fig. 4. The tracking error is not perfectly cancelled out due to the added filter and the fact that the disturbance is not sinusoidal. Note that the controller $C(s)$ of the ODD is not known to us. However, we have a good agreement between the simulation and experiment result.

5. CONCLUSIONS

In this paper, an adaptive output regulator was proposed for cancelling sinusoidal disturbance

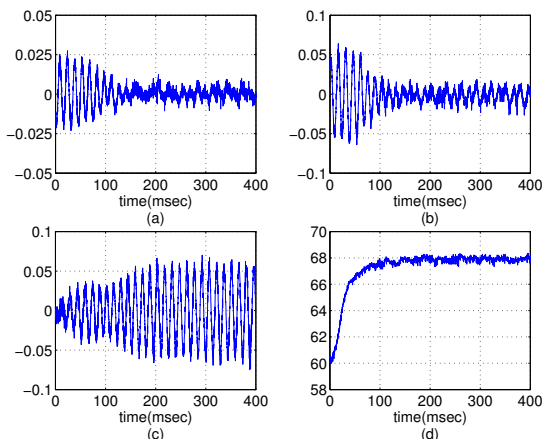


Fig. 5. Experiment result.

with unknown frequency. This result is an extended version of our previous work for output regulator. By adding the adaptive algorithm to add-on output regulator, the output regulator is able to reject disturbance even though its frequency is unknown while preserving the advantages of the output regulator. It was shown that the proposed method is stable for frequency within a connected compact set. In addition, its stability was proven under the assumption that the frequency of disturbance varies slowly in time. The proposed method was applied to a commercial optical disk drive as an add-on type controller even without knowing the structure of pre-installed controller. We confirm that there is a good agreement between simulation and experiment results.

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