TRACKING CONTROL FOR UNCERTAIN TAKAGI SUGENO FUZZY SYSTEMS WITH EXTERNAL DISTURBANCES

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Abstract: In this paper a fuzzy tracking control is proposed for non linear dynamic system with bounded external disturbances and admissible uncertainties described as Takagi-Sugeno (T-S) fuzzy model. Using a tracking reference model, a stability condition scheme based on an augmented system with a guaranteed H∞ performance is proposed to obtain tracking performances. The main result of this paper is to provide, for the overall system in the presence of uncertainties, a sufficient stability condition for the robust fuzzy H∞ controller expressed in terms of linear matrix inequalities (LMIs). An example of robust fuzzy H∞ controller for uncertain nonlinear system is given for illustration.

Keywords: Fuzzy control, linear matrix inequality (LMI), quadratic stability, tracking, relaxed condition, H∞ robust control, uncertain systems.

1. INTRODUCTION

An essential problem in control theory is that of controlling a system in order to have its output asymptotically tracking reference signals. For nonlinear dynamic systems, it is not obvious to solve tracking problems. Hence, stabilisation problems are often treated first. In general, these problems for such kind of systems use special control strategies like sliding mode control, fuzzy adaptive control and so on. Several works have recently focused on the stabilization of nonlinear dynamic systems using Takagi-Sugeno (T-S) fuzzy models (Kim, *et al*., 1995; Narendra and Annaswamy, 1989; Sastry and Bodson, 1989; Takagi and Sugeno, 1985; Wang, *et al*., 1996).

It is well known that T-S models can provide an efficient representation of complex nonlinear systems in terms of fuzzy sets and fuzzy reasoning (Takagi and Sugeno, 1985). These kinds of models have been proved to be a very good representation of a certain class of non linear systems. The appeal of the T-S model is that the stability and performance characteristics of the system can be analysed using a Lyapunov approach. It has been demonstrated (Kim, *et al*., 1995; Wang, *et al*., 1996) that sufficient conditions for the stability and performances of a system are stated in terms of a linear matrix inequalities (LMIs) set. The control design is carried out based on the fuzzy model via the so-called parallel-distributed compensation scheme (PDC) (Wang, *et al., 1995*; Wang, *et al*., 1996). On the one hand, most of the results obtained with such an approach have focused only on the stabilization problem and cannot handle the design objectives such as attenuation of the disturbances effect, tracking control, structured uncertainties and so on (Tanaka, *et al*., 1998). On the other hand, only few results are available in the literature concerning the tracking control problem (Kung and Li, 1997; Ying., 1999; Tseng, *et al*., 2001; Tseng and Chen, 2001) and very few of them are concerned with the uncertain systems.

In (Kung and Li, 1997) a fuzzy tracking controller design for discrete time systems is proposed using the feedback linearization technique. In the same way (Ying. 1999) has established a simple necessary and sufficient condition for determining local stability of the fuzzy systems and then derived a fuzzy tracking controller. In (Tseng, *et al*., 2001), the authors deal with the tracking problem for nonlinear systems described by T-S model, with external disturbances and measurement noise. In their work, an H∞ performance related to the tracking error for bounded reference inputs is formulated and a fuzzy observer-based fuzzy controller is also developed. In (Tseng and Chen, 2001) the same strategy has been developed for a particular class of the nonlinear interconnected systems.

Concerning uncertain systems, the tracking problem still remains open in the case of uncertain T-S fuzzy model. For this class of systems, we find some works dealing only with the stabilisation problem and stability analysis (Tanaka, *et al*., 1996; Lee, *et al*., 2001). This work is on the line of the one presented by (Tseng, *et al*., 2001), but our contribution concerns the case of uncertain T-S Fuzzy models with external disturbances.

Hence, this paper presents a novel approach based on the T-S fuzzy model representation to deal with the problem of tracking control for non linear dynamic system with bounded external disturbances and admissible uncertainties using T-S fuzzy model. In this approach, LMI-based design problems are defined and employed to find feedback gains of a fuzzy controller and common positive definite matrices P satisfying a stability criterion derived in terms of Lyapunov direct method. Based on this control scheme and that criterion, a fuzzy controller is then designed to stabilize an augmented system and to achieve the H∞ control performance.
This paper is organised as follows: Section 2 introduces the uncertain T-S fuzzy model and gives the proposed control law. In Section 3, a sufficient condition is developed to ensure the stability of the augmented system with a reference model. Following this, a method and some manipulations are proposed to improve the LMI feasibility of the main result presented in the second theorem. A simulation example is proposed in Section 4.

2. PROBLEM FORMULATION

The Takagi-Sugeno fuzzy dynamic model is described by fuzzy IF-THEN rules, which locally represent linear input-output relations of nonlinear systems. Consider the i *th* rule of uncertain and perturbed fuzzy model described as follows (Takagi and Sugeno, 1985):

Plant Rule *i* :

If $z_1(t)$ is F_{i1} and … and $z_p(t)$ is F_{ip}
Then $\dot{x} = (A_i + \Delta A_i)\, x(t) + (B_i + \Delta B_i)\, u(t) + \varphi(t)$

$$i = 1, 2, \dots, r. \tag{1}$$

where F_{ij} is the fuzzy set and r is the number of **If-Then** rules. $x(t) \in R^n$ is the state vector, $u(t) \in \boldsymbol{R}^m$ is the input vector, $\varphi(t)$ is the unknown external disturbance with a known upper bound $\varphi_{up} \geq \|\varphi(t)\|$, $z_1(t) \sim z_p(t)$ are the premise variables that are functions of the states, $A_i \in \boldsymbol{R}^{n \times n}$, and $B_i \in \boldsymbol{R}^{n \times m}$ describe the nominal system and ΔA_i, ΔB_i represent the time varying parameter uncertainties which are defined as follows:

$$\begin{bmatrix}\Delta A_i(t) & \Delta B_i(t)\end{bmatrix} = H\ F(t)\ \begin{bmatrix}E_{1i} & E_{2i}\end{bmatrix} \quad i = 1, 2, \dots, r. \tag{2}$$

where H, E_{1i} and E_{2i} $i = 1, 2, \dots, r$, are known constant real matrices with appropriate dimensions, and $F(t)$ is unknown matrix function which is bounded by :

$$F(t) \in \Omega := \left\{ F(t) / F(t)^T F(t) \leq I \right\} \tag{3}$$

Note that the element of $F(t)$ are Lebesgue measurable.
Given a pair of ($x(t)$, $u(t)$), the final outputs of the fuzzy systems are inferred as follows :

$$\begin{aligned}\dot{x}(t) &= \frac{\sum_{i=1}^{r} w_i\left(z(t)\right)\left\{(A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t) + \varphi(t)\right\}}{\sum_{i=1}^{r} w_i\left(z(t)\right)} \\ &= \sum_{i=1}^{r} h_i\left(z(t)\right)\left\{(A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t)\right\} + \varphi(t)\end{aligned} \tag{4}$$

where

$$z(t) = \left[z_1(t)\ \ z_2(t)\ \dots\ z_p(t)\right],$$

$$w_i\left(z(t)\right) = \prod_{j=1}^{p} F_{ij}\left(z_j(t)\right)$$

and for all t,

$$\begin{cases}\sum_{i=1}^{r} w_i\left(z(t)\right) > 0 \\ w_i\left(z(t)\right) \geq 0\end{cases} \quad i = 1, 2, \dots, r \tag{5}.$$

$$h_i\left(z(t)\right) = \frac{w_i\left(z(t)\right)}{\sum_{i=1}^{r} w_i\left(z(t)\right)} \tag{6}$$

where, $F_{ij}(z_j(t))$ is the degree of membership of $z_j(t)$ to F_{ij}, and for all t, we have (Kim, *et al*., 1995; Tanaka, *et al*., 1998; Wang, *et al.*, 1995; Wang, *et al*., 1996)

$$\begin{cases}\sum_{i=1}^{r} h_i\left(z(t)\right) = 1 \\ h_i\left(z(t)\right) \geq 0\end{cases} \quad i = 1, 2, \dots, r \tag{7}$$

To deal the tracking problem for such system, we consider the following reference model (Narendra and Annaswamy, 1989; Sastry and Bodson, 1989):

$$\dot{x}_r = A_r\, x_r(t) + r(t)\,, \tag{8}$$

where $x_r(t)$ is the reference state, A_r is a specific asymptotically stable matrix, and $r(t)$ is a bounded reference input.
Let us consider the H∞ tracking performance related to tracking error $x_r(t) - x(t)$ as follows (Tseng, *et al*., 2001; Tseng and Chen, 2001; Chen, 1996) (Essounbouli, et *al*., 2002):

$$\frac{\int_0^{tf} \left\{ \left[x_r(t) - x(t)\right]^T Q \left[x_r(t) - x(t)\right] \right\} dt}{\int_0^{tf} \left\{ r(t)^T r(t) + \varphi(t)^T \varphi(t) \right\} dt} \leq \eta^2$$

or

$$\begin{aligned}\int_0^{tf} \left\{ \left[x_r(t) - x(t)\right]^T Q \left[x_r(t) - x(t)\right] \right\} dt \\ \leq \eta^2 \int_0^{tf} \left\{ r(t)^T r(t) + \varphi(t)^T \varphi(t) \right\} dt\end{aligned} \tag{9}$$

Where t_f is the terminal time of control, Q a positive definite weighting matrix, and η is a prescribed attenuation level.
If we choose the following control law for the *i th* rule

Control Rule *i* :

If $z_1(t)$ is F_{i1} and … and $z_p(t)$ is F_{ip}
Then $u(t) = -K_i\left[x_r(t) - x(t)\right]$ $\quad i = 1, 2, \ldots, r$ (10)

Then the overall fuzzy controller is given by

$$u(t) = -\frac{\sum_{i=1}^{r} w_i\left(z(t)\right)K_i\left[x_r(t)-x(t)\right]}{\sum_{i=1}^{r} w_i\left(z(t)\right)} = -\sum_{i=1}^{r} h_i\left(z(t)\right)K_i\left[x_r(t)-x(t)\right] \tag{11}$$

By applying the control law above and considering the condition (3), the dynamics $x(t)$ and $x_r(t)$ (i.e. the state and the reference variable respectively) lead to the following augmented system:

$$\dot{\tilde{x}} = \left[A_a + H_a F_a(t) E_a\right]\tilde{x}(t) + \phi(t) \tag{13}$$

where

$$A_a = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \begin{bmatrix} A_i + B_i K_j & -B_i K_j \\ 0 & A_r \end{bmatrix}$$

$$E_a = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \begin{bmatrix} E_{1i} + E_{2i}K_j & -E_{2i}K_j \\ 0 & 0 \end{bmatrix}$$

$$H_a = \begin{bmatrix} H & 0 \\ 0 & 0 \end{bmatrix} \qquad F_a(t) = \begin{bmatrix} F(t) & 0 \\ 0 & 0 \end{bmatrix}$$

$$\phi(t) = \begin{bmatrix} \varphi(t) \\ r(t) \end{bmatrix} \quad \text{and} \quad \tilde{x} = \begin{bmatrix} x \\ x_r \end{bmatrix}$$

Hence, the H∞ tracking performance in (9) can be modified as follows:

$$\int_0^{tf}\left\{\left[x_r(t)-x(t)\right]^T Q\left[x_r(t)-x(t)\right]\right\}dt = \int_0^{tf}\tilde{x}^T(t)\,\tilde{Q}\,\tilde{x}(t)\,dt \le \eta^2\int_0^{tf}\phi(t)^T\phi(t)\,dt \tag{14}$$

where Q is a symmetric positive definite weighting matrix and

$$\tilde{Q} = \begin{bmatrix} Q & -Q \\ -Q & Q \end{bmatrix}$$

and,

$$\phi(t)^T\phi(t) = r(t)^T r(t) + \varphi(t)^T\varphi(t)$$

3. FUZZY TRACKING CONTROL DESIGN FOR AN UNCERTAIN SYSTEM :

In this study, the objective is to determine the gains of the fuzzy control law (11) that achieve the H∞ tracking control performance in (14). Consequently we can state the following results.

Theorem 1
If there exist a symmetric positive definite matrix $P = P^T > 0$, positive constants τ and η, and the feedback gains K_i shown in (11), such that the following condition (15) hold :

$$\begin{bmatrix} A_a^T P + PA_a + \tilde{Q} + \tau E_a^T E_a & PH_a & P \\ H_a^T P & -\tau I & 0 \\ P & 0 & -\eta^2 I \end{bmatrix} \le 0 \tag{15}$$

then the closed loop fuzzy system (13) is quadratically stable and the H∞ tracking control performance in (14) is guaranteed for a prescribed attenuation η.

Proof :

The augmented system (13) can be modified as follow:

$$\dot{\tilde{x}} = A_a\tilde{x}(t) + H_a q(t) + \phi(t) \tag{16}$$

with :

$$q(t) = F_a(t)E_a\,\tilde{x}(t) \tag{17}$$

Note that

$$F(t)^T F(t) \le I \iff F_a(t)^T F_a(t) \le I \tag{18}$$

From (17) and (18), we can write

$$q(t)^T q(t) \le \left(E_a\tilde{x}(t)\right)^T\left(E_a\tilde{x}(t)\right) = \tilde{x}(t)^T E_a^T E_a\tilde{x}(t) \tag{19}$$

Let us now consider the following candidate Lyapunov function for the augmented system (13) :

$$V = \tilde{x}^T(t)\,P\,\tilde{x}(t) \tag{20}$$

with $P = P^T > 0$.
then, the augmented system (13) is quadratically stable and the H∞ tracking performance in (14) is guaranteed if

$$\frac{dV(\tilde{x},t)}{dt} + \tilde{x}\,\tilde{Q}\,\tilde{x} - \eta^2\phi^T\phi \le 0 \tag{21}$$

Using (16) and the derivative of (20), the condition above becomes.

$$\tilde{x}^T(A_a^T P + PA_a + \tilde{Q})\tilde{x} + q(t)^T H_a^T P\tilde{x} + \tilde{x}^T PH_a q(t) + \phi^T P\tilde{x} + \tilde{x}^T P\phi - \eta^2\phi^T\phi \le 0 \tag{22}$$

From (17) and (20), the conditions (19) and (21) can be written as :

$$\begin{bmatrix} \tilde{x}(t) \\ q(t) \\ \phi(t) \end{bmatrix}^T \begin{bmatrix} -E_a^T E_a & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{x}(t) \\ q(t) \\ \phi(t) \end{bmatrix} \le 0 \tag{23}$$

$$\begin{bmatrix} \tilde{x}(t) \\ q(t) \\ \phi(t) \end{bmatrix}^T \begin{bmatrix} A_a^T P + PA_a + \tilde{Q} & PH_a & P \\ H_a^T P & 0 & 0 \\ P & 0 & \eta^2 I \end{bmatrix} \begin{bmatrix} \tilde{x}(t) \\ q(t) \\ \phi(t) \end{bmatrix} \le 0 \tag{24}$$

By using the S-procedure (Boyd, *et al.*, 1994) subject to the existence of a positive scalar $\tau > 0$, the conditions (23) and (24) lead to:

$$\begin{bmatrix} A_a^T P + PA_a + \widetilde{Q} + \tau E_a^T E_a & PH_a & P \\ H_a^T P & -\tau I & 0 \\ P & 0 & -\eta^2 I \end{bmatrix} \le 0$$

■

This condition that ensures the quadratic stability of the closed loop fuzzy system, is not easily taken in to deduce the gains control. Thus, in view to find a more exploitable mathematical condition for the robust performance problem for the uncertain system, we choose:

$$A_{ij} = \begin{bmatrix} A_i + B_i K_j & -B_i K_j \\ 0 & A_r \end{bmatrix} \quad (25)$$

$$E_{aij} = \begin{bmatrix} E_{1i} + E_{2i} K_j & -E_{2i} K_j \\ 0 & 0 \end{bmatrix} \quad (26)$$

In this case, the following result can be announced

Theorem 2

If there exist a symmetric positive definite matrix $P = P^T > 0$, positive constants τ and η, and the feedback gains K_i shown in (11), such that the following condition (27) hold:

$$\begin{bmatrix} A_{ij}^T P + PA_{ij} + \widetilde{Q} & PH_a & P & E_{aij}^T \\ H_a^T P & -\tau I & 0 & 0 \\ P & 0 & -\eta^2 I & 0 \\ E_{aij} & 0 & 0 & -\tau^{-1} I \end{bmatrix} \le 0$$

$$i, j = 1, 2, .., r \quad (27)$$

then the closed loop fuzzy system (13) is quadratically stable and the H∞ tracking control performance in (14) is guaranteed for a prescribed attenuation η.

Proof

By using the terms (25) and (26), the condition (15) in theorem 1 will be written as

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t)) h_j(z(t)) \begin{bmatrix} A_{ij}^T P + PA_{ij} + \widetilde{Q} + \tau E_{aij}^T E_{aij} & PH_a & P \\ H_a^T P & -\tau I & 0 \\ P & 0 & -\eta^2 I \end{bmatrix} \le 0 \quad (28)$$

Tthis is equivalent to

$$\begin{bmatrix} A_{ij}^T P + PA_{ij} + \widetilde{Q} + \tau E_{ij}^T E_{ij} & PH_a & P \\ H_a^T P & -\tau I & 0 \\ P & 0 & -\eta^2 I \end{bmatrix} \le 0$$

$$i, j = 1, 2, .., r \quad (29)$$

With some manipulation and using the Schur complement (Boyd, *et al*., 1994) the condition above is equivalent to that in theorem 2 (27).

■

Solving the conditions of theorem 2 remains a difficult task. Therefore, in this section we will first show how to fulfil this last inequality in terms of LMIs (standard LMI). Note that, for the convenience of design, we assume that:

$$P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \text{ with } P_1 = P_1^T > 0 \text{ and } P_2 = P_2^T > 0$$

The condition (27) of theorem 2 becomes

$$\begin{bmatrix} H_{11} & H_{12} & H_{13} & 0 & H_{15} & 0 & H_{17} & 0 \\ H_{21} & A_r^T P_2 + P_2 A_r + Q & 0 & 0 & 0 & P_2 & H_{27} & 0 \\ H_{31} & 0 & -\tau I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\tau I & 0 & 0 & 0 & 0 \\ H_{51} & 0 & 0 & 0 & -\eta^2 I & 0 & 0 & 0 \\ 0 & P_2 & 0 & 0 & 0 & -\eta^2 I & 0 & 0 \\ H_{71} & H_{72} & 0 & 0 & 0 & 0 & -\tau^{-1} I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\tau^{-1} I \end{bmatrix} \le 0$$

$$i, j = 1, 2, .., r \quad (30)$$

with

$$H_{11} = (A_i + B_i K_j)^T P_1 + P_1 (A_i + B_i K_j) + Q$$

$$H_{12} = H_{21}^T = -P_1 B_i K_j - Q$$

$$H_{13} = H_{31}^T = P_1 H$$

$$H_{15} = H_{51} = P_1$$

$$H_{17} = H_{71}^T = (E_{1i} + E_{2i} K_j)^T$$

$$H_{27} = H_{72}^T = (E_{2i} K_j)^T$$

P_1, P_2 and K_i can be solved following these two steps
First, note that (30) implies that

$$H_{11} = (A_i + B_i K_j)^T P_1 + P_1 (A_i + B_i K_j) + Q \le 0$$

$$i, j = 1, 2, .., r \quad (31)$$

After pre-post-multiply (31) by $N_1 = P_1^{-1}$ and letting $Y_i = K_i N_1$, we obtain

$$N_1 A_i^T + Y_j^T B_i^T + A_i N_1 + B_i Y_j + N_1 Q N_1 \le 0 \quad i, j = 1, 2, .., r$$

By using the Schur complement, we get

$$\begin{bmatrix} N_1 A_i^T + Y_j^T B_i^T + A_i N_1 + B_i Y_j & N_1 \\ N_1 & -Q^{-1} \end{bmatrix} \le 0$$

$$i, j = 1, 2, .., r \quad (32)$$

The variables N_1 and Y_i (thus $P_1 = N_1^{-1}$ and $K_i = Y_i N_1^{-1}$) can be obtained by solving the LMI (32).

In the second step, by substituting P_1 and K_i into (30) which will be a standard linear matrix inequality (LMI's) (Gahinet, *et al*., 1995), we can easily solve P_2 and τ from (30). If there exist positive definite solutions P_1, P_2 and a positive scalar τ such that (30) holds, then the closed loop system (12) is stable and the H∞ tracking performance in (14) can be achieved for a prescribed attenuation level η.

4. EXAMPLE

Let us consider the problem of the tracking on an inverted pendulum with a cart. The equations of motion for the pendulum are (Cannon, 1967)

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \frac{g\sin(x_1) - amlx_2^2\sin(2x_1)/2 - a\cos(x_1)u}{4l/3 - aml\cos^2(x_1)}$$

where x_1 denotes the angle (in radians) of the pendulum from the vertical, x_2 is the angular velocity, $g = 9.8\ m/s^2$ is the constant gravity, m is the mass of the pendulum, M is the mass of the cart, $2l$ is the length of the pendulum, and u is the force applied to the cart (in Newtons). $a = 1/(m+M)$. Let $m = 0.1 Kg$, $M = 1\ Kg$, $2l = 1.0\ m$.

To minimize the design effort and complexity, we try to use as few rules as possible. Notice that when $x_1 = \pm\pi/2$, the system is uncontrollable. Hence, we approximate the system by the following two-rule T-S fuzzy model, where the membership functions are shown in Figure 1.

Rule *1* : **If** $x_1(t)$ is about 0
Then
$$\dot{x}(t) = (A_1 + \Delta A_1)\, x(t) + (B_1 + \Delta B_1)\, u(t) + \varphi(t)$$

Rule *2* : **If** $x_1(t)$ is about $x_1 = \pm\pi/2$ ($|x_1(t)| < 2$)
Then
$$\dot{x}(t) = (A_2 + \Delta A_2)\, x(t) + (B_2 + \Delta B_2)\, u(t) + \varphi(t)$$

where

$$A_1 = \begin{bmatrix} 0 & 1 \\ g/4l/3 - aml & 0 \end{bmatrix}$$
$$B_1 = \begin{bmatrix} 0 \\ -a/4l/3 - aml \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 1 \\ 2g/\pi\left(4l/3 - aml\beta^2\right) & 0 \end{bmatrix}$$
$$B_2 = \begin{bmatrix} 0 \\ -a\beta/4l/3 - aml\beta^2 \end{bmatrix}$$

with $\beta = \cos(88^\circ)$ and

$$\Delta A_1 = H\ F(t)\ E_{11} \qquad \Delta A_2 = H\ F(t)\ E_{12}$$
$$\Delta B_1 = H\ F(t)\ E_{21} \qquad \Delta B_2 = H\ F(t)\ E_{22}$$
$$H = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad F(t) = \sin(2t) \in \Omega$$
$$E_{11} = [1 \quad 0], \qquad E_{21} = 0.5 \qquad (i = 1)$$
$$E_{12} = [1 \quad 0], \qquad E_{22} = 0.25 \qquad (i = 2).$$
$$\varphi(t) = [0.01\sin(2t) \quad 0.02\cos(2t)]^T .$$

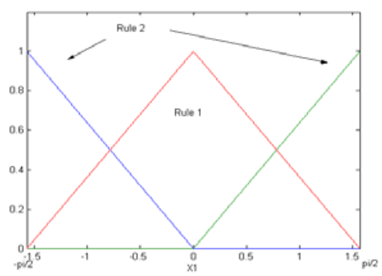

Fig. 1. Membership functions of two-rule model

Afterwards, we compute the gains K_i according to the steps defined in section 3 for the conditions (27). Hence, by choosing

$$Q = \begin{bmatrix} 1e^{-8} & 0 \\ 0 & 9e^{-7} \end{bmatrix}, \quad A_r = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}$$

$$\text{and} \qquad r(t) = \begin{bmatrix} 0 \\ 0.7381\sin(t) \end{bmatrix},$$

The following gains are obtained

$$K_1 = [416.0733 \quad 300], \quad K_2 = [416.0733 \quad 300]$$

and the attenuation level is $\eta = 0.5$.

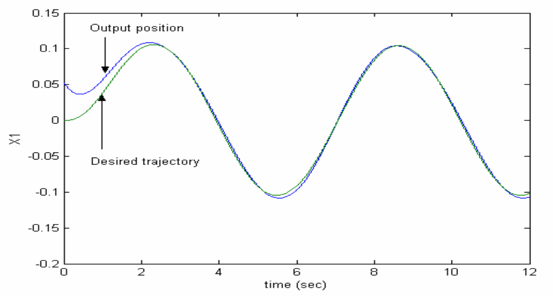

Fig. 2. Position tracking

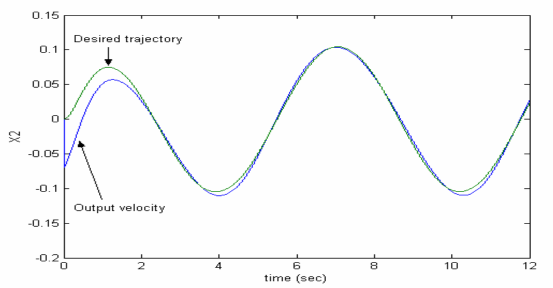

Fig. 3. Velocity Tracking

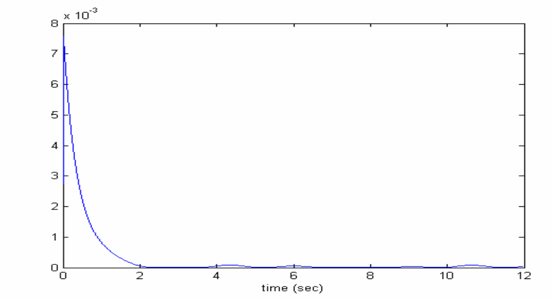

Fig. 4. Quadratic error tracking

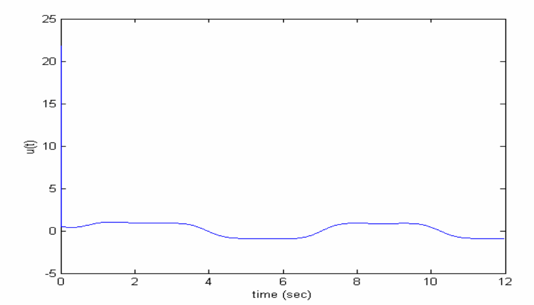

Fig. 5. Control force

Figure 2 shows the position tracking trajectory with as initial conditions $x_1(0)=0.05$ and $x_2(0)=0$ for the system, Figure 3 illustrates the tracking velocity signal. Figure 4 shows the quadratic error signal whereas the force control signal is given in figure 5.

These obtained results show a good tracking performance for the case of an uncertain and a perturbed system.

5. CONCLUSION

In this paper, we have developed via H∞ a controller design method for nonlinear uncertain systems with external disturbances described by Takagi-Sugeno model. Using a tracking reference model, a stability condition scheme based on an augmented system with a guaranteed H∞ performance was proposed to ensure good tracking performances. The main results are concerning the two developed theorems to ensure a sufficient condition in terms of LMI for the problem of uncertain systems tracking, expressed in Takagi Sugeno form.

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