COMPOSITE ADAPTIVE FUZZY CONTROL

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Abstract: Adaptive fuzzy control has been a topic of active research over the last decade. However, most efforts have been directed toward one goal: achieving asymptotic stability and tracking. Little attention has been paid to the accuracy of the identified fuzzy models and to their transparency and interpretability whereas these should be the key aspects motivating the use of fuzzy models in adaptive control. The main contribution of this paper is to present an adaptive fuzzy controller with composite adaptive laws based on both tracking and prediction error. Compared to other adaptive fuzzy controllers, the proposed controller achieves smoother parameter adaptation, better accuracy and improved performance. It overcomes some of the drawbacks of similar schemes described in the literature on adaptive fuzzy control. The limitations of the proposed approach are also discussed. *Copyright © 2005 IFAC*

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1. INTRODUCTION

In the last decade, there has been an increasing interest in adaptive fuzzy control (AFC) for input-affine nonlinear dynamic systems. This interest has been motivated by the demands for high control performance in situations in which an accurate model of the controlled plant is not available, or when the plant is time-varying. Fuzzy systems have the potential to play an important role in adaptive control, mainly thanks to their *universal function approximation* property and their amenability to (linguistic) interpretation of the input-output relationships. However, so far, research in AFC has been mainly focused on the following two fundamental requirements:

– *stability* of the closed-loop system (all the signals in closed-loop must be bounded),
– *asymptotic convergence* of the tracking error to zero or to a neighborhood of zero.

Another desirable feature, though not always explicitly stated, is the convergence of the adapted parameters to some optimal values (for a time-invariant process).

The most common stable AFC schemes are based on *feedback linearization* (Wang, 1993; Wang, 1996). They apply to input-affine models in the controllable canonical form and mostly employ singleton fuzzy systems to approximate the unknown system (*indirect schemes*) or the unknown control law (*direct schemes*).

The antecedent parameters (membership functions) are usually fixed and the consequent parameters are adapted, based on the tracking error, by means of stable adaptive laws derived through Lyapunov synthesis.

The design of such adaptive fuzzy controllers must inherently consider the *robustness* issue since any finite dimensional fuzzy approximator unavoidably introduces an *approximation error*. Such an error is usually handled as a disturbance acting on the system by means of standard modifications:

– *An additional nonlinear damping term*, usually in a sliding mode framework, (Su and Stepanenko, 1994; Han *et al.*, 2001; Fishle and Schroder, 1999; Spooner and Passino, 1996; Chen *et al.*, 1996; Tong *et al.*, 2000; Chang, 2000).

– *A modified adaptive law* such as projection (Wang, 1993; Wang, 1996), dead-zones (Koo, 2001), $\sigma-$modification, $\epsilon-$modification (Skrjanc *et al.*, 2002).

Another important design issue is the interpretability of fuzzy systems. In the fuzzy modeling literature, it has been recognized that fuzzy systems are not *transparent* and *interpretable* by default, contrary to what is often stated as an implicit advantage of these systems (Setnes *et al.*, 1998*a*; Setnes *et al.*, 1998*b*; Jin, 2000; Babuška, 2002). Nonetheless, in the literature on AFC, the relevance of the identified fuzzy systems in terms of accuracy and interpretability is usually disregarded and the benefits of fuzzy logic are thus only partially exploited. We argue that using fuzzy systems instead of a black-box technique is advantageous not only because fuzzy systems can provide a good guess for the initial system model (by the inclusion of prior qualitative knowledge), but also to gather more insight about the unknown systems dynamics and/or control law during or at the end of adaptation.

Adaptive controllers with *composite adaptive laws*, based on two sources of information (*tracking* and *prediction errors*) can potentially be beneficial with respect to both the aforementioned fundamental design issues. In the context of classical adaptive control (Slotine and Li, 1991; Duarte and Narendra, 1989), it has been shown for linear plants that a significant improvement in the control performance and in the robustness can be achieved, essentially as a consequence of a smoother and quicker adaptation process. In the AFC literature (Yin and Lee, 1995; Hojati and Gazor, 2002), it has been also claimed that composite adaptation can provide better performance and improved parameter convergence (although under the assumption that the approximation error is sufficiently small).

The main contribution of this paper is to introduce a novel indirect model reference adaptive controller with a composite adaptive law, combining the parameter estimation scheme described in (Wang, 1995) and the MRAC described in (Wang, 1996). The proposed scheme (i) assures the *global stability* of the closed-loop system, (ii) provides improved control performance compared to adaptive controllers with standard adaptive laws driven only by the tracking error and (iii) addresses some of the drawbacks of the controllers described in (Yin and Lee, 1995; Hojati and Gazor, 2002). Contrary to the controllers in the aforementioned references, it does not require the knowledge of the n^{th} order derivative of the output and it provides a smoother parameter adaptation with reduced oscillations.

The remainder of this paper is structured as follows. Section 2 gives background on indirect model reference adaptive fuzzy control. In Section 3, the composite adaptation idea is discussed in the context of the references (Yin and Lee, 1995; Hojati and Gazor, 2002). In Section 4, the proposed composite adaptive controller is described and its advantages and limitations are discussed. Section 5 presents a simulation example and Section 6 concludes the paper.

2. INDIRECT MODEL REFERENCE AFC

In this section, the basic elements of indirect model reference AFC schemes are introduced. First, the controller structure is described; then the standard adaptive laws based on tracking error are presented.

2.1 Controller structure

Consider indirect adaptive fuzzy controllers (Wang, 1996) for systems in the controllable canonical form:

$$x^{(n)} = f(\mathbf{x}) + g(\mathbf{x})u \tag{1}$$

$$y = x \tag{2}$$

where $\mathbf{x} = \left[x, \dot{x}, \ldots, x^{(n-1)}\right]^T \in \mathbb{R}^n$ is the state vector. We assume that $g(\mathbf{x}) > 0$ for all $\mathbf{x} \in X \subset \mathbb{R}^n$. The *control goal* is to track a desired trajectory y_m while keeping all the signals in the closed-loop bounded. The tracking error $e = y_m - y$ is the difference between the trajectory y_m generated by a *reference model* and the output y of the system. Further, introduce the vector of the tracking error and its $n-1$ derivatives $\boldsymbol{e} = \left[e, \dot{e}, \ldots, e^{(n-1)}\right]^T$ and the feedback gain vector $\boldsymbol{k} = \left[k_n, \ldots, k_1\right]^T$. If the functions $f(\mathbf{x})$ and $g(\mathbf{x})$ are known, the gains k_i can be chosen such that the roots of the polynomial $h(s) = s^n + k_1 s^{n-1} + \ldots + k_n$ are in the open left-half of the complex plane. The *feedback linearizing* control law

$$u^* = \frac{1}{g(\mathbf{x})}\left[-f(\mathbf{x}) + y_m^{(n)} + \boldsymbol{k}^T\boldsymbol{e}\right] \tag{3}$$

then produces the desired linear *error dynamic*:

$$e^{(n)} + \boldsymbol{k}^T\boldsymbol{e} = 0 \tag{4}$$

or equivalently

$$\dot{\boldsymbol{e}} = \boldsymbol{\Lambda}_c \boldsymbol{e} \tag{5}$$

where the matrix $\mathbf{\Lambda}_c \in \mathbb{R}^{n\times n}$ is given by

$$\mathbf{\Lambda}_c = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \dots & 0 & 1 \\ -k_n & -k_{n-1} & \dots & \dots & -k_1 \end{bmatrix}. \qquad (6)$$

The ideal control law (3) guarantees that $\lim_{t\to\infty} e(t) = 0$. The basic idea of indirect AFC is to approximate the unknown functions $f(\mathbf{x})$ and $g(\mathbf{x})$ in the control law (3), by using two singleton fuzzy systems:

$$\hat{f}(\mathbf{x}) = \boldsymbol{\theta}_f^T \boldsymbol{\xi}_f(\mathbf{x}) \qquad (7)$$

$$\hat{g}(\mathbf{x}) = \boldsymbol{\theta}_g^T \boldsymbol{\xi}_g(\mathbf{x}) \qquad (8)$$

where $\boldsymbol{\theta}_f$ and $\boldsymbol{\theta}_g$ are the *consequent* parameters to be adapted, $\boldsymbol{\xi}_f(\mathbf{x})$ and $\boldsymbol{\xi}_g(\mathbf{x})$ are the *normalized degrees of fulfillment* of the (fixed) fuzzy rules. If we replace the functions $f(\mathbf{x})$ and $g(\mathbf{x})$ in (3) with their fuzzy approximations, we have the control law

$$u = \frac{1}{\hat{g}(\mathbf{x})}\left[-\hat{f}(\mathbf{x}) + y_m^{(n)} + \boldsymbol{k}^T\boldsymbol{e}\right]. \qquad (9)$$

In general, functions $f(\mathbf{x})$ and $g(\mathbf{x})$ cannot be exactly approximated by the two fuzzy systems $\hat{f}(\mathbf{x})$ and $\hat{g}(\mathbf{x})$. If we introduce the *optimal parameters*

$$\boldsymbol{\theta}_f^* = \arg\min_{\theta_f}\left(\sup_{x\in X}|\hat{f}(\mathbf{x}) - f(\mathbf{x})|\right)$$

$$\boldsymbol{\theta}_g^* = \arg\min_{\theta_g}\left(\sup_{x\in X}|\hat{g}(\mathbf{x}) - g(\mathbf{x})|\right) \qquad (10)$$

then the *inherent approximation error* can be defined as

$$w = [\boldsymbol{\theta}_f^{*T}\boldsymbol{\xi}_f(\mathbf{x}) - f(\mathbf{x})] + [\boldsymbol{\theta}_g^{*T}\boldsymbol{\xi}_g(\mathbf{x}) - g(\mathbf{x})]u \qquad (11)$$

The above definitions for the optimal parameters and for the approximation error represent the common choice in the literature on AFC. The use of the operator $\sup(*)$ is motivated by the subsequent stability analysis: w acts as a disturbance and its magnitude critically affects closed-loop stability. Substituting the control law (9) into (1) and using the definition of the inherent approximation error (11), after some manipulations, we obtain the error dynamics

$$\dot{\boldsymbol{e}} = \mathbf{\Lambda}_c\boldsymbol{e} + \boldsymbol{b}_c\left[\boldsymbol{\phi}_f^T\boldsymbol{\xi}_f(\mathbf{x}) + \boldsymbol{\phi}_g^T\boldsymbol{\xi}_g(\mathbf{x})u\right] + \boldsymbol{b}_c w \qquad (12)$$

where $\boldsymbol{b}_c = [0,...,0,1]^T$ and $\boldsymbol{\phi}_f = \boldsymbol{\theta}_f - \boldsymbol{\theta}_f^*$ is the difference between the actual parameters $\boldsymbol{\theta}_f$ of the fuzzy system $\hat{f}(\mathbf{x})$ and the optimal parameters $\boldsymbol{\theta}_f^*$ (analogously for $\boldsymbol{\phi}_g = \boldsymbol{\theta}_g - \boldsymbol{\theta}_g^*$).

2.2 *Standard adaptive laws*

The adaptive laws are derived using Lyapunov synthesis with the following Lyapunov function

$$V = \frac{1}{2}\,\boldsymbol{e}^T\mathbf{P}\boldsymbol{e} + \frac{1}{2\gamma_f}\boldsymbol{\phi}_f^T\,\boldsymbol{\phi}_f + \frac{1}{2\gamma_g}\boldsymbol{\phi}_g^T\boldsymbol{\phi}_g \qquad (13)$$

which is the sum of the contributions of the tracking error $\boldsymbol{e}$ and the parameter errors $\boldsymbol{\phi}_f$ and $\boldsymbol{\phi}_g$. Matrices $\mathbf{P} \in \mathbb{R}^{n\times n}$ and $\mathbf{Q} \in \mathbb{R}^{n\times n}$ are positive-definite matrices that fulfill the Lyapunov equation

$$\mathbf{\Lambda}_c^T\mathbf{P} + \mathbf{P}\mathbf{\Lambda}_c = -\mathbf{Q}. \qquad (14)$$

This V simultaneously guarantees the boundedness of the tracking error and the parameter errors. The time-derivative of V is obtained by differentiating (13), substituting for $\dot{\boldsymbol{e}}$ from (12) and using (14)

$$\dot{V} = -\frac{1}{2}\,\boldsymbol{e}^T\mathbf{Q}\boldsymbol{e} + \frac{1}{\gamma_f}\boldsymbol{\phi}_f^T\left[\dot{\boldsymbol{\theta}}_f + \gamma_f e_s\boldsymbol{\xi}_f(\mathbf{x})\right] + \frac{1}{\gamma_g}\boldsymbol{\phi}_g^T\left[\dot{\boldsymbol{\theta}}_g + \gamma_g e_s\boldsymbol{\xi}_g(\mathbf{x})\,u\right] + e_s\,w \qquad (15)$$

where γ_f and γ_g are the *learning rates*, $e_s = \boldsymbol{e}^T\boldsymbol{p}_n$ and $\boldsymbol{p}_n$ is the last column of $\mathbf{P}$. If the parameters $\boldsymbol{\theta}_f$ and $\boldsymbol{\theta}_g$ are adapted according to the following laws

$$\dot{\boldsymbol{\theta}}_f = -\gamma_f\;e_s\boldsymbol{\xi}_f(\mathbf{x}) \qquad (16)$$

$$\dot{\boldsymbol{\theta}}_g = -\gamma_g\;e_s\boldsymbol{\xi}_g(\mathbf{x})\,u \qquad (17)$$

the terms in the brackets in (15) are zero and $\dot{V}$ becomes

$$\dot{V} = -\frac{1}{2}\,\boldsymbol{e}^T\mathbf{Q}\boldsymbol{e} + e_s\,w\,. \qquad (18)$$

If the second term in (18) due to the inherent approximation error w can be neglected or somehow neutralized, the derivative of Lyapunov function is negative-semi-definite thus assuring the boundedness of the tracking error and the parameter errors.

Remark. The adaptive law (17) must be modified for preventing $\hat{g}(\mathbf{x})$ from being zero (e.g. by a projection operator).

3. COMPOSITE ADAPTATION

The basic motivation for the design of *composite adaptive laws* is that a faster and smoother parameter adaptation can be achieved using two sources of information, namely the tracking and the prediction errors (see (Slotine and Li, 1991) for low-pass filter interpretation of composite adaptive laws). The improved adaptation, in turn, leads to a faster reduction of the tracking error.

A smoother parameter adaptation has the advantage that high-frequency unmodeled dynamics are not excited and this results in a more robust control scheme. In the context of AFC, the smoothness of parameter update is a highly desirable feature also in view of the transparency of the identified fuzzy models. Widely oscillating singletons, in fact, may significantly reduce the chances of a clear linguistic interpretation of the fuzzy rules.

However, the combination of two sources of information in the adaptive laws is not necessary beneficial under all circumstances: its effectiveness relies on the adaptation ability of the individual methods and on how these methods interact. Recently, (Hojati and Gazor, 2002) proposed the adaptive laws

$$\dot{\boldsymbol{\theta}}_f = -\gamma_f \; [\gamma\epsilon + e_s]\, \boldsymbol{\xi}_f(\mathbf{x}) \tag{19}$$

$$\dot{\boldsymbol{\theta}}_g = -\gamma_g \; [\gamma\epsilon + e_s]\, u\, \boldsymbol{\xi}_g(\mathbf{x}) \tag{20}$$

where $\epsilon \equiv \hat{x}^{(n)} - x^{(n)}$ is the prediction error, and γ_f, γ_g, γ are the learning rates. The estimated state $\hat{\mathbf{x}}$ is provided by a *serial-parallel* estimation model of the plant in which the first $n-1$ component of the estimated state are given by $\dot{\hat{x}}_i = x_{i+1}$ and the n^{th} component (the only one actually estimated) is given by

$$\hat{x}^{(n)} = \hat{f}(\mathbf{x}) + \hat{g}(\mathbf{x})u\,. \tag{21}$$

Clearly, the evaluation of the prediction error requires the knowledge of $x^{(n)}$. Hence, the assumption that the full state is measurable does not suffice, unless $x^{(n)}$ is determined by direct differentiation. The requirement that the full state is available is quite restrictive in itself, but the requirement that also the derivative of the state is a physically measurable quantity is generally not practical. Thus, in the subsequent section, novel adaptive laws are proposed, which rely on the knowledge of the state but not of its derivative.

By using equations (1), (12) and (21), the prediction error ϵ can be written in the form

$$\begin{aligned}\epsilon &= \left[\boldsymbol{\phi}_f^T \boldsymbol{\xi}_f(\mathbf{x}) + \boldsymbol{\phi}_g^T \boldsymbol{\xi}_g(\mathbf{x})u\right] + w \\ &= e^{(n)} + \boldsymbol{k}^T \boldsymbol{e}\,. \end{aligned} \tag{22}$$

From Eq. (22), it can be seen that the prediction error ϵ is the sum of two contributions. The first term $e^{(n)}$ can have significant variations even if the tracking error is small (especially for a high-order system). The second term $\boldsymbol{k}^T\boldsymbol{e}$ (defining a stable manifold) is similar to e_s and hence its effect is somehow equivalent to an increment in the learning rate. Moreover, Eq. (22) also shows that the modelling error is directly injected in the adaptive laws by the prediction error without any filtering. We can then expect that the composite adaptation scheme does not generally assure a smooth parameter convergence, contrary to the general remarks in (Hojati and Gazor, 2002). In this reference, the closed-loop stability analysis and the justification of the improved performance rely on the assumption of negligible unmodelled dynamics. However, this assumption is contradicted if the non-smooth parameter adaptation excites unmodeled dynamics.

It is also useful to remark that there are earlier research contributions (Yin and Lee, 1995) that lead to similar results for a different class of systems (n^{th} order linear systems with scheduled parameters) and a different error model (the prediction error with regards to the estimated control input). In fact, it can be shown that a relation between the prediction error and the tracking error similar to Eq. (22) (except for a multiplicative factor which is the estimate of a model parameter) can be obtained, after straightforward manipulations of the equations in (Yin and Lee, 1995). Hence, our previous remarks about (Hojati and Gazor, 2002) can be extended also to this case.

4. PROPOSED ADAPTIVE LAWS

The basic idea for deriving the new composite adaptive laws is (i) to define the *prediction error* as $\epsilon \equiv \hat{x}^{(n-1)} - x^{(n-1)}$ thus avoiding the use of $x^{(n)}$ and (ii) to estimate the state with a slightly different serial-parallel estimation model (Wang, 1995). The first $n-1$ state variables are again measured and not estimated whereas Eq. (21) is replaced by

$$\hat{x}^{(n)} = -\alpha\epsilon + \hat{f}(\mathbf{x}) + \hat{g}(\mathbf{x})u \tag{23}$$

the parameter α being a user-defined positive constant. Subtracting (1) and (23), after some manipulations, we have

$$\begin{aligned}\dot{\epsilon} + \alpha\epsilon &= \left[\boldsymbol{\phi}_f^T \boldsymbol{\xi}_f(\mathbf{x}) + \boldsymbol{\phi}_g^T \boldsymbol{\xi}_g(\mathbf{x})u\right] + w \\ &= e^{(n)} + \boldsymbol{k}^T \boldsymbol{e}\,. \end{aligned} \tag{24}$$

It can be seen that the prediction error ϵ is, in this case, the output of a first-order stable *low-pass filter* (with breaking frequency α), whose input is a linear combination of the tracking error e and its derivatives (or alternatively the sum of the two terms in the square brackets depending on the parameter errors and of the approximation error w). The smoothing action of such a filter entails mainly two advantages: faster tracking error convergence and smoother parameter adaptation. A smooth adaptation is a necessary condition (although not sufficient) for fuzzy rules interpretability. Such condition is obviously critical if the antecedent parameter are also adapted, but it is highly desirable even in the case of fixed antecedents.

The composite adaptive laws are determined by Lyapunov synthesis, following the same steps described for standard adaptation, but with a different Lyapunov function

$$V = \frac{1}{2}\,\boldsymbol{e}^T\mathbf{P}\boldsymbol{e} + \frac{1}{2}\,\epsilon^2 + \frac{1}{2\gamma_f}\boldsymbol{\phi}_f^T\,\boldsymbol{\phi}_f + \frac{1}{2\gamma_g}\boldsymbol{\phi}_g^T\boldsymbol{\phi}_g\,. \tag{25}$$

The time-derivative of V is obtained by differentiating (25), substituting for $\dot{\boldsymbol{e}}$ from (24) and for $\dot{\epsilon}$ from (12) and finally using (14)

$$\dot{V} = -\frac{1}{2}\,\boldsymbol{e}^T\mathbf{Q}\boldsymbol{e} + \frac{1}{\gamma_f}\boldsymbol{\phi}_f^T\left[\dot{\boldsymbol{\theta}}_f + \gamma_f(e_s+\epsilon)\boldsymbol{\xi}_f(\mathbf{x})\right]$$

$$+\frac{1}{\gamma_g}\boldsymbol{\phi}_g^T\left[\dot{\boldsymbol{\theta}}_g+\gamma_g(e_s+\epsilon)\boldsymbol{\xi}_g(\mathbf{x})\,u\right]$$
$$-\frac{1}{2}\,\alpha\epsilon^2+(e_s+\epsilon)\,w\,. \tag{26}$$

With adaptive laws of the same form as (19)-(20), but with $\gamma = 1$ and with the new definition of ϵ, we end up with

$$\dot{V}=-\frac{1}{2}\,\boldsymbol{e}^T\mathbf{Q}\boldsymbol{e}-\frac{1}{2}\,\alpha\epsilon^2+(e_s+\epsilon)\,w\,. \tag{27}$$

If the disturbance term in (27) due to w can be neglected (as in the considered simulation example) or its effects compensated by a standard robust modification, the closed-loop stability is guaranteed. For example, a sliding-mode-like compensation term can be conveniently used, if upper bounds on the approximation errors and a lower bound on $g(\mathbf{x})$ are available (Fishle and Schroder, 1999).

Remark 1. The proposed adaptive controller relies on a serial-parallel estimation model and thus the knowledge of the full state vector is still required. Moreover, the system model is needed in canonical form.

Remark 2. The assumption that the inherent approximation error w can be easily made arbitrarily small, by simply increasing the number of basis functions, without any drawback other than increased computational costs, in general is not fully acceptable. In fact, w as expressed by (11) also depends on the control input (9) and this can exhibit very high peaks thus making w not negligible.

5. SIMULATION EXAMPLE

Consider a polytopic system represented by a singleton fuzzy system which linearly interpolates two first-order transfer functions with static gains and time constants, given respectively by $K_1 = 3$, $K_2 = 1$ and $\tau_1 = 1$, $\tau_2 = 2$ (Babuška and Oosterom, 2003). The fuzzy system has two symmetric triangular membership functions in the domain $[1,\ 10]$ and it can be recast in normal form (1)-(2) with $n = 1$, by choosing

$$f(x)=-\frac{x}{\tau(x)},\quad g(x)=\frac{K(x)}{\tau(x)} \tag{28}$$

where $K(x)$ and $\tau(x)$ are the scheduled static gain and time-constant respectively. The simplicity of the chosen example allows a transparent illustration of the issues discussed in the previous sections. The *reference model* is a first-order linear system with a time constant $\tau_m = 1/5$ and a static gain $K_m = 1$. The *reference signal* $\boldsymbol{r}_m$ is a repeating sequence with values in the range $[1, 10]$. The feedback gain has been set to $k = 5$. Fuzzy approximators with 4 rules have been considered and no additional robust modifications have been used. The parameter α has been set to 5, the learning rates γ_f, γ_g have been set equal to 10 and $\gamma = 1$.

We have compared AFC schemes with (i) standard adaptive laws (Std), (ii) with the adaptive laws described in (Hojati and Gazor, 2002) (Hoj02) and with the proposed composite adaptation (CompAd). In Fig. 1 it is reported the tracking error for the three schemes. It can be seen that both the composite schemes definitely improve the performance of the standard scheme. The performance of the composite schemes are almost comparable: the controller of (Hojati and Gazor, 2002) slightly outperforms the proposed controller but this is substantially due to the fact that the former uses also $x^{(n)}$. In Fig. 2 it is presented

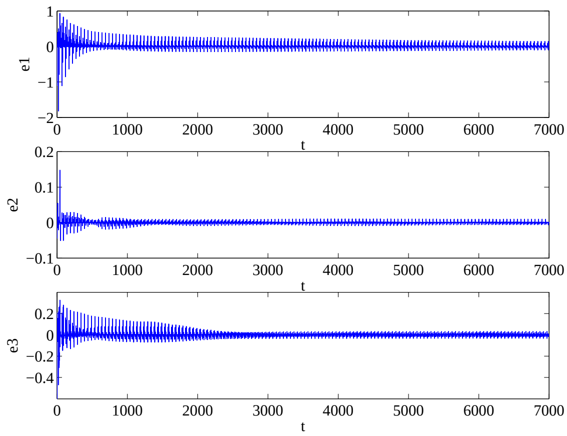

Fig. 1. Tracking error: (i) Std, (ii) Hoj02, (iii) CompAd

the temporal evolution of the singletons of $\hat{f}(x)$ for the three control schemes (similar results hold for $\hat{g}(x)$). It is evident that the proposed control scheme offers a better trade-off between convergence speed and smoothness of the parameter adaptation. Finally,

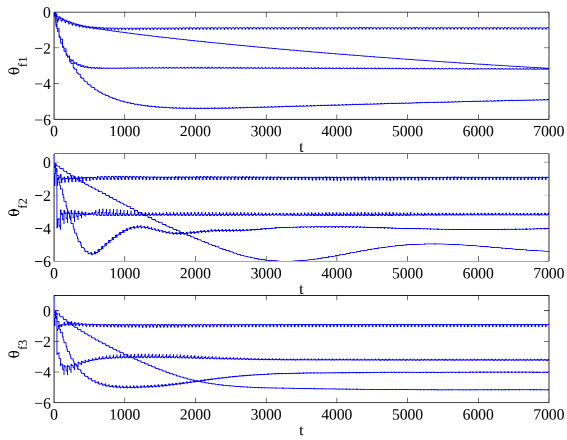

Fig. 2. Estimated singletons for $\hat{f}(x)$: (i) Std, (ii) Hoj02, (iii) CompAd

for testing the accuracy of the identified models, the temporal evolution of $\hat{f}(x)$ has been reported (see Fig. 3). The proposed composite adaptation clearly offers a more accurate instantaneous estimate, compared to the standard scheme and to the controller in (Hojati and Gazor, 2002).

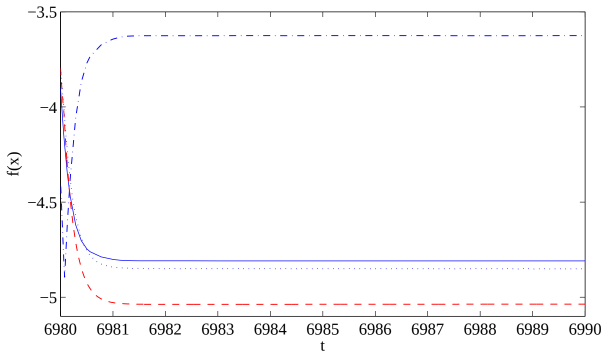

Fig. 3. Estimated $\hat{f}(x)$: true value (solid), Std (dash-dot), Hoj02 (dashed), CompAd (dotted)

6. CONCLUSIONS

In this paper, a novel indirect model reference adaptive fuzzy controller with composite adaptive laws has been presented. It has been shown that such a controller can enhance the performance of standard adaptive controllers while preserving the closed-loop stability. Furthermore, it offers some advantages with respect to other similar controllers with composite adaptive laws, namely, it does not require the knowledge of $x^{(n)}$ and leads to a smoother parameter adaptation. The latter feature positively affects the interpretability of the identified fuzzy systems and possibly the robustness of the control scheme with regards to unmodeled dynamics. The proposed scheme should be tested on high-order systems and on experimental benchmarks in order to fully assess its performance.

REFERENCES

Babuška, R. (2002). Accuracy and transparency of fuzzy systems. In: *Advances in Computational Intelligence and Learning: Methods and Applications* (H-J. Zimmerman, G. Tselentis, M. van Someren and G. Dounias, Eds.). pp. 3–16. Kluwer Academic Publishers. Boston, USA.

Babuška, R. and M. Oosterom (2003). Design of optimal membership functions for fuzzy gain-scheduled control. In: *Fuzzy Systems, 2003. FUZZ '03. The 12th IEEE International Conference on*. Vol. 1. pp. 476–481.

Chang, Y. C. (2000). Robust tracking control of nonlinear MIMO systems via fuzzy approaches. *Automatica* (36), 1535–1545.

Chen, B. S., C. H. Lee and Y. C. Chang (1996). H_∞ tracking design of uncertain nonlinear siso systems: Adaptive fuzzy approach. *IEEE Trans. on Fuzzy Systems* **4**(1), 32–43.

Duarte, M. A. and K. S. Narendra (1989). Combined direct and indirect approach to adaptive control. *IEEE Trans. on Automatic Control* **34**(10), 1071–1075.

Fishle, K. and D. Schroder (1999). An improved adaptive fuzzy control method. *IEEE Trans. on Fuzzy Systems* **7**(1), 27–40.

Han, H., C. Y. Su and Y. Stepanenko (2001). Adaptive control of a class of nonlinear systems with nonlinearly parameterized fuzzy approximators. *IEEE Trans. on Fuzzy Systems* **9**(2), 315–323.

Hojati, M. and S. Gazor (2002). Hybrid adaptive fuzzy identification and control of nonlinear systems. *IEEE Trans. on Fuzzy Systems* **10**(2), 198–210.

Jin, Y. (2000). Fuzzy modeling of high-dimensional systems: Complexity reduction and interpretability improvement. *IEEE Transactions on Fuzzy Systems* **8**(-), 212–22.

Koo, K. M. (2001). Stable adaptive fuzzy controller with time-varying dead-zone. *Fuzzy Sets and Systems* (121), 161–168.

Setnes, M., R. Babuška and H.B. Verbruggen (1998*a*). Rule-based modeling: Precision and transparency. *IEEE Transactions on Systems, Man and Cybernetics – Part C: Applications and Reviews* **28**(1), 165–169.

Setnes, M., R. Babuška and H.B. Verbruggen (1998*b*). Transparent fuzzy modeling: Data exploration and fuzzy sets aggregation. *International Journal of Human Computer Studies* **49**(2), 159–179.

Skrjanc, I., S. Balzic and D. Matko (2002). Direct fuzzy model-reference adaptive control. *International Journal of Intelligent Systems* **17**, 943–963.

Slotine, J. J. E. and W. Li (1991). *Applied Nolinear Control*. Prentice Hall.

Spooner, J. T. and K. M. Passino (1996). Stable adaptive control using fuzzy systems and neural networks. *IEEE Trans. on Fuzzy Systems* **4**(3), 339–359.

Su, C. Y. and Y. Stepanenko (1994). Adaptive control of a class of nonlinear systems with fuzzy logic. *IEEE Trans. on Fuzzy Systems* **2**(4), 285–294.

Tong, S., J. Tang and T. Wang (2000). Fuzzy adaptive control of multivariable nonlinear systems. *Fuzzy Sets and Systems* (111), 153–167.

Wang, L. X. (1993). Stable adaptive fuzzy control of nonlinear systems. *IEEE Trans. on Fuzzy Systems* **1**(2), 146–154.

Wang, L. X. (1995). Design and analysis of fuzzy identifiers of nonlinear dynamic systems. *IEEE Trans. on Automatic Control* **40**(1), 11–23.

Wang, L. X. (1996). Stable adaptive fuzzy controllers with application to inverted pendulum tracking. *IEEE Trans. on Systems, Man, and Cybernetics* **26**(5), 677–691.

Yin, T. K. and C. S. G. Lee (1995). Fuzzy model reference adaptive control. *IEEE Trans. on Systems, Man and Cybernetics* **25**(12), 1606–1615.