AN OPTIMAL EXTRAPOLATOR FOR REDUCING PHASE DELAY OF SAMPLE DATA-HOLD

Reza Shahnazi [*], Hamid Khaloozadeh [**]

[*]Department of Electrical Engineering Ferdowsi University of Mashhad, Mashhad, Iran
[*]re_sh50@stu-mail.um.ac.ir
[**]Department of Electrical Engineering, K.N.T University of Technology, Tehran, Iran
[**]h_khaloozadeh@eetd.kntu.ac.ir

Abstract: The objective of this paper is to minimize the phase delay of the zero-order hold (ZOH) an usual sample-data hold device. In this paper an optimized data holding scheme whose phase delay is significantly less than of the ZOH is presented. The method is based on using optimal extrapolators, which has a better result than extrapolators proposed by Yekutiel (1980) and Belicynski and Kozinski (1984). This method is more general than the other methods.

Keywords: Sample and hold, Optimization, Bandwidth, Discrete systems, Digital filter.

1. INTRODUCTION

Discrete time control systems may operate partly in discrete time and partly in continuous time. Thus in such control systems some signals appear as discrete-time functions and other signals as continuous-time functions. In most cases continuous time systems controlled by a digital computer, so a data hold circuit will be used to convert the discrete-time signal into a continuous-time signal (Ogata, 1987). The most frequently employed holding device is the "zero-order-hold" (ZOH), which implies that the circuit holds the amplitude of the sample from one sampling instant to the next.

A technique which often was employed for designing digital controllers is designing a continuous controller in S-plane and transfers it to Z-plane by bilinear transformation. In such cases the intrinsic phase delay by the ZOH has direct reduction in phase margin. To reduce phase delay Yekutiel (1980), Belicynski and Kozinski (1984) and Leonard (1999) proposed a digital filter, which can be achieved by modification the digital controller software. The method which Yekutiel (1980) proposed was named Piece-wise Constant Higher-Order-Hold (PC-HOH), which includes a standard ZOH hardware and modifying the output level of it by the digital filter. The filter is obtained by using Taylor's series expansion (order m) as an extrapolator polynomial for the new output value, $y(nT+\Delta)$, and is as follows

$$G_m(z) = \frac{y_m(z)}{y(z)} = \sum_{i=0}^{m} \frac{\Delta^i}{i!}\left(\frac{1-z^{-1}}{T}\right)^i \tag{1}$$

which $y^{(i)}(nT)$ denotes the i-th time derivative of $y(t)$. In PC-HOH, $\Delta = T/2$ is assumed, so $G_m(z)$ will be as follows

$$G_m(z) = \sum_{i=0}^{m} \frac{1}{i!}\left(\frac{1-z^{-1}}{2}\right)^i \tag{2}$$

Table 1 shows the transfer functions for m=0, 1, 2. It is obvious m=0 represents the standard ZOH.

Table 1 Digital filter transfer functions related to PC-HOH

Order (*m*)	$G_i(z)$ for $\Delta=T/2$
0	1
1	$1.5-0.5z^{-1}$
2	$1.625-0.75z^{-1}+0.125z^{-2}$

Belicynski and Kozinski (1984) proposed another method based on Newton Extrapolation Polynomial. The Newton Extrapolation Polynomial Method (NEPM) sets its output at the mT-th moment to a level equal to the predicted value $y[(m+q)T]$ where $0 \le q < 1$ and holds it for one sampling period. The prediction value of $y[(m+q)T]$, denoted $\tilde{y}[(m+q)T]$, can be obtained by evaluating the mth-order Newton Extrapolation Polynomial by defining $y(iT)=y_i$ and $y[(m+q)T]=\tilde{y}_{m+q}$, and finally, the proposed filter is obtained as follows

$$G_f(z)=\frac{\tilde{y}(z)}{y(z)}=\sum_{k=0}^{m} B_k^m(q)z^{-k} \tag{3}$$

where

$$B_k^m(q)=(-1)^k\binom{q-1+k}{k}\binom{q+m}{m-k} \tag{4}$$

Repeatedly Belicynski and Kozinski (1984) assumed that q=0.5 or $\Delta=qT=T/2$. Table 2 represents filter transfer functions for m=0, 1, 2 using NEPM. It is shown that the NEPM has better result than PC-HOH.

Table 2 Digital filter transfer functions related to NEPM

Order(*m*)	$G_i(z)$ for $q=\Delta/T=0.5$
0	1
1	$1.5-0.5z^{-1}$
2	$1.875-1.25z^{-1}+0.375z^{-2}$

The method which Leonard (1999) proposed, is not based on extrapolation polynomials but he used the digital phase lead filter as follows

$$F(z)=\frac{1+bz^{-1}}{1+b} \qquad -1<b<1 \tag{5}$$

Condition $-1<b<1$ guarantees that $F(z)$ is a minimum phase filter. Since $F(1)=1$, the low frequencies are not changed with $F(z)$.

As it is known the ZOH transfer function is as follows

$$ZOH(j\omega)=\frac{1-\exp(-j\omega T)}{j\omega} \tag{6}$$

see for example (Ogata, 1987; Phillips and Harbor, 1988) that has a phase lag of $0.5\omega T$.

The usual Digital to Analog Converters (DAC) contains a sampler that has a gain $1/T$, so, compounding the frequency responses of (5) and (6) reach to:

$$H(j\omega)=\frac{1}{T}ZOH(j\omega)\frac{1+b\exp(-j\omega T)}{1+b} \tag{7}$$

In order to eliminate the phase delay of ZOH efficiently, Leonard (1999) proposed minimizing a cost function $J(b)$ which is defined as follows

$$J(b)=\int_0^{\omega_{BW}}\left|\Phi(\omega,b)\right|^2 d\omega \tag{8}$$

where $\Phi(\omega,b)$ is the phase of $H(j\omega)$ and ω_{BW} is the closed-loop bandwidth of the plant under control. This method is called Optimal Filter Method (OFM), which is compounding standard ZOH and optimal filter. From Sampling Theorem and Aliasing Phenomenon, the relation between sampling frequency and ω_{BW} can be found as

$$\omega_s = 6 \ \ to\ 25\ \omega_{BW}=k\omega_{BW} \tag{9}$$

Table 3 represents the optimal value of b for different k. It is shown that OFM has better result than PC-HOH and NEPM, (for m=1). One advantage of OFM besides of a better phase delay compensation is, the OFM take into account the ω_{BW} and the ratio of sampling-time to ω_{BW} ($k=\omega_s/\omega_{BW}$) of the system to its cost function.

In the next section the proposed method which is called Optimal Extrapolator is presented. Section 3 presents the stability analysis of the proposed method. Frequency responses comparison among the different methods is done in section 4. An example is simulated in section 5.

2. MAIN RESULT

The main contribution of this paper is in defining a cost function which has been involved by $\Delta(0\le\Delta<T)$ or $q(0\le q<1)$ and the best Δ or q is computed by minimization this cost function. As it was seen in (1) the digital filter transfer function related to PC-HOH in Z-plane is as follow

$$G_m(z)=\frac{y_m(z)}{y(z)}=\sum_{i=0}^{m}\frac{\Delta^i}{i!}\left(\frac{1-z^{-1}}{T}\right)^i$$

The frequency behavior of $G_m(z)$ $m=1,2,...$ is obtained by substitution of $z=\exp(j\omega T)$ in (1), so

$$G_m(\exp(j\omega T))=\sum_{i=0}^{m}\frac{\Delta^i}{i!}\left(\frac{1-\exp(-j\omega T)}{T}\right)^i \tag{10}$$

Considering (6), since a DAC contains a sampler which has a gain $1/T$, so the compound transfer function of ZOH and proposed method for m=1,2 is as follows

$$H_{11}(j\omega) = \frac{1}{T} ZOH(j\omega) \sum_{i=0}^{1} \frac{\Delta_1^i}{i!} \left(\frac{1-\exp(-j\omega T)}{T}\right)^i \qquad (11)$$

$$H_{21}(j\omega) = \frac{1}{T} ZOH(j\omega) \sum_{i=0}^{2} \frac{\Delta_2^i}{i!} \left(\frac{1-\exp(-j\omega T)}{T}\right)^i \qquad (12)$$

The phase of $H_{i1}(j\omega)$ has been shown by $\Phi_{i1}(\omega, \Delta_i)$ for i=1, 2, respectively as below:

$$\Phi_{11}(\omega, \Delta_1) = -0.5\omega T + \tan^{-1}\left(\frac{\Delta_1 \sin(\omega T)}{T + \Delta_1 (1-\cos(\omega T))}\right) \qquad (13)$$

$$\Phi_{21}(\omega, \Delta_2) = -0.5\omega T + \tan^{-1}\left(\frac{2T\Delta_2 \sin(\omega T) + \Delta_2^2 (2\sin(\omega T) - \sin(2\omega T))}{2T^2 + 2T\Delta_2 (1-\cos(\omega T)) + \Delta_2^2 ((1+\cos(2\omega T)) - 2\cos(\omega T))}\right) \qquad (14)$$

Like (8) a cost function as follows can be defined

$$J_{i1}(\Delta_i) = \int_0^{\omega_{BW}} \left|\Phi_{i1}(\omega, \Delta_i)\right|^2 d\omega \qquad i = 1,2 \qquad (15)$$

The objective is to minimize $J_{i1}(\Delta_i)$ for i=1, 2.

Now, for minimizing $J_{i1}(\Delta_i)$ i=1, 2 with respect to variation of Δ_i in the interval [0,T] the function ***quad*** for integration and function ***min*** for finding the minimum from MATLAB software has been used. For better comparison Δ_i was normalized in interval [0,1], which the normalized value of Δ_i is denoted by Δ_{i_n} . Table 4 shows Δ_{i_n} for i=1,2 and for different k in the interval [0,1]. It can be seen from Table 4 that for small k the optimal Δ is far from T/2 but when k goes larger it will be close to T/2. This proposed holding device is denoted by P_1.

The above approach can be repeated for NEPM. From (3) we have

$$G_f(z) = \frac{\tilde{y}(z)}{y(z)} = \sum_{k=0}^{m} B_k^m(q) z^{-k}$$

It was mentioned that in NEPM for finding transfer functions q=0.5 was assumed, but it is not an optimal q. Like (11) and (12) the compound transfer functions for m=1,2 are as follows

$$H_{12}(j\omega) = \frac{1}{T} ZOH(j\omega) \sum_{k=0}^{1} B_k^m(q_1) \exp(-kj\omega T) \qquad (16)$$

$$H_{22}(j\omega) = \frac{1}{T} ZOH(j\omega) \sum_{k=0}^{2} B_k^m(q_2) \exp(-kj\omega T) \qquad (17)$$

The phase of $H_{i2}(j\omega)$ has been shown by $\Phi_{i2}(\omega, q_i)$ for i=1,2, respectively, therefore

$$\Phi_{12}(\omega, q_1) = -0.5\omega T + \tan^{-1}\left(\frac{q_1 \sin(\omega T)}{q_1 (1-\cos(\omega T)) + 1}\right) \qquad (18)$$

$$\Phi_{22}(\omega, q_2) = -0.5\omega T + \tan^{-1}\left(\frac{q_2(q_2+1)\sin(\omega T) - 1/2 q_2 (q_2+1)\sin 2\omega T}{1/2(q_2+1)(q_2+2) - q_2(q_2+2)\cos(\omega T) + 1/2 q_2 (q_2+1)\cos(2\omega T)}\right) \qquad (19)$$

Like (15) a cost function as follows can be defined:

$$J_{i2}(q_i) = \int_0^{\omega_{BW}} \left|\Phi_{i2}(\omega, q_i)\right|^2 d\omega \qquad i = 1,2 \qquad (20)$$

Again the objective is to minimize $J_{i1}(q_i)$ for i=1,2, with respect to q_i . Table 5 shows optimal q_i for different values of k. This proposed holding device is denoted by P_2.

Since $\Delta = qT$ so (13) is the same as (18). Therefore P_1 and P_2 are the same for m=1.

Investigating the Table 3,4 it is seen that P_1, P_2 and the OFM have the same zero (zero at $z= q/(q+1)=-b$) and the same pole (pole at z=0) for m=1.

One advantage of these two methods is to permit of using lower sampling rate. It can be seen that the closed-loop bandwidth of system is involved in P_1 and P_2. Table 6 and 7 represents the digital filter transfer functions related to P_1 and P_2 using the optimal Δ_i and q_i for k=6, and m=0, 1, 2.

The higher order filters can be obtained as above, easily.

Table 3 The parameters of the OFM method for different k

k	**2**	**3**	**4**	**5**	**6**	**8**	**10**	**15**	**20**	**25**
b	-0.619	-0.547	-0.477	-0.432	-0.405	-0.374	-0.359	-0.345	-0.340	-0.338

Table 4 Proposed method parameters for PC-HOH and different *k* (m=1,2)

K	2	3	4	5	6	8	10	15	20	25	30
Δ_{1n}	0.9997	0.9995	0.9110	0.7582	0.6769	0.6151	0.5601	0.5350	0.5020	0.5086	0.5152
Δ_{2n}	0.7503	0.6182	0.5326	0.4967	0.4835	0.4798	0.4888	0.5231	0.5020	0.4954	0.5073

Table 5 The parameters of the proposed method for NEPM and different *k* (m=1,2)

k	2	3	4	5	6	8	10	15	20	25	30
q_1	0.9997	0.9995	0.9110	0.7582	0.6769	0.6151	0.5601	0.5350	0.5020	0.5086	0.5152
q_2	0.67	0.53	0.45	0.42	0.421	0.43	0.45	0.45	0.55	0.49	0.50

3. STABILITY ANALYSIS

As mentioned in section 2 and from equation (1) the transfer functions for m=1 and 2 are:

$$1+\frac{1-z^{-1}}{T}\Delta \qquad 0\le\Delta<T \tag{21}$$

$$1+\frac{1-z^{-1}}{T}\Delta+\left(\frac{1-z^{-1}}{T}\right)^2\frac{\Delta^2}{2} \qquad 0\le\Delta<T \tag{22}$$

respectively. As it is known $\Delta = qT$ which $0\le q<1$, so equations (21) and (22) will become as follows

$$\frac{(q+1)z\text{-}q}{z} \qquad 0\le q<1 \tag{23}$$

$$\frac{(q^2+q+2)z^2-2q(q+1)z+q^2}{2z^2} \qquad 0\le q<1 \tag{24}$$

Obviously (23) is the same as (3) for m=1. The transfer function of (3) for m=2 is

$$\frac{(q+1)(q+2)z^2-2q(q+2)z+q(q+1)}{2z^2} \tag{25}$$

It is seen, the poles of the transfer functions are in the unit circle since $0\le q<1$. The zero of (23) is in the unit circle because $z= q/(q+1)$. Considering Jury's lemma it is obvious that the numerator polynomial of (24) and (25) satisfies the stability conditions. It can be seen that the transfer functions of the proposed methods for *m*=1,2 are both minimum phase for any *q* in the interval [0,1], so they are easy to implement. Also from Table 6 and 7 it can be seen that they are close to each other, for *m*=2, so it can be said that the optimization methods reach to the same results.

Table 6 Digital filter transfer functions related to P_1

Order(*m*)	$G_i(z)$ for k=6
0	1
1	$1.679-0.669z^{-1}$
2	$1.7173-0.9510z^{-1}+0.2338z^{-2}$

Table 7 Digital filter transfer functions related to P_2

Order(*m*)	$G_i(z)$ for k=6
0	1
1	$1.679-0.669z^{-1}$
2	$1.7182-1.0164z^{-1}+0.2982z^{-2}$

4. FREQUENCY RESPONSE

In this part the comparison of the frequency responses of different methods is shown. The closed-loop bandwidth $\omega_{BW}=1.66$ and *k*=6 has been assumed.

Figs. 1 and 2 show the Bode plots of ZOH, P_1, P_2 for m=1 and OFM. It can be seen that P_1, P_2 for *m*=1 and OFM as mentioned in section 2 have same frequency response. It is also can be seen that the phase delay has been compensated. Figs. 3 and 4 represent the Bode plots of ZOH, PC-HOH, NEPM, P_1 and P_2 for *m*=1. It can be seen that P_1 and P_2 has better phase delay compensation.

Figs. 5 and 6 represents the Bode plots of ZOH, PC-HOH and P_1 for *m*=2, and Figs. 7 and 8 represent the Bode plots of ZOH, NEPM and P_2 for *m*=2.
It can be seen that, although for m=2, PC-HOH and NEPM has higher phase than P1 and P2 but they are not optimal with respect to cost functions defined in (15) and (20), so optimal extrapolators have better result than PC-HOH, NEPM and it concludes OFM, too.

Magnitude distortion as will be seen in Figs. 1, 3, 5 and 7 is acceptable, because in most applications an anti-aliasing filter is employed to attenuate high frequencies and also frequencies near Nyquist frequencies in order to avoid aliasing consequences, see for example (Astorm and Wittenmark, 1990; Landau, 1993; Leonard, 1999), therefore reducing the phase lag on bandwidth, will be more important. Clearly, as can be seen in Figs. 2, 4, 6 and 8 by increasing the extrapolation order (*m*) this aim can be achieved.

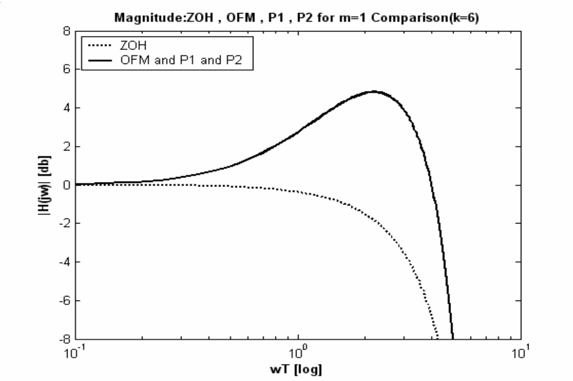

Fig. 1. Frequency Response-Magnitude P_1, P_2 and OFM

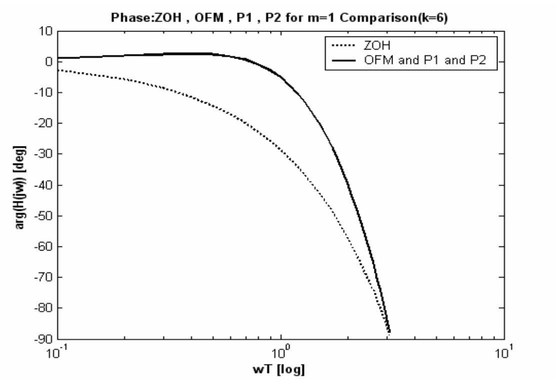

Fig. 2. Frequency Response-phase P_1, P_2 and OFM

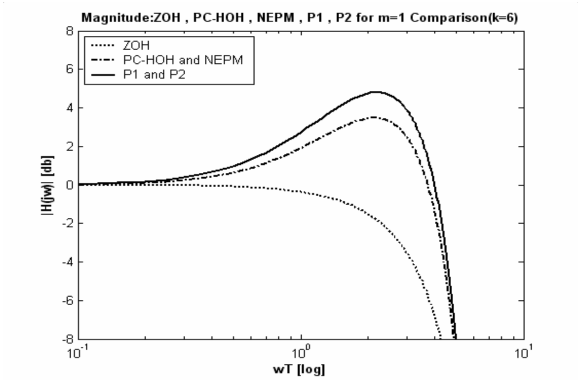

Fig. 3. Frequency Response-Magnitude P_1, P_2 , PC-HOH and NEPM for m=1

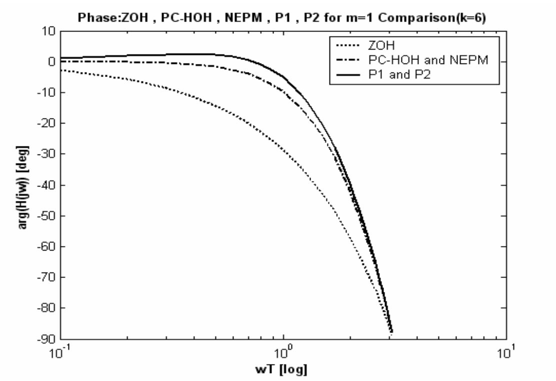

Fig. 4. Frequency Response-Phase P_1, P_2, PC-HOH NEPM for m=1

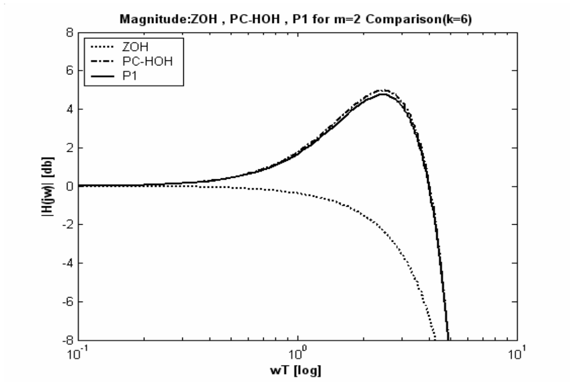

Fig. 5. Frequency Response-Magnitude P_1, PC-HOH for m=2

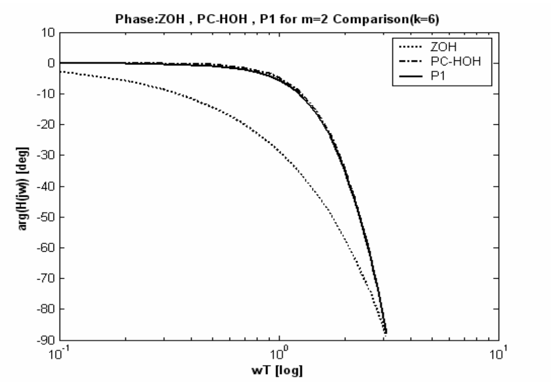

Fig. 6. Frequency Response-Phase P_1, PC-HOH for m=2

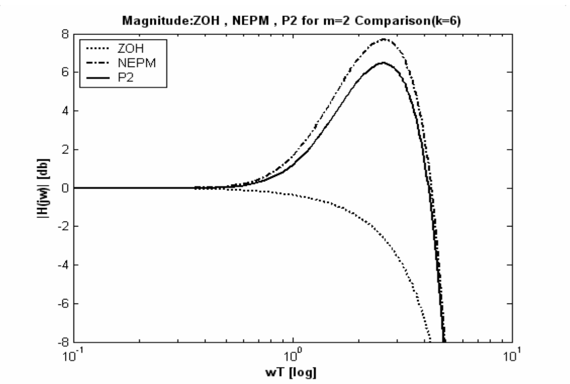

Fig. 7. Frequency Response-Magnitude P_2, NEPM for m=2

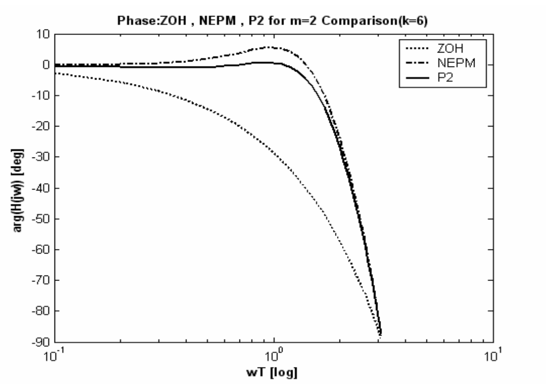

Fig. 8. Frequency Response-Phase P_2, NEPM for m=2

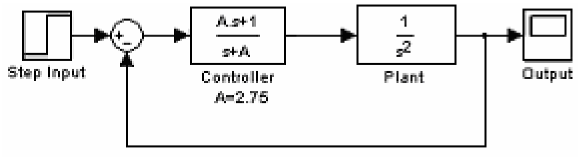

Fig. 9. Plant and continuous controller

Table 8 Results of system using different controllers

System	Phase Margin [deg]	Gain Margin [dB]	Open Loop BW (0 dB) [rad/s]	Closed Loop BW (0 dB) [rad/s]	Closed Loop Peak/Freq. [dB]/[rad/s]	Step Over shoot [%]
Continuous	50.03	Inf	1	1.24	2.58/0.63	27.72
Digital + ZOH	31.78	8.65	1.01	1.71	5.2/1.05	51.94
Digital+P1(m=1)	48.49	6.79	1.24	2.6	2.2/0.660	29.57
Digital+P1(m=2)	50.14	7.49	1.08	1.58	2.54/0.666	24

5. EXAMPLE

Consider the lead/lag compensated double-integrator plant which has been shown in Fig. 9. This system is used as a benchmark in such cases. If the controller digitized by Tustin transformation then it leads to $G(z) = 1.644(z-0.7950)/(z-0.037)$.

The open loop bandwidth is $\omega_{co} = 1\ rad/s$ .The sampling frequency has been selected as $\omega_s = 10\ rad/s.$ So the closed-loop bandwidth $\omega_{BW} = \omega_s / 6$ =1.66>1 is obtained. The controller is followed by one of three holding devices 1) The standard ZOH 2) P_1 for m=1 3) P_1 for m=2. Table 8 demonstrates the systems characteristic. Phase delay compensation can be deduced when optimal extrapolator is added to the classical ZOH.

6. CONCLUSIONS

In this paper a new method for reducing the phase delay of ZOH was presented. The proposed method that is called Optimal Extrapolator provides better result than PC-HOH and NEPM. The proposed method for *m*=1 has as the same results as OFM. Furthermore the proposed method has a general framework for higher order extrapolator polynomials.

REFERENCES

Astorm, K.J., and B. Wittenmark (1990) *Computer controlled systems.* Englewood Cliffs, NJ: Prentice-Hall.

Beliczynski, B. and W. Kozinski (1984). *A reduced-delay sampled-data hold*. IEEE TRANS. AUTOMAT. CONTR., VOL.AC-29, NO.2, 179-181.

Landau, I.L. (1993). *Identification et commande des systems*. FRANCE HERMES.

Leonard, F. (1999). *First-order optimal reduced-delay sample-data holds*. IEEE TRANS. AUTOMAT. CONTR., VOL.44, NO. 7, 1446-1449.

Ogata, K. (1987). *Discrete-Time Control Systems.* Prentice-Hall, Inc. 1987.

Phillips C.L. and R. D. Harbor (1988). *Feedback Control Systems* Prentice-Hall, Inc.

Yekutiel, F. (1980). *A reduced-delay sampled-data hold*. IEEE TRANS. AUTOMAT. CONTR., VOL.AC-25, NO. 4, 847-850.