

# NONLINEAR CONTROL OF BOOST AC/DC CONVERTERS OUTPUT VOLTAGE REGULATION AND POWER FACTOR CORRECTION

Abdelmajid ABOULOIFA, Fouad GIRI, Ibtissam LACHKAR

GREYC, ISMRA, 6 Bd Maréchal Juin, 14050 Caen

Abstract: We are considering the problem of controlling *AC/DC* switched power converters of the *Boost* type. The control objectives are twofold: (i) regulating the output voltage to a desired reference value, (ii) assuring a unitary power factor by enforcing the voltage and the current delivered by the electric network to be in phase. The considered problem is dealt with by designing a *nonlinear* controller involving a *cascade-structure*. The inner loop regulates the active power; it is built-up using the backstepping design approach. The outer loop regulates the converter squared output voltage using a filtered PI regulator. The controller thus obtained is shown, using tools from the averaging theory, to achieve its objectives. *Copyright © 2005 IFAC*

Keywords: Power converters, Voltage control, power factor correction.

## 1. INTRODUCTION

Static power converters have a very wide domain of applications. However, these converters still have an important drawback as they contribute to the pollution of the electric network. Indeed, these converters constitute *nonlinear* loads for the distribution network and, consequently, generate harmonic currents that may cause some annoying effects such as extra power losses in the network. Therefore, converter controllers should not only have as objective output voltage regulation, but also rejection of the mentioned current harmonics. Most of previous works have focussed only on voltage regulation (Sira, *et al.*, 1997).

In the present paper, we are considering the problem of controlling a whole *AC/DC* converter (Mecha and Funabiki, 1993;). We will particularly focus on *AC/DC* converters with boost chopper (Fig.1). Our objective is to regulate the output voltage while ensuring a unitary power factor (PF). The last objective amounts to rejecting the whole current harmonics at the converter input. On the other hand, *AC/DC* converters are featured by their variable-structure and their nonlinear dynamics. To deal with

the considered control problem a nonlinear controller including two loops is built-up.

The inner loop is first developed, using the *backstepping* technique, in such a way that the converter input current be *sinusoidal* and *in phase* with the network supply voltage. The converter variable-structure feature is coped with basing the above regulator design upon an *average model* of the system. It is worth noting, that model averaging is widely used in the literature (Krein, 1990).

The natural purpose of the outer loop would be the regulation of the converter output voltage  $v_o$ . However, we will choose to perform regulation of  $v_o^2$  rather than  $v_o$ . Actually,  $v_o^2$  undergoes a (first-order) *linear* differential equation while  $v_o$  undergoes a *nonlinear* equation. Using this variable transformation ( $v_o \rightarrow v_o^2$ ), reference tracking on  $v_o^2$  can be achieved using a filtered PI regulator.

A theoretical analysis, involving tools from the Lyapunov stability and the averaging theory, shows that the nonlinear cascade controller thus constructed actually achieves, in the mean, its objectives. The

controller performances and robustness (with respect to load changes) are further illustrated by many simulated examples.

## 2. MODELLING OF THE CONVERTER

The converter under study is represented by Fig.1 It includes three main parts, namely a LC-filter, a diode bridge rectifier and a boost chopper. The latter operates according to the Pulse Width Modulation (PWM) principle, (Mahdavi, *et al.*, 1997). This means that time is shared in intervals of length  $T$ . Within any period, the IGBT-switch is ON during  $\alpha T$ , for some  $0 \leq \alpha \leq 1$ . Then, energy is stored in the inductance  $L_o$  and the diode  $D_o$  is blocked. During the rest of the period, i.e.  $(1-\alpha)T$ , the switch IGBT is OFF and, consequently, the inductance discharges in the load resistance  $R_o$ . The value of  $\alpha$  varies from a period to another and its variation law determines the trajectory of output voltage  $v_o$ . The variable  $\alpha$ , called duty cycle, then turns out to be the control input for the converter.

Mathematical modeling of the converter is completed applying Kirchhoff's laws. So doing, one gets:

$$\frac{di_n}{dt} = -\frac{v_{rect}}{L} + \frac{v_n}{L} \quad (1)$$

$$\frac{dv_{rect}}{dt} = -\frac{i_{rect}}{C} + \frac{i_n}{C} \quad (2)$$

$$\frac{di_{L_o}}{dt} = -\frac{v_{IGBT}}{L_o} + \frac{|v_{rect}|}{L_o} \quad (3)$$

$$C_o \frac{dv_o}{dt} = i_{D_o} - \frac{v_o}{R_o} \quad (4)$$

The current  $i_{rect}$  flowing in the input rectifier is obtained from (5):

$$i_{rect} = i_o \text{sign}(v_{rect}) \quad (5)$$

The voltage  $v_{IGBT}$  takes undergoes different equations depending on the state of the IGBT-switch. These equations can be given a unique mathematical expression by introducing a binary variable  $\mu$  :

$$\mu = \begin{cases} 1 & \text{if IGBT is ON} \\ 0 & \text{if IGBT is OFF} \end{cases} \quad (6)$$

Then, one has for  $v_{IGBT}$  the following expressions:

$$v_{IGBT} = (1-\mu)v_o \quad (7)$$

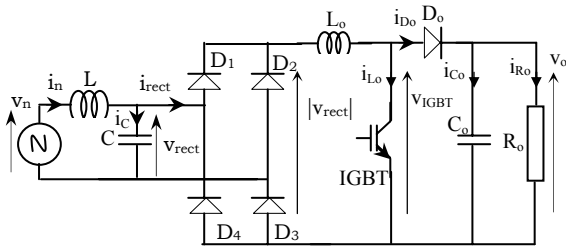


Fig.1. AC/DC Boost converter

Similarly, the current  $i_{D_o}$  in the diode  $D_o$  undergoes different laws, depending on the states of the IGBT-switch. It is given by:

$$i_{D_o} = (1-\mu)i_{L_o} \quad (8)$$

Substituting (5), (7) and (8) in (1)-(4), yields the final form of the (instantaneous) converter model:

$$\frac{di_n}{dt} = -\frac{v_{rect}}{L} + \frac{v_n}{L} \quad (9a)$$

$$\frac{dv_{rect}}{dt} = -\frac{i_{L_o} \text{sign}(v_{rect})}{C} + \frac{i_n}{C} \quad (9b)$$

$$\frac{di_{L_o}}{dt} = \frac{|v_{rect}|}{L_o} - \frac{(1-\mu)}{L_o} v_o \quad (9c)$$

$$\frac{dv_o}{dt} = \frac{(1-\mu)}{C_o} i_{L_o} - \frac{v_o}{R_o C_o} \quad (9d)$$

This model is useful to build-up an accurate simulator for the converter. However, it cannot be based upon to design a continuous control law as it involves a binary control input, namely  $\mu$ . To overcome this difficulty, it is usually resorted to the averaging process over cutting intervals, (Krein, 1990). This process is shown to give rise to average versions (of the above model) involving as a control input the mean value of  $\mu$  which is nothing other than the duty cycle  $\alpha$ . The average model turns out to be the following:

$$\frac{di_n}{dt} = -\frac{v_{rect}}{L} + \frac{v_n}{L} \quad (10a)$$

$$\frac{dv_{rect}}{dt} = -\frac{i_{L_o} \text{sign}(v_{rect})}{C} + \frac{i_n}{C} \quad (10b)$$

$$\frac{di_{L_o}}{dt} = \frac{|v_{rect}|}{L_o} - \frac{(1-\alpha)}{L_o} v_o \quad (10c)$$

$$\frac{dv_o}{dt} = \frac{(1-\alpha)}{C_o} i_{L_o} - \frac{v_o}{R_o C_o} \quad (10d)$$

## 3. CONTROLLER DESIGN

The controller synthesis will be performed in two major steps. First, a current inner loop is designed to cope with the PFC issue. In the second step, an outer voltage loop is built-up to achieve voltage regulation.

### 3.1 Current inner loop design

The PFC objective means that the converter input current should be *sinusoidal* and *in phase* with the network supply voltage. We therefore seek a regulator that enforces the current  $i_n$  to track a reference signal of the form  $i_{nref} \stackrel{\text{def}}{=} \beta v_n$ . At this point the parameter  $\beta$  is any real number. The regulator will now be designed using the backstepping technique (Krstic, 1995), based on the (partial) model (10a-c).

### 3.1.1 Step 1: Stabilization of the sub-system (10a)

Let us introduce the following tracking error:

$$z_1 = i_n - i_{nref} \quad (11)$$

where  $i_{nref} \stackrel{\text{def}}{=} \beta v_n$  denote the corresponding reference signal. Using (10a), time-derivation of (11) yields the following error dynamics:

$$\frac{dz_1}{dt} = -\frac{v_{rect}}{L} + \frac{v_n}{L} - \frac{di_{nref}}{dt} \quad (12)$$

In (12),  $(-v_{rect}/L)$  stands as a (virtual) control variable. Then,  $z_1$  can be regulated to zero if  $-(v_{rect}/L) = \sigma$  where  $\sigma$  is a stabilising function defined by:

$$\sigma = -\frac{v_n}{L} + \frac{di_{nref}}{dt} - c_1 z_1 \quad (13)$$

Indeed, this choice would imply that:  $\dot{z}_1 = -c_1 z_1$  (where  $c_1 > 0$  is a design parameter) which clearly establishes asymptotic stability of (12) with respect to the Lyapunov function :

$$W_1 = 0.5z_1^2 \quad (14)$$

Because:

$$\dot{W}_1 = -c_1 z_1^2 < 0 \quad (15)$$

As  $(-v_{rect}/L)$  is not the actual control input, a new error variable  $z_2$  between the virtual control and its desired value  $\sigma$  (stabilising function) is introduced:

$$z_2 = -v_{rect}/L - \sigma \quad (16)$$

Then, equation (12) becomes, using (13) and (16):

$$\dot{z}_1 = -c_1 z_1 + z_2 \quad (17)$$

Also, the Lyapunov function derivative becomes :

$$\dot{W}_1 = -c_1 z_1^2 + z_1 z_2 \quad (18)$$

### 3.1.2 Step 2: Stabilization of the sub-system (10a-b)

Achieving the PFC objective amounts to enforcing the error variables  $(z_1, z_2)$  to vanish. To this end, one needs the dynamics of  $z_2$ . Deriving (16), it follows from (10b) that:

$$\frac{dz_2}{dt} = \frac{i_{Lo} \text{sign}(v_{rect})}{LC} - \frac{i_n}{LC} - \frac{d\sigma}{dt} \quad (19)$$

In the above equation, the quantity  $\frac{i_{Lo} \text{sign}(v_{rect})}{LC}$  stands as a ‘‘virtual control input’’. We now need to select a Lyapunov function  $W_2$  for the  $(z_1, z_2)$ -system. As the objective is to drive its states  $(z_1, z_2)$  to zero, it is natural to choose the following function:

$$W_2 = W_1 + 0.5z_2^2 \quad (20)$$

Using (18) and (19), this implies:

$$\dot{W}_2 = -c_1 z_1^2 + z_2(z_1 + \dot{z}_2) \quad (21)$$

This shows that, for the  $(z_1, z_2)$ -system to be globally asymptotically stable, it is sufficient to choose the virtual control input so that  $\dot{W}_2 = -c_1 z_1^2 - c_2 z_2^2$  (for any  $c_2 > 0$ ). Then, it follows from (21) that:

$$\dot{z}_2 = -z_1 - c_2 z_2 \quad (22)$$

Replacing in (22)  $\dot{z}_2$  by its expression (19) and solving the resulting equation with respect to  $\frac{i_{Lo} \text{sign}(v_{rect})}{LC}$ , yields the following stabilising function:

$$\delta = -z_1 - c_2 z_2 + \frac{i_n}{LC} + \frac{d\sigma}{dt} \quad (23)$$

As  $\frac{i_{Lo} \text{sign}(v_{rect})}{LC}$  is not the actual control input, a new error variable  $z_3$  between the virtual control and its desired value  $\delta$  is introduced:

$$z_3 = \frac{i_{Lo} \text{sign}(v_{rect})}{LC} - \delta \quad (24)$$

Then, equation (19) becomes, using (23) and (24):

$$\dot{z}_2 = -z_1 - c_2 z_2 + z_3 \quad (25)$$

Also, the Lyapunov function derivative becomes:

$$\dot{W}_2 = -c_1 z_1^2 - c_2 z_2^2 + z_2 z_3 \quad (26)$$

### 3.1.3 Step 3: Stabilization of the sub-system (10a-c)

Time-derivation of  $z_3$  gives, using (24) and (10c):

$$\frac{dz_3}{dt} = \frac{\text{sign}(v_{rect})}{LC} \left( \frac{|v_{rect}|}{L_o} - \frac{(1-\alpha)}{L_o} v_o \right) - \frac{d\delta}{dt} \quad (27)$$

The actual control variable, namely  $\alpha$ , appears for the first time in equation (27). An appropriate control law for generating  $\alpha$ , has now to be found for the system (17), (25), (27) whose state vector is  $(z_1, z_2, z_3)$ . Let us consider the Lyapunov function  $W_3$ :

$$W_3 = W_2 + 0.5z_3^2 \quad (28)$$

Its time-derivative along trajectory of (26) and (27) is:

$$\begin{aligned} \frac{dW_3}{dt} = & -c_1 z_1^2 - c_2 z_2^2 \\ & + z_3 \left( \frac{\text{sign}(v_{rect})}{LC} \left( \frac{|v_{rect}|}{L_o} - \frac{(1-\alpha)}{L_o} v_o \right) - \frac{d\delta}{dt} + z_2 \right) \end{aligned} \quad (29)$$

This shows that, for the  $(z_1, z_2, z_3)$ -system to be globally asymptotically stable, it is sufficient to choose the control  $\alpha$  so that  $\dot{W}_3 = -c_1 z_1^2 - c_2 z_2^2 - c_3 z_3^2$  which in view of (29) amounts to ensuring that:

$$\frac{\text{sign}(v_{rect})}{LC} \left( \frac{|v_{rect}|}{L_o} - \frac{(1-\alpha)}{L_o} v_o \right) - \frac{d\delta}{dt} + z_2 = -c_3 z_3 \quad (30)$$

Solving the resulting equation with respect to  $\alpha$ , yields the following backstepping control law:

$$\alpha = 1 - \frac{L_o LC \text{sign}(v_{rect})}{v_o} \left( z_2 + c_3 z_3 - \frac{d\delta}{dt} \right) - \frac{|v_{rect}|}{v_o} \quad (31)$$

**Proposition 3.1.** Consider the system, next called *inner closed-loop*, consisting of the subsystem (10a-d) and the control law (31). If the ratio  $\beta$  and its three first derivatives are available, then the inner closed-loop system undergoes, in the  $(z_1, z_2, z_3)$ -coordinates, the following equation:

$$\frac{d}{dt} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} = \begin{pmatrix} -c_1 & 1 & 0 \\ -1 & -c_2 & 1 \\ 0 & -1 & -c_3 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} \quad (32)$$

Consequently, the error vector  $(z_1, z_2, z_3)$  is globally asymptotically vanishing.

### 3.2 Outer voltage loop design

The aim of the outer loop is to generate a tuning law for the ratio  $\beta$  in such a way that the output voltage  $v_o$  be regulated to a given reference value  $v_{oref}$ .

Relation between  $\beta$  and  $v_o$ . The first step in designing such a loop is to establish the relation between the ratio  $\beta$  (control input) and the output voltage  $v_o$ . This is the object of the following proposition.

**Proposition 3.2.** Consider the power converter described by (10a-d) and the inner control law defined by (31). Under the same assumptions as in Proposition 3.1, one has the following properties:

1°) The output voltage  $v_o$  varies in response to the tuning ratio  $\beta$  according to the following equation:

$$\frac{dv_o}{dt} = -\frac{v_o}{R_o C_o} + \frac{\hat{v}_n^2}{2C_o v_o} (1 - \cos(2\omega_n t)) \beta \quad (33)$$

where  $\hat{v}_n$  denotes the magnitude of the network (sinusoidal) voltage  $v_n$ .

2°) The squared voltage  $v_o^2$  varies, in response to the tuning ratio  $\beta$ , according to the linear equation:

$$\frac{dy}{dt} = -ay + k_o (1 - \cos(2\omega_n t)) \beta \quad (34)$$

where  $a = \frac{R_o C_o}{2}$  and  $k_o = \frac{\hat{v}_n^2}{C_o}$

**Proof.** 1) The first step consists in replacing the circuit part above the set  $C_o$ - $R_o$ , by an equivalent current generator, as shown by Fig. 2. In view of equation (10d), the underlying current value  $i_{equ}$  coincides with  $(1 - \alpha)i_{L_o}$ . So, (10d) becomes:

$$\frac{dv_o}{dt} = \frac{i_{equ}}{C_o} - \frac{v_o}{R_o C_o} \quad (35)$$

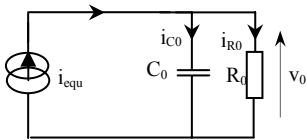


Fig.2. Equivalent current generator

The equivalent current  $i_{equ}$  will now be expressed in function of the tuning ratio  $\beta$ , using power conservation arguments. Using the fact that  $i_n = \beta v_n$  (because of Proposition 3.1), the instantaneous power entering the converter turns out to the following:

$$P_n = \beta \hat{v}_n^2 \sin^2(\omega_n t) = \frac{\beta \hat{v}_n^2}{2} (1 - \cos(2\omega_n t)) \quad (36)$$

On the other hand, the power that is actually transmitted to the load is  $P_{load} = v_o i_{equ}$ . But, the entering instantaneous power is integrally transmitted to the load (which is the only dissipative element). Then, the quantity  $P_{load}$  does coincide with  $P_n$ , this yields:  $i_{equ} = \beta \frac{\hat{v}_n^2}{2v_o} (1 - \cos(2\omega_n t))$ , which together with

(35) establishes (33).

2) Let us introduce the variable change  $y = v_o^2$  in (33). Deriving  $y$  with respect to time and using (33), yields the first-order linear model (34) and completes the proof of Proposition 3.2.

Squared output voltage control. The ratio  $\beta$  stands as a control input in the system (34). The problem at hand is to design for  $\beta$  a tuning law so that the squared voltage  $y = v_o^2$  tracks a given reference

signal  $y^* \stackrel{\text{def}}{=} v_{oref}^2$ . Ignoring time-varying feature, of

the controlled (first order) system, a PI control law should be sufficient. Bearing in mind the fact that the third first derivatives of  $\beta$  should be available (Propositions 3.1 and 3.2), we rather use a filtered PI,

$$\beta = \left( \frac{b}{b+s} \right)^3 (k_p e_1 + k_i e_2) \quad (37)$$

$$e_1 = y^* - y, \quad e_2 = \int_0^t e_1 dt, \quad (38)$$

where  $s$  may denote as well the derivative operator ( $s=d/dt$ ). At this point, the regulator parameters ( $b, k_p, k_i$ ) are any positive real constants.

### 3.3 Control system analysis

In the following Theorem, it is shown that the control objectives are achieved, in the mean, for a specific class of reference signals, with an accuracy that depends on the network frequency  $\omega_n$ . To formulate the results the following notations are needed:

$$\varepsilon = \frac{1}{2\omega_n}, \quad a_0 = k_o k_i b^3, \quad a_1 = (a + k_o k_p) b^3$$

$$a_2 = 3ab^2 + b^3, \quad a_3 = 3ab + 3b^2, \quad a_4 = (a + 3b)$$

**Theorem 3.1** (main result). Consider the AC/DC Boost power converter shown by Fig.1. together with the controller consisting of the inner-loop regulator (31) and the outer-loop regulator (37-38). Then, the resulting closed-loop system has the following properties:

- 1) The error  $i_n - i_{nref}$  vanishes asymptotically,
- 2) Let the reference signal  $y^*$  be nonnegative and periodic with period  $N\pi/\omega_n$ , where  $N$  any positive integer. Furthermore, let the regulator parameters

(b, k<sub>p</sub>, k<sub>i</sub>) be any positive real constants that satisfy the following inequalities:

$$(a_4 a_3 - a_2) a_2 - (a_4 a_1 - a_0) a_0 > 0 \quad (39)$$

$$\left[ (a_4 a_3 - a_2) a_2 - (a_4 a_1 - a_0) a_0 \right] (a_4 a_1 - a_0) - (a_4 a_3 - a_2)^2 a_0 > 0 \quad (40)$$

Then, there exists a positive  $\varepsilon^*$  such that for  $\varepsilon \leq \varepsilon^*$ , the tracking error is a harmonic signal which depends continuously on  $\varepsilon$ , i.e.  $e_1 = e_1(t, \varepsilon)$ . Moreover:

$$\lim_{\varepsilon \rightarrow 0} e_1(t, \varepsilon) = 0$$

*Remarks 3.2.*

- The first part of the Theorem says that the power factor objective is actually achieved.
- The second part shows that the tracking objective is achieved, in the mean, with an accuracy that depends on the voltage network frequency ( $\omega_n$ ). The larger  $\omega_n$  the better tracking objective.
- The period of the reference signal is any multiple of that of the power network (which is equal to  $\pi/\omega_n$ ). That is, the reference signal is slower than the network voltage. This particularly includes constant references.

**Proof of Theorem 3.1** (Outline).

Part 1 of the Theorem is a direct consequence of Proposition 3.1, using (37) which shows that  $\beta$  and its derivatives (up to the third order) are available. This also guarantees that equation (34), in Proposition 3.2, holds too. In order to prove the second part of the Theorem 1, let us introduce the following state variables:

$$x_1 = e_1, \quad x_2 = e_2, \\ x_3 = \frac{b}{b+s} (k_p e_1 + k_i e_2), \quad x_4 = \frac{b}{b+s} x_3, \quad x_5 = \frac{b}{b+s} x_4$$

Then, it follows from (34) and (37-38) that the state vector  $X = [x_1, x_2, x_3, x_4, x_5]^T$  undergoes the following state equation:

$$\frac{dX}{dt} = f(t, X) \quad (41)$$

where:

$$f(X, t) = \begin{bmatrix} -ax_1 - k_0(1 - \cos(2\omega_n t))x_5 + ay^* + \frac{dy^*}{dt} \\ x_1 \\ b(-x_3 + k_p x_1 + k_i x_2) \\ b(-x_4 + x_3) \\ b(-x_5 + x_4) \end{bmatrix} \quad (42)$$

Stability of the above system will now be dealt with resorting to averaging analysis tools. First, as  $y^*$  is periodic with period  $N/2\omega_n$ , it will prove to be useful introducing the following auxiliary reference function:

$$y^r(t) = y^* \left( \frac{N}{2\omega_n} t \right) \quad (43)$$

This readily implies that  $y^*(t) = y^r \left( \frac{2\omega_n}{N} t \right)$  and that  $y^r$

is periodic, with period  $2\pi$ . Let us now introduce the time-scale change:  $\tau = 2\omega_n t$ . Then the term in  $y^*$  in (42) becomes:

$$-ay^*(t) \frac{dy^*(t)}{dt} = -y^r \left( \frac{\tau}{N} \right) - 2\omega_n \frac{dy^r \left( \frac{\tau}{N} \right)}{d\tau} \quad (44)$$

It also follows from (41-42) that  $X$  undergoes the differential equation:

$$\frac{dX}{d\tau} = \varepsilon g(\tau, X, \varepsilon) \quad (45)$$

where:

$$g(\tau, X, \varepsilon) = \begin{bmatrix} -ax_1 - k_0(1 - \cos(\tau))x_5 + ay^r \left( \frac{\tau}{N} \right) + 2\omega_n \frac{dy^r \left( \frac{\tau}{N} \right)}{d\tau} \\ x_1 \\ b(-x_3 + k_p x_1 + k_i x_2) \\ b(-x_4 + x_3) \\ b(-x_5 + x_4) \end{bmatrix} \quad (46)$$

Now, let us introduce the average function

$$G(X) = \frac{1}{2\pi N} \int_0^{2\pi N} g(\tau, X, 0) d\tau. \text{ It follows from (45) that:}$$

$$G(X) = \begin{bmatrix} -ax_1 - k_0 x_5 - a\bar{y}^r \\ x_1 \\ b(-x_3 + k_p x_1 + k_i x_2) \\ b(-x_4 + x_3) \\ b(-x_5 + x_4) \end{bmatrix} \quad (47)$$

where  $\bar{y}^r$  denotes the mean value of  $y^r$ , which the same as that of  $y^*$ . Note that the mean value, over  $[0, 2\pi N]$ , of the derivative in the first line of (46), is zero because  $y^r$  is periodic with period  $2\pi$ . The averaging theory suggests to get stability results regarding the system of interest (41-42), from analyzing the following averaged system:

$$\frac{dZ}{dt} = \varepsilon G(Z) \quad (48)$$

To this end, notice that (48) has a unique equilibrium at:

$$Z^* = \left[ 0, \frac{-a\bar{y}^r}{k_0 k_i}, \frac{-a\bar{y}^r}{k_0}, \frac{-a\bar{y}^r}{k_0}, \frac{-a\bar{y}^r}{k_0} \right]^T \quad (49)$$

On the other hand, as (48) is linear, the stability properties of its equilibrium are full determined by the following state-matrix:

$$A = \begin{bmatrix} -a & 0 & 0 & 0 & -k_0 \\ 1 & 0 & 0 & 0 & 0 \\ bk_p & bk_i & -b & 0 & 0 \\ 0 & 0 & b & -b & 0 \\ 0 & 0 & 0 & b & -b \end{bmatrix} \quad (50)$$

The equilibrium  $Z^*$  will be globally asymptotically stable if all eigenvalues of  $A$  have negative real parts. The mentioned eigenvalues are the zeros of the following characteristic polynomial:

$$\det(\lambda I - A) = \lambda^5 + a_4 \lambda^4 + a_3 \lambda^3 + a_2 \lambda^2 + a_1 \lambda^1 + a_0 \quad (51)$$

It can be checked, applying for instance the well known Routh's algebraic criteria, that all zeros of polynomial (51) will have negative real parts if its coefficients ( $a_0$  to  $a_4$ ) satisfy (39-40).

Now applying averaging theory (see e.g. Theorem 4.1 in (Zhi-fen, et al 1992)), one concludes that there exists a  $\varepsilon^* > 0$  such that for  $\varepsilon < \varepsilon^*$ , the differential equation (41-42) has a harmonic solution,  $X = X(t, \varepsilon)$ , that depends continuously on  $\varepsilon$ . Moreover, one has:  $\lim_{\varepsilon \rightarrow 0} X(t, \varepsilon) = Z^*$

This, together with (48), yields in particular that  $\lim_{\varepsilon \rightarrow 0} e_1(t, \varepsilon) = 0$ . The Theorem is thus established.

#### 4. SIMULATIONS

Performances and design aspects of the controller will now be illustrated by simulations performed in the Matlab/Simulink environment. The controlled AC/DC converter has the following characteristics:  $\hat{v}_n = 60V$ ,  $L = 2mH$ ,  $C = 10\mu F$ ,  $L_o = 20mH$ ,  $C_o = 4000\mu F$ ,  $R_o = 20\Omega$  and it operates at the cutting frequency  $f_c = 10kHz$ . The reference squared output voltage  $v_{oref}^2$  is a step signal of amplitude 2500 (Volts)<sup>2</sup>. The values  $c_1 = 10000$ ,  $c_2 = 10000$  and 15000 proved to be appropriate for the inner loop design parameters. Bearing in mind Remark 2a, the outer loop parameters have been chosen as follows:  $k_p = 0.005$ ,  $k_i = 0.0011$  and  $b = 1000$ .

Figures 3 to 6 illustrate the controller performances. As expected (Remark 2b),  $v_o^2$  converge in the mean to its reference value (see Fig. 3.). Furthermore, it is checked that the observed voltage ripple oscillate at the frequency  $2\omega_n$  (Remark 2b) and is much smaller than the average value of the signals.

Comparing Fig. 3 and Fig. 5, one particularly sees that the magnitude variation of the input current  $i_n$  is correlated to the (mean) value of the squared output voltage  $v_o^2$ . This confirms the power conservation through the circuit. Fig. 4 shows that the outer-loop control  $\beta$  is practically unaffected by the ripple phenomena. Finally, Fig. 6 shows that the input current  $i_n$  and the output voltage  $v_o$  are in phase, ensuring a unitary power factor.

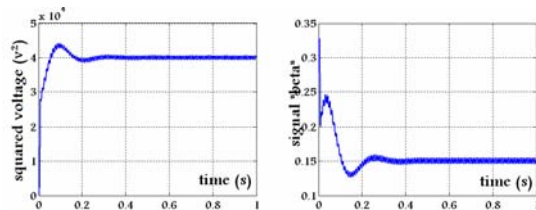


Fig. 3. squared voltage

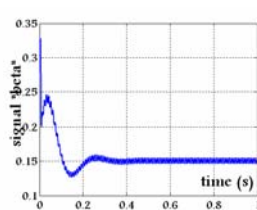


Fig. 4. signal  $\beta$

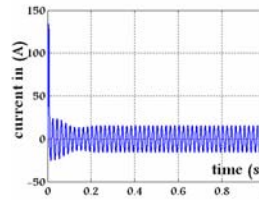


Fig. 5. current  $i_n$

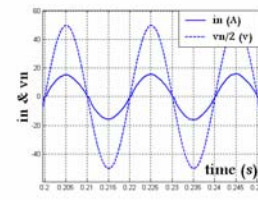


Fig. 6.  $v_n$  and  $i_n$

To analyze the robustness capability of the proposed controller a load change is operated according to Fig.7. Except for this change, the rest of the converter characteristics are the same as previously. The resulting closed-loop system behavior is illustrated by figures 8 to 10. The first figure shows that the effect of the resistance changes on the output voltage is well compensated. Fig. 9 shows that the PFC property is preserved. Finally, Fig. 10 shows that loop mode separation is still satisfied.

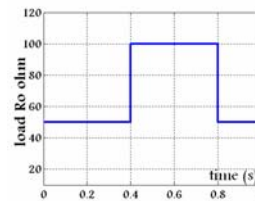


Fig. 7. Load resistance  $R_o$

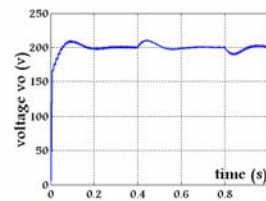


Fig. 8. voltage  $v_o$

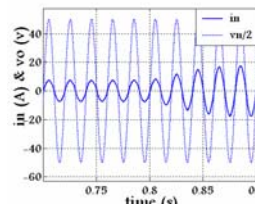


Fig. 9. current  $i_n$  and voltage  $v_n$

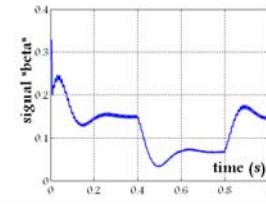


Fig. 10. signal  $\beta$

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