ASPECTS OF POLE PLACEMENT TECHNIQUE IN SYMMETRICAL OPTIMUM METHOD FOR PID CONTROLLER DESIGN

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Abstract: The paper presents an analytical approach to the design of PID controllers by combining pole placement with symmetrical optimum method, for the integration plus first-order plant model. The desired closed-loop transfer function (c.l.t.f.) contains a second-order oscillating system and a lead-delay compensator. It is shown that the zero value of c.l.t.f. depends on the real-pole value of c.l.t.f. and in addition, there is only one pole value, which satisfies the assumptions of symmetrical optimum method. In these conditions, the analytical expressions of the controller parameters can be simplified. The method is applied to design a PID autopilot for heading control of a ship with first-order Nomoto model. *Copyright © 2005 IFAC*

Keywords: PID control, pole placement, symmetrical optimum method, autopilot

1. INTRODUCTION

Due to widespread industrial use of PID controllers, it is clear that even a small percentage improvement in PID design could have a major impact worldwide (Silva, *et al.*, 2002). Tuning of PID controllers is a difficult task, as a three-parameter model should be defined and it must be accurate at higher frequencies (Astrom and Hagglund, 1995). Although, analytical methods are more convenient than graphical methods based on frequency diagrams, in industry, most controllers are tuned using frequency response methods (Tang and Ortega, 1993).

Analytical methods rely on low-order models characterized by a small number of parameters. The most employed models are the integration plus first-order model, which is used for thermal and electromechanical processes, and the first-order plus dead-time model, which is used for chemical processes (Datta, *et al.*, 2000).

In this paper, the integration plus first-order model type is used for ship dynamics modelling. Also, analytical design of PID autopilot for heading control is considered.

If pole placement method (PPM) is used to synthesize the PID controller, the first step is to specify some performance conditions of the closed-loop system, which lead to the expression of the closed-loop transfer function (c.l.t.f.) (Yuz and Salgado, 2003). In this paper, the desired c.l.t.f. contains a second-order oscillating system and a pole-zero pair, with real and negative values. Applying PPM, the zero value depends on the pole value, and the controller parameters depend on the parameters of c.l.t.f. The resulting open-loop transfer function (o.l.t.f.) contains a double-integrator element and a pole-zero pair, with real and negative values. Hence, the symmetrical optimum method (SOM) can be used (Kessler, 1958).

Imposing symmetrical characteristics of the open-loop transfer function, the analytical expressions of the controller parameters can be simplified.

The goal of this paper is to find the pole-zero values of c.l.t.f. and the simplified analytical expressions of PID controller parameters, which satisfy two simultaneous conditions: the desired close-loop transfer function and symmetrical characteristics of the open-loop transfer function.

The paper is organized as follows. Section 2 provides mathematical models used in simulations. In section 3, the analytical expressions of PID controller parameters are obtained using PPM. In section 4, the expressions of the controller parameters are simplified, imposing symmetrical characteristics of the o.l.t.f. Section 5 describes the simulation results, using a PID autopilot for heading control of a ship. Conclusions are presented in section 6.

2. MATHEMATICAL MODELS

Consider the classical structure of the control loop without disturbances, as shown in Fig. 1. The plant model contains an integrator and the controller is of PID type.

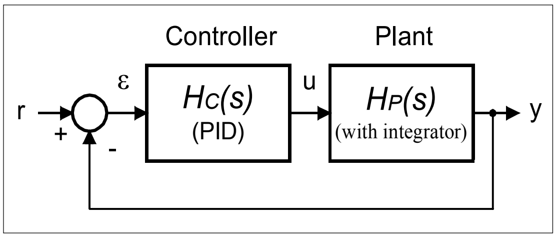

Fig. 1. Classical structure of the control loop

The performance conditions of the closed-loop system can be specified imposing the expression of system transfer function. In general, a second order reference model is chosen to approximate the behaviour of the closed-loop system:

$$H_0(s) = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2} \ , \tag{1}$$

where $\omega_0 > 0$ is the natural frequency and $\zeta > 0$ is the damping coefficient.

Because the plant model contains an integrator and another one is included into the PID controller, the open-loop transfer function contains a double-integrator, which can not be obtained with c.l.t.f. given in (1). Therefore, the reference model must be completed with a lead-delay compensator, which contains a pole-zero pair, with real and negative values (Ceanga, *et al.*, 2001):

$$H_0(s) = \frac{\omega_0^2 \cdot \dfrac{p}{z} \cdot (s+z)}{\left(s^2 + 2\zeta\omega_0 s + \omega_0^2\right) \cdot (s+p)} \tag{2}$$

where $z > 0$ and $p > 0$.

The plant model contains an integrator, and it is characterized by a dominant time constant (T_P) and a gain coefficient (k_P). The expression of the model depends on the process type.

a) If the process is fast, then the small time constants can not be neglected and the model contains an equivalent small time constant (T_Σ), corresponding to the sum of parasitic time constants:

$$H_P(s) = \frac{k_P}{s \cdot (sT_P + 1) \cdot (sT_\Sigma + 1)} \ , \tag{3}$$

where $T_\Sigma < T_P$.
In this case, the PID controller is of the form:

$$H_C(s) = \frac{k_C}{sT_C} \cdot (sT_C + 1) \cdot \left(sT_C^{'} + 1\right) \ , \tag{4}$$

where $T_\Sigma < T_C^{'} < T_C$.

The open-loop transfer function is:

$$H(s) = \frac{k_C}{sT_C} \cdot (sT_C + 1) \cdot \left(sT_C^{'} + 1\right) \cdot \frac{k_P}{s \cdot (sT_P + 1) \cdot (sT_\Sigma + 1)} \tag{5}$$

Using pole cancellation, the non-zero dominant pole of the plant model is cancelled by choosing:

$$T_C^{'} = T_P \tag{6}$$

Thus, only two controller parameters must be determined: k_C and T_C.

It can be observed that, if the process does not have any non-zero dominant pole (model without time constant T_P), then the controller is of PI type (without time constant $T_C^{'}$) and the same parameters must be determined (k_C and T_C).

b) If the process is slow, the equivalent small time constant (T_Σ) can be neglected and the plant model is:

$$H_P(s) = \frac{k_P}{s \cdot (sT_P + 1)} \tag{7}$$

The PID controller contains a supplementary degree of freedom and it is of the form:

$$H_C(s) = \frac{k_C}{sT_C} \cdot (sT_C + 1) \cdot \frac{sT_C^{'} + 1}{sT_1 + 1} \ , \tag{8}$$

where $T_\Sigma < T_1 < T_C^{'} < T_C$.
The open-loop transfer function is:

$$H(s) = \frac{k_C}{sT_C} \cdot (sT_C + 1) \cdot \frac{sT_C^{'} + 1}{sT_1 + 1} \cdot \frac{k_P}{s \cdot (sT_P + 1)} \tag{9}$$

Again, using pole cancellation, the non-zero dominant pole of the plant model is cancelled, resulting equation (6): $T_C^{'} = T_P$.
In this case, three controller parameters must be determined: k_C, T_C and T_1.

This is a more general case because the time constant T_1 is not imposed by the process and it can be chosen. So, in this paper, the plant model given in (7) is used for computations, but discussions are made also for model given in (3).

In both cases, the open-loop transfer functions, given in (5) and (9), have similar expressions:

$$H(s) = H_C(s)\cdot H_P(s) = \frac{k_C \cdot k_P \cdot (sT_C + 1)}{s^2 T_C \cdot (sT + 1)} \qquad (10)$$

With this open-loop transfer function, the symmetrical optimum method (SOM) can be used.

The time constant T has different meanings: in the first case, it represents the equivalent small time constant (T_Σ) imposed by the process, while in the second case, it is a controller parameter (T_I).

3. PID CONTROLLER DESIGN USING POLE PLACEMENT METHOD

Consider the control system illustrated in Fig. 1 with the desired closed-loop transfer function given in (2).

Proposition 1: For every pole value ($s = -p$) of the desired closed-loop transfer function given in (2), there is only one zero value ($s = -z$) for which the open-loop transfer function has a double-pole in origin, and in addition, the zero frequency is smaller than the pole frequency: $z < p$.

The proposition demonstration includes the next lemma results.

Lemma 1: The necessary and sufficient condition, for the existence of a double-pole in origin for the open-loop transfer function of a control system illustrated in Fig. 1, starting from a desired c.l.t.f. of the form given in (2), is:

$$z = \frac{\omega_0 p}{2\zeta\, p + \omega_0}, \quad \forall p, \omega_0, \zeta > 0 \qquad (11)$$

Proof L1: The transfer function of the open-loop system can be computed starting from the desired closed-loop transfer function (Nicolau, 2004):

$$H(s) = H_C(s)\cdot H_P(s) = \frac{H_0(s)}{1 - H_0(s)} =$$

$$= \frac{\omega_0^2 \cdot \frac{p}{z} \cdot (s + z)}{s^3 + s^2(p + 2\zeta\omega_0) + s\left[2\zeta\omega_0 p + \omega_0^2\left(1 - \frac{p}{z}\right)\right]} \qquad (12)$$

From (12) it can be observed that a double-pole in origin is obtained if the equation below is satisfied:

$$2\zeta\omega_0 p + \omega_0^2\left(1 - \frac{p}{z}\right) = 0, \qquad (13)$$

which is equivalent with:

$$2\zeta\, pz + \omega_0 z = \omega_0 p \qquad (14)$$

From (14) it results the necessary and sufficient condition indicated in (11) (q.e.d.).

Implicitly, the unique zero value results, whose expression depends on the selected pole value:

$$s = -z = -\frac{\omega_0 p}{2\zeta\, p + \omega_0}, \quad \forall p, \omega_0, \zeta > 0 \qquad (15)$$

For every frequency (p) of the pole, the corresponding frequency (z) of the zero is smaller:

$$\forall\, p > 0 \;\Rightarrow\; z = \frac{\omega_0 p}{2\zeta\, p + \omega_0} < p \qquad (16)$$

So, for the lead-delay compensator introduced into the desired c.l.t.f. given in (2), the phase-lead effect is dominantly.

In this case, the real values of pole-zero pair and conjugate complex poles, of the desired c.l.t.f. given in (2), are illustrated in Fig. 2.

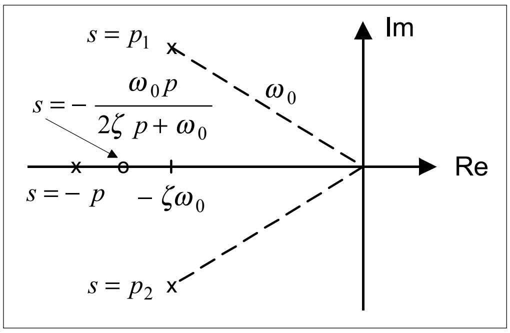

Fig. 2. Poles and zero of the desired c.l.t.f.

Using (11) and (13) in (12), the expression of the open-loop transfer function is obtained:

$$H(s) = \frac{\omega_0 \cdot (2\zeta\, p + \omega_0)\cdot\left(s + \frac{\omega_0 p}{2\zeta\, p + \omega_0}\right)}{s^2 \cdot [s + (p + 2\zeta\omega_0)]} \qquad (17)$$

Denote by ω_z and ω_p, respectively, the zero and pole frequencies of the open-loop transfer function:

$$\omega_z = \frac{\omega_0 p}{2\zeta\, p + \omega_0} \qquad \omega_p = 2\zeta\,\omega_0 + p \qquad (18)$$

The open-loop transfer function can be rewritten:

$$H(s) = \frac{\omega_0 (2\zeta\, p + \omega_0)\cdot (s + \omega_z)}{s^2 \cdot [s + \omega_p]} \qquad (19)$$

Putting into evidence the time constants, the open-loop transfer function can be rewritten, like in (10):

$$H(s) = \frac{\omega_0^2 p \cdot \left(s\frac{2\zeta\, p + \omega_0}{\omega_0 p} + 1\right)}{s^2 (2\zeta\,\omega_0 + p)\left(s\frac{1}{2\zeta\,\omega_0 + p} + 1\right)} \qquad (20)$$

From (20), using (5) or (9) corresponding to the plant model indicated in (3) or (7), respectively, it results:

$$H(s) = \frac{k_C}{sT_C}\cdot (sT_C + 1)\cdot \frac{sT_C' + 1}{sT + 1}\cdot \frac{k_P}{s\cdot (sT_P + 1)} =$$

$$= \frac{\omega_0^2 p \cdot \left(s\frac{2\zeta\, p + \omega_0}{\omega_0 p} + 1\right)}{s^2 (2\zeta\,\omega_0 + p)\left(s\frac{1}{2\zeta\,\omega_0 + p} + 1\right)}, \qquad (21)$$

where $T = T_\Sigma$ or $T = T_I$.

Equation (21) can be reduced to an equality of two polynomials of 3rd order in s variable:

$$k_P \cdot k_C \cdot (2\zeta\,\omega_0 + p)\cdot(sT_C + 1)\cdot\left(sT_C^{'} + 1\right)\cdot\left(s\frac{1}{2\zeta\,\omega_0 + p} + 1\right) =$$

$$= T_C \cdot \omega_0^2 p \cdot (sT + 1)\cdot(sT_P + 1)\cdot\left(s\frac{2\zeta\,p + \omega_0}{\omega_0 p} + 1\right), \quad (22)$$

where no pole cancellation was considered.

The equality must be true for every frequency, resulting a four equation system:

$$\begin{cases} k_P \cdot k_C \cdot (2\zeta\,\omega_0 + p) = T_C \cdot \omega_0^2 p & (23.1) \\ T_C \cdot T_C^{'} \cdot \dfrac{1}{2\zeta\,\omega_0 + p} = T \cdot T_P \cdot \dfrac{2\zeta\,p + \omega_0}{\omega_0 p} & (23.2) \\ T_C \cdot T_C^{'} + (T_C + T_C^{'}) \cdot \dfrac{1}{2\zeta\,\omega_0 + p} = & \\ \quad = T \cdot T_P + (T + T_P) \cdot \dfrac{2\zeta\,p + \omega_0}{\omega_0 p} & (23.3) \\ T_C + T_C^{'} + \dfrac{1}{2\zeta\,\omega_0 + p} = T + T_P + \dfrac{2\zeta\,p + \omega_0}{\omega_0 p} & (23.4) \end{cases}$$

The solutions of the equation system are the PID controller parameters (Nicolau, 2004):

$$\begin{cases} k_C = \dfrac{\omega_0}{k_P} \cdot \dfrac{2\zeta\,p + \omega_0}{2\zeta\,\omega_0 + p} & (24.1) \\ T_C = \dfrac{2\zeta\,p + \omega_0}{\omega_0 p} = \dfrac{1}{\omega_z} & (24.2) \\ T_C^{'} = T_P & (24.3) \\ T = \dfrac{1}{2\zeta\,\omega_0 + p} = \dfrac{1}{\omega_p} & (24.4) \end{cases}$$

It can be observed that the solution (24.3) represents the pole cancellation condition, considered in (6), and it does not depend on the pole value.

If the process is slow and the equivalent small time constant (T_Σ) is ignored, the time constant $T = T_1$ represents a controller parameter given in (24.4).
The time constants must satisfy the inequalities:

$$T_\Sigma < T_1 < T_C^{'} < T_C\,, \quad (25)$$

which can be transposed into frequency domain:

$$\omega_z < \frac{1}{T_P} < \omega_p < \frac{1}{T_\Sigma} \quad (26)$$

Therefore, the reference model must be chosen so that the parameters of c.l.t.f. (ω_0 , ζ and p) to satisfy the system of inequalities:

$$\begin{cases} 2\zeta\,\omega_0 + p < \dfrac{1}{T_\Sigma} & (27.1) \\ 2\zeta\,\omega_0 + p > \dfrac{1}{T_P} & (27.2) \\ 2\zeta\,p + \omega_0 > p\omega_0 T_P & (27.3) \end{cases}$$

If the time constant T_Σ is imposed by the process and the plant model given in (3) is considered, then the solution (24.4) becomes:

$$T_\Sigma = \frac{1}{2\zeta\,\omega_0 + p} = \frac{1}{\omega_p}\,, \quad (28)$$

which represents a supplementary condition for parameters of c.l.t.f. (ω_0 , ζ and p).
In addition, the time constants must satisfy the inequalities:

$$T_\Sigma < T_C^{'} < T_C\,, \quad (29)$$

which can be transposed into frequency domain:

$$\omega_z < \frac{1}{T_P} < \omega_p = \frac{1}{T_\Sigma} \quad (30)$$

In this case, the reference model must be chosen so that the parameters of c.l.t.f. (ω_0 , ζ and p) to satisfy the following conditions:

$$\begin{cases} 2\zeta\,\omega_0 + p = \dfrac{1}{T_\Sigma} & (31.1) \\ 2\zeta\,\omega_0 + p > \dfrac{1}{T_P} & (31.2) \\ 2\zeta\,p + \omega_0 > p\omega_0 T_P & (31.3) \end{cases}$$

It can be observed that the first condition in (31) is more restrictive than the corresponding one in (27), while the last two conditions are the same.

4. SYMMETRICAL CHARACTERISTICS OF OPEN-LOOP TRANSFER FUNCTION

The PID controller parameters depend on the parameters of c.l.t.f. (ω_0 , ζ and p). In general, ω_0 and ζ characterize the desired system behaviour and they have fixed values, while the pole value can be chosen. Specific pole values can be imposed by using supplementary conditions.

In this paper, the conditions for choosing the pole value refer to the symmetrical optimum method, which simplify the expressions of PID parameters.
The goal is to find that pole value of the c.l.t.f., which satisfies the assumptions of symmetrical optimum method around natural frequency ω_0, for the transfer function of open-loop system given in (19). Using this value, the expressions of PID parameters in (24) are simplified.

Proposition 2: There is only one admissible value for the pole (s = - p) of c.l.t.f. given in (2), so that the corresponding o.l.t.f. given in (17) to have symmetrical characteristics around ω_0 :

$$p = \omega_0\,, \quad \forall\, p, \omega_0, \zeta > 0 \quad (32)$$

Proof: For the specified open-loop transfer function, the symmetry of magnitude-frequency characteristic around natural frequency ω_0 implies the symmetry of phase-frequency characteristic. Therefore, only the symmetry of former characteristic must be imposed.

The general form of the symmetrical optimum method imposes two conditions for magnitude-frequency characteristic:

a) the central frequency ω_0 must be equally placed between zero and pole frequencies on the 10-base logarithmic scale:

$$\frac{\omega_0}{\omega_z} = \frac{\omega_p}{\omega_0} \tag{33}$$

b) for central frequency ω_0 , the magnitude-frequency characteristic of o.l.t.f. must have *0 dB*:

$$\left| H(j\omega_0) \right| = 1 \tag{34}$$

Using (18) in (33), the first condition becomes:

$$\frac{\omega_0 p}{2\zeta\, p + \omega_0} \cdot (2\zeta\, \omega_0 + p) = \omega_0^2 \tag{35}$$

From (35), it results:

$$p^2 = \omega_0^2 \,, \quad \forall\, p, \omega_0, \zeta > 0 \quad \Rightarrow \quad p = \omega_0 \tag{36}$$

So, the condition (33) is satisfied if $p = \omega_0$.

For the second condition in (34), the magnitude of open-loop transfer function in frequency ω_0 is computed from (19):

$$\left| H(j\omega_0) \right| = \frac{(2\zeta\, p + \omega_0) \cdot \sqrt{\omega_z^2 + \omega_0^2}}{\omega_0 \cdot \sqrt{\omega_p^2 + \omega_0^2}} \tag{37}$$

The frequencies ω_z and ω_P are replaced with their expressions from (18), resulting:

$$\left| H(j\omega_0) \right| = \sqrt{\frac{p^2 + \omega_0^2 + 4\zeta\, p\omega_0 + 4\zeta^2 p^2}{p^2 + \omega_0^2 + 4\zeta\, p\omega_0 + 4\zeta^2 \omega_0^2}} \tag{38}$$

Using (38) in (34), the same solution in (36) is obtained: $\quad p^2 = \omega_0^2 \quad \Rightarrow \quad p = \omega_0$

Concluding, there is only one admissible value for the pole of c.l.t.f., so that the corresponding o.l.t.f. to have symmetrical characteristics around ω_0 (q.e.d.).

From (11), it results:

$$p = \omega_0 \quad \Rightarrow \quad z = \frac{\omega_0}{2\zeta + 1} \tag{39}$$

The expression of c.l.t.f. becomes:

$$H_0(s) = \frac{\omega_0^2 \cdot (2\zeta + 1) \cdot \left(s + \dfrac{\omega_0}{2\zeta + 1} \right)}{\left(s^2 + 2\zeta\omega_0 s + \omega_0^2 \right) \cdot \left(s + \omega_0 \right)} \tag{40}$$

Also, from (18), the zero and pole frequencies of o.l.t.f. are obtained:

$$\omega_z = \frac{\omega_0}{2\zeta + 1}, \qquad \omega_p = \omega_0 \cdot (2\zeta + 1) \tag{41}$$

The open-loop transfer function can be rewritten:

$$H(s) = \frac{\omega_0^2 (2\zeta + 1) \cdot \left(s + \dfrac{\omega_0}{2\zeta + 1} \right)}{s^2 \cdot \left[s + \omega_0 (2\zeta + 1) \right]} \tag{42}$$

The real values of pole-zero pair and conjugate complex poles, of the c.l.t.f. given in (40), are illustrated in Fig. 3.

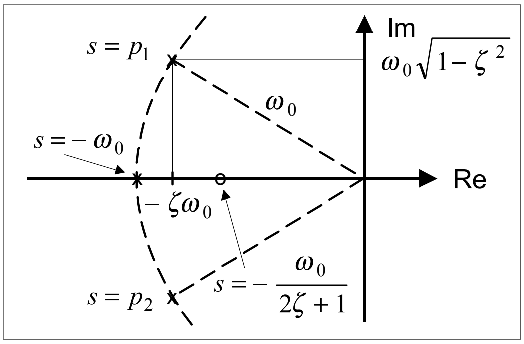

Fig. 3. Poles and zero of the c.l.t.f. with $p = \omega_0$

The position of the zero $s = -z = -\dfrac{\omega_0}{2\zeta + 1}$ depend on the parameter ζ:

- if $\zeta \in \left(0, \dfrac{1}{2}\right)$, then $-\omega_0 < -z < -\zeta\omega_0$ and the zero is placed between the two points: $s = -\omega_0$ and $s = -\zeta\omega_0$, respectively;
- if $\zeta = \dfrac{1}{2}$, then $-z = -\zeta\omega_0$. This is the particular case of the Kessler's symmetrical optimum method;
- if $\zeta > \dfrac{1}{2}$, then $0 > -z > -\zeta\omega_0$ and the zero is placed to the right of the point $s = -\zeta\omega_0$. This is the case illustrated in Fig. 3.

Knowing the pole value of c.l.t.f. ($p = \omega_0$), the PID controller parameters result from (24):

$$\begin{cases} k_C = \dfrac{\omega_0}{k_P} & (43.1) \\[2mm] T_C = \dfrac{2\zeta + 1}{\omega_0} & (43.2) \\[2mm] T_C^{'} = T_P & (43.3) \\[2mm] T_1 = \dfrac{1}{(2\zeta + 1) \cdot \omega_0} & (43.4) \end{cases}$$

The parameters in (43) correspond to the plant model given in (7) and PID controller given in (8).

In this case, the conditions from (27) depend on the parameters ω_0 and ζ. Hence, the reference model must be chosen so that the parameters of c.l.t.f. (ω_0 and ζ) to satisfy the system of inequalities:

$$\begin{cases} (2\zeta + 1) \cdot \omega_0 < \dfrac{1}{T_\Sigma} & (44.1) \\[2mm] (2\zeta + 1) \cdot \omega_0 > \dfrac{1}{T_P} & (44.2) \\[2mm] \dfrac{2\zeta + 1}{\omega_0} > T_P & (44.3) \end{cases}$$

Proposition 3: In the case of symmetrical characteristics of the o.l.t.f. given in (42) around the natural frequency ω_0, the phase margin and the distance between the frequency points on the 10-base logarithmic scale depend only on the parameter ζ.

Proof: The distance between the frequency points on the 10-base logarithmic scale can be easily obtained, using (41) in (33):

$$\frac{\omega_0}{\omega_z} = \frac{\omega_p}{\omega_0} = 2\zeta + 1 \tag{45}$$

The phase margin is:

$$\varphi_m = \pi + \arg\left(H(j\omega_0)\right) \tag{46}$$

Using the o.l.t.f. given in (42), results:

$$\varphi_m = \mathrm{arctg}(2\zeta + 1) - \mathrm{arctg}\left(\frac{1}{2\zeta + 1}\right) \tag{47}$$

It can be observed that, for particular case of the Kessler's symmetrical optimum method ($\zeta = 0.5$), the distance between frequency points is equal with an octave and the phase margin is $\varphi_m = 36.87$ *[deg]*.

5. SIMULATION RESULTS

For simulations, the heading control problem of a ship is considered, using a PID autopilot.

The ship model is linear, being identified for a ship speed of 22 knots (Nicolau, 2004). It is a first order Nomoto model of the form given in (7):

$$H_P(s) = \frac{\psi(s)}{\delta(s)} = \frac{k_P}{s \cdot \left(sT_P + 1\right)}, \tag{48}$$

where $\psi(s)$ and $\delta(s)$ represent the Laplace transforms of yaw angle and rudder angle, respectively.

The ship model parameters are:

$$k_P = -0.0834 \ [s^{-1}], \quad T_P = 5.98 \ [s] \tag{49}$$

The autopilot model is given in (8) and the desired c.l.t.f. is given in (2). The parameters ω_0 and ζ are chosen from performance conditions (Fossen, 1994):

$$\zeta = 0.9, \quad \omega_0 = 0.1 \ [rad/s] \tag{50}$$

Starting from the desired c.l.t.f. and imposing symmetrical characteristics of the o.l.t.f., the expressions in (40) and (42) are obtained. The step response of the c.l.t.f. is illustrated in Fig. 4.

From (43), the autopilot parameters are obtained:

$k_C = -1.2$, $T_C = 28\,[s]$, $T_C' = 5.98\,[s]$, $T_1 = 3.57\,[s]$

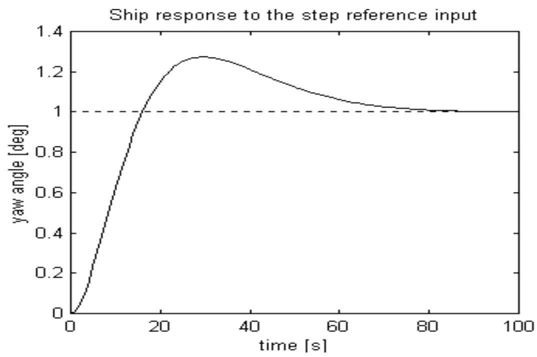

Fig. 4. Step response of the closed-loop system

Considering $T_\Sigma = 1$ *[s]*, the conditions in (44) are satisfied. The symmetrical characteristics of the o.l.t.f. are illustrated in Fig. 5. It can be observed that the phase margin is $\varphi_m = 50.69$ *[deg]*.

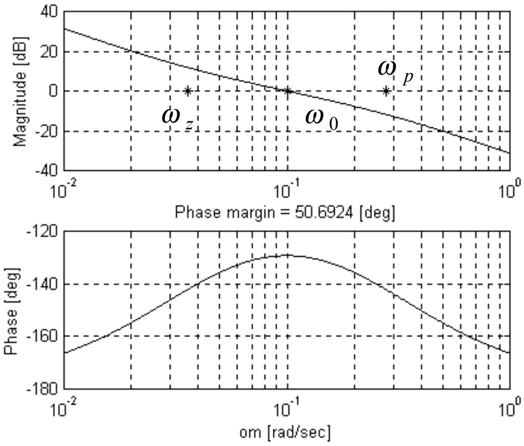

Fig. 5. Symmetrical characteristics of the o.l.t.f.

6. CONCLUSIONS

There is only one possible pair for the pole-zero values of c.l.t.f. so that the corresponding parameters of PID controller to satisfy two simultaneous conditions: the desired behaviour of close-loop system and symmetrical characteristics of the open-loop transfer function.

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