AN OPTIMISATION-BASED APPROACH TO INTERVAL MODEL IDENTIFICATION IN THE FREQUENCY DOMAIN

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Abstract: In this paper, an Optimisation-based Approach to Interval Model Identification in the Frequency Domain is proposed. As a result of its application, an interval LTI model with a prefixed structure that satisfies that its worst-case frequency response contains all the measured given data is obtained. The optimisation problem is formulated using, as constraints, that this inclusion property is explicitly satisfied making use of some auxiliary results used to prove the Kharitonov Theorem and, as objective function, the minimisation of the size of parameter space box according to some metric and the width of model frequency envelopes. Finally, an example of application of this approach is proposed.

Keywords: Frequency Response, Identification, Uncertain Systems, Robustness.

1. INTRODUCTION

The characteristics of the performance of a control system will depend on the accuracy how the model describes the behaviour of the process. For this premise to be accomplished, it is necessary to know that one of the principles of system modelling is simplification. This consists of obtaining a model that catch the main features of the process under analysis by the simplest way. However, a real process can be extremely complex to be described in an absolutely accurate way by a mathematical model. This inaccuracy due to modelling errors is known as uncertainty. If the behaviour of the process has to be described by a LTI model, it implies an additional set of simplifying hypotheses that increases the original modelling errors. The main factors that cause modelling errors are working point modification respect to the nominal model, unconsidered non-linear dynamics, high frequency dynamics not modelled, time delays not contemplated and parameter inaccuracy due to identification or modelling methods used. This uncertainty can be divided in two main categories: parametric (structured) and structural (non-structured) uncertainty. This work deals with models with structured uncertainty, more concretely LTI models with interval coefficients.

In Bhattacharyya (1995) and Keel (1996) interval model identification algorithms are proposed. These algorithms introduce a set of artificial parameters that produces a family of frequency response functions that include the experimental ones. The aim of such identification procedures is to provide a model to be used in the design of robust controllers using parametric methods. In the case of Bhattacharyya algorithm a single measured frequency response is used while in the case of Keel algorithm a set of measured frequency response obtained at different operating points is used. This last approach is followed and extended to the case that an interval of measurements for each frequency is available.

The aim of the present approach is also to provide an interval model that satisfies the inclusion property and also is intended to be used in Robust Control using parametric methods. This approach translates the problem of interval model identification to an optimisation problem explicitly guarantying the inclusion property. This requirement introduces a set of constraints in the frequency response of the interval model. These constraints are not easily expressed using mathematical formulas but instead a set of rules that can be derived from some auxiliary results used to prove the Kharitonov Theorem being these the main contribution of this paper. Once the optimisation problem is formulated, it should be solved numerically in a guaranteed way in order to obtain the global optimal solution. Finally, an example of application of this approach is proposed.

2. PROBLEM STATEMENT

2.1 The problem

Given a set of interval frequency responses $[D_k(j\omega)]$ $k=1,...,N$ measured at the following test frequencies $\omega_1, \omega_2, \cdots, \omega_n$, that contain real frequency system responses obtained at different operating points which interval model should be obtained. The objective is to develop an identification algorithm that allows to identify the uncertainty intervals of the following interval model $G(j\omega)$:

$$G(j\omega)=\frac{B(j\omega)}{A(j\omega)}=\frac{\left[b_m^-,b_m^+\right](j\omega)^m+}{\left[a_n^-,a_n^+\right](j\omega)^n+}$$

$$\frac{+\left[b_{m-1}^-,b_{m-1}^+\right](j\omega)^{m-1}+\cdots+\left[b_0^-,b_0^+\right]}{+\left[a_{n-1}^-,a_{n-1}^+\right](j\omega)^{n-1}+\cdots+\left[a_0^-,a_0^+\right]}=$$

$$=\frac{\left\{\left[b_0^-,b_0^+\right]-\left[b_2^-,b_2^+\right]\omega^2+\cdots\right\}+}{\left\{\left[a_0^-,a_0^+\right]-\left[a_2^-,a_2^+\right]\omega^2+\cdots\right\}+} \tag{1}$$

$$\frac{+j\left\{\left[b_1^-,b_1^+\right]\omega-\left[b_3^-,b_3^+\right]\omega^3-\cdots\right\}}{+j\left\{\left[a_1^-,a_1^+\right]\omega-\left[a_3^-,a_3^+\right]\omega^3-\cdots\right\}}$$

such that the following requirements are satisfied[1]:

(a) *Membership Requirement*: $D_k(j\omega_i)\subseteq[G(j\omega_i]$ for all $i=1,..,n$ and $k=1,..,N$

(b) *Minimum Size Requirement*: the size of parameter space box should be minimum according to some metric.

(c) *Frequency Response Requirement*: the sum of the squared difference between model and data envelopes has to be as minimum as possible.

These requirements are imposed since the obtained interval model is intended to be used for tuning the parameters of a robust controller being necessary that all the observed behaviours from the real plant are included and explained by it. Therefore, measured frequency response have to be bounded by the frequency response of the estimated interval model.

2.2 The data

According to what has been written above, at each sample frequency, ω_i there is a set of N measured frequency responses $D_k(j\omega_i)$ since $k=1,..,N$ .Let $|D(j\omega_i)|^+=\max_k\{|D_k(j\omega_i)|\}$ $|D(j\omega_i)|^-=\min_k\{|D_k(j\omega_i)|\}$, $\angle D(j\omega_i)^+=\max_k\{\angle D(j\omega_i)\}$ and $\angle D(j\omega_i)^-=\min_k\{\angle D_k(j\omega_i)\}$, be the corresponding intervals in which the module and phase of the measured frequency response lie. The set of tests frequencies and the measured limits for module and phase at each frequency conforms the available data and can be stored in a five-row matrix establishing the set-membership requirements described in (a):

$$\begin{pmatrix} \omega_1 & \omega_2 & \cdots & \omega_n \\ |D(j\omega_1)|^+ & |D(j\omega_2)|^+ & \cdots & |D(j\omega_n)|^+ \\ |D(j\omega_1)|^- & |D(j\omega_2)|^- & \cdots & |D(j\omega_n)|^- \\ \angle D(j\omega_1)^+ & \angle D(j\omega_2)^+ & \ddots & \angle D(j\omega_n)^+ \\ \angle D(j\omega_1)^- & \angle D(j\omega_2)^- & \cdots & \angle D(j\omega_n)^- \end{pmatrix} \tag{2}$$

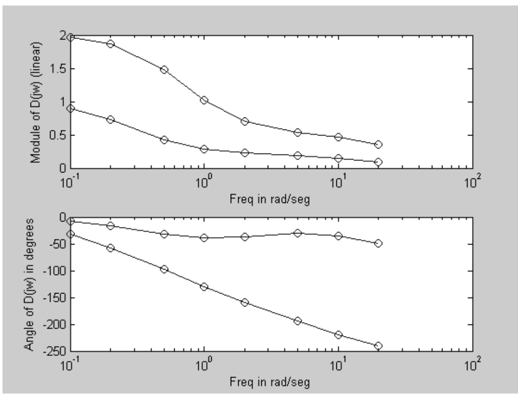

Figure 1. Maximum and minimum responses for module and angle of $D(j\omega)$

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[1] In the literature there are two approaches to connect the available uncertain data (2) and the model (1) according: to let the model to be explained by uncertain data or to let the data to be explained by the calibrated model.The first approach is called ***consistency*** or ***bounded-error identification*** (Milanese, 1996) and the resulting parameter space would be the interval values for the coefficients of the LTI model that produces a frequency response bounded by the module and the angle data. But our interest along this paper will lie on the second approach, the ***worst-case*** or ***robust identification*** (Bhattacharyya, 1995).

The reader can observe that the proposed algorithm begins with only the maximum and the minimum values for the modulus and the angle of the empirical frequency response, as it is illustrated by an example in Figure 1, instead of a set of trajectories as in Keel algorithm (1996).

2.3 The associated optimisation problem

The problem of interval model identification will be translated in an optimisation problem. Requirements (a) will establish the problem constraints, while (b) and (c) will define the objective function.

3. PROBLEM CONSTRAINTS AND ITS TRANSLATION INTO A SET OF RULES

In order to satisfy the set-membership requirement (a) described in Section 2.1, the following set of constraints should be satisfied at each single frequency test ω_k by the obtained interval model:

$$\begin{aligned} \left|D(j\omega_k)\right|^+ &\le \left|G(j\omega_k)\right|^+ \\ \left|G(j\omega_k)\right|^- &\le \left|D(j\omega_k)\right|^- \\ \angle D(j\omega_k)^+ &\le \angle G(j\omega_k)^+ \\ \angle G(j\omega_k)^- &\le \angle D(j\omega_k)^- \end{aligned} \quad (3)$$

i.e., available data $D(j\omega)$ is explained by the linear model $G(j\omega)$ with interval parameters.

These set of constraints should be included in the optimisation problem associated to the interval model identification problem. However, inclusion constraints (3) cannot be directly expressed as mathematical formulas since involve the maximization and minimization of module and phase of the interval model to be identified at each test frequency. But using some auxiliary results used to prove Kharitonov's Theorem these constraints can be translated in a set of rules.

3.1 Frequency envelopes of an interval model

Let

$$G(j\omega,\theta) = \frac{P_1(j\omega,\theta)}{P_2(j\omega,\theta)} \quad (4)$$

be a transfer function which maximum and minimum values for module and angle have to be determined with respect to a set of uncertain parameters θ[2]. Then:

$$\begin{aligned} \max\left(\left|G(j\omega)\right|\right) &= \frac{\max\left(\left|P_1(j\omega)\right|\right)}{\min\left(\left|P_2(j\omega)\right|\right)} \\ \min\left(\left|G(j\omega)\right|\right) &= \frac{\min\left(\left|P_1(j\omega)\right|\right)}{\max\left(\left|P_2(j\omega)\right|\right)} \end{aligned} \quad (5)$$

and

$$\begin{aligned} \max\left(\angle G(j\omega)\right) &= \max\left(\angle P_1(j\omega)\right) - \min\left(\angle P_2(j\omega)\right) \\ \min\left(\angle G(j\omega)\right) &= \min\left(\angle P_1(j\omega)\right) - \max\left(\angle P_2(j\omega)\right) \end{aligned} \quad (6)$$

It is clear that it is necessary to know how to determine the maximum and the minimum values for the module and the angle of a general interval polynomial $P(j\omega)$. The auxiliary results to Kharitonov's Theorem give some rules or conditions to compute such maximum and minimum values. At a given frequency ω_k it is correct to write that all the possible images of the interval polynomial

$$\begin{aligned} P(j\omega_k) &= \left[p_q^-, p_q^+\right](j\omega_k)^q + \left[p_{q-1}^-, p_{q-1}^+\right](j\omega_k)^{q-1} + \\ &+ \cdots + \left[p_1^-, p_1^+\right](j\omega_k) + \left[p_0^-, p_0^+\right] \end{aligned} \quad (7)$$

are contained in a rectangle in the complex plane which vertices are the four Kharitonov polynomials:

$$\begin{aligned} P_1(j\omega_k) &= p_0^+ + p_1^+(j\omega_k) + p_2^-(j\omega_k)^2 \\ &+ p_3^-(j\omega_k)^3 + p_4^+(j\omega_k)^4 + \cdots, \\ P_2(j\omega_k) &= p_0^- + p_1^-(j\omega_k) + p_2^+(j\omega_k)^2 \\ &+ p_3^+(j\omega_k)^3 + p_4^-(j\omega_k)^4 + \cdots, \\ P_3(j\omega_k) &= p_0^- + p_1^+(j\omega_k) + p_2^+(j\omega_k)^2 \\ &+ p_3^-(j\omega_k)^3 + p_4^-(j\omega_k)^4 + \cdots, \\ P_4(j\omega_k) &= p_0^+ + p_1^-(j\omega_k) + p_2^-(j\omega_k)^2 \\ &+ p_3^+(j\omega_k)^3 + p_4^+(j\omega_k)^4 + \cdots. \end{aligned} \quad (8)$$

If these vertices are plotted in complex plane $\mathbb{C}$ it will result in the rectangle presented in Figure 2:

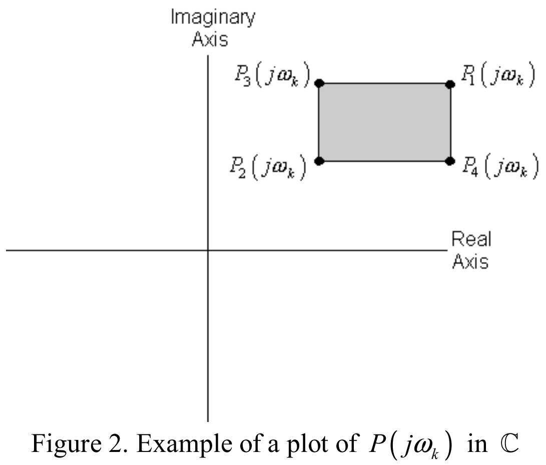

Figure 2. Example of a plot of $P(j\omega_k)$ in $\mathbb{C}$

[2] For simplicity in the notation the dependecy of all transfer functions and polynomials from here to the end of this section will be omitted

Note that there are more than nine possible relative situations between the Kharitonov rectangle and the axis of the complex plane:

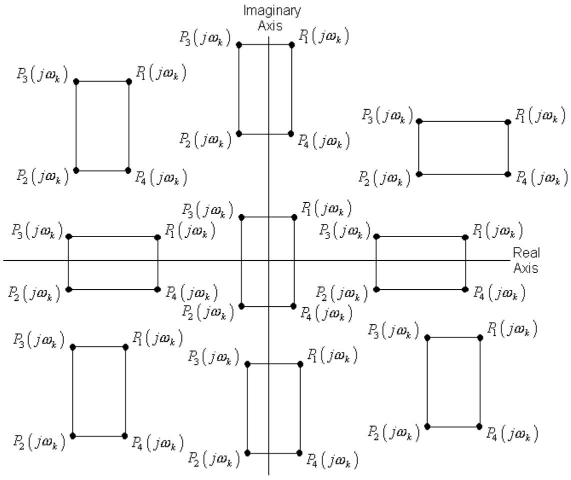

Figure 3. Possible relative situations between a plot of $P\left(j\omega_k\right)$ and axis in $\mathbb{C}$

We say "more than" nine situations because when a rectangle intersects with at least one axis it does not do it in a symmetric way.

According to this consideration, the set of rules is presented in the Appendix of this paper.

Once it is known how to determine the maximum and the minimum value of the module and angle of a given interval polynomial, it will be easy to determine the maximum and minimum module and angle of an interval transfer function. In consequence, the set of inclusion rules as constraints for the optimisation problem can now be established.

4. FORMULATION OF THE OPTIMISATION PROBLEM

The proposed goal is to minimize the squared diagonal of the hyper-rectangular parameter space and the squared difference between frequency domain data envelopes and model envelopes. Then:

$$\min \ f\left(\Theta\right)=\alpha\left(\theta_1^2+\theta_2^2+\cdots+\theta_p^2\right)+$$
$$+\left\{\begin{array}{l}\beta_1\left(j\omega_k\right)\left(\left|D\left(j\omega_k\right)\right|^+-\left|G\left(j\omega_k,\Theta\right)\right|^+\right)^2+\\ +\beta_2\left(j\omega_k\right)\left(\left|G\left(j\omega_k,\Theta\right)\right|^--\left|D\left(j\omega_k\right)\right|^-\right)^2+\\ +\beta_3\left(j\omega_k\right)\left(\angle D\left(j\omega_k\right)^+-\angle G\left(j\omega_k,\Theta\right)^+\right)^2+\\ +\beta_4\left(j\omega_k\right)\left(\angle G\left(j\omega_k,\Theta\right)^--\angle D\left(j\omega_k\right)^-\right)^2\end{array}\right\}\Bigg|_{k=1..n}$$
$$subject\ \ to:$$
$$\left[D(j\omega_i)\right]\subseteq\left[G(j\omega_i,\boldsymbol{\theta})\right]\ \ i=1,...,n$$
$$\boldsymbol{\theta}\in\left[\boldsymbol{\theta}\right] \qquad (9)$$

where $\left[D(j\omega_i)\right]\subseteq\left[G(j\omega_i,\boldsymbol{\theta})\right]$ $i=1,...,n$ should be implemented using the rules described in (3), $\left[\theta\right]$ is the interval vector of uncertain parameters, $\left[D(j\omega_i)\right]$ and $\left[G(j\omega_i,\theta)\right]$ are the intervals containing the measured and modelled frequency responses, respectively. α is a scalar value and $\beta_1\left(j\omega_k\right)$, $\beta_2\left(j\omega_k\right)$, $\beta_3\left(j\omega_k\right)$ and $\beta_4\left(j\omega_k\right)$ are weight vectors.

Due to the nature of the set of constraints the convexity of this problem is not assured, so the existence of a unique global solution is neither assured. If the global solution needs to be guaranteed it will be necessary to use global optimisation tools.

A good starting point seems to be Tyler and Morari (1999) that will allow to transform the set of rules into binary expressions.

5. EXAMPLE

Let $D\left(j\omega\right)$ be a synthetic process which provide the source data in the frequency domain with $\omega=\left\{0.1,0.2,0.5,1,2,5,10,20\right\}$ rad/sec:

$$D\left(j\omega\right)=\frac{[-0.1,0.1]j\omega+15}{1\left(j\omega\right)^2+[1,5]j\omega+8} \qquad (10)$$

It is clear that the subset of the parameter space that defines the model is a two-dimensional rectangle $\mathbf{QR_0}$:

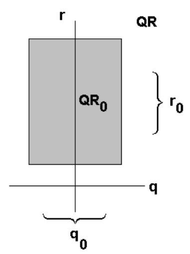

Figure 4. Subset of parameter space for $D\left(j\omega\right)$

The maximum and the minimum module and angle of the frequency response of this process, obtained using the set of rules described in Section 3, are plotted as the source data for the algorithm:

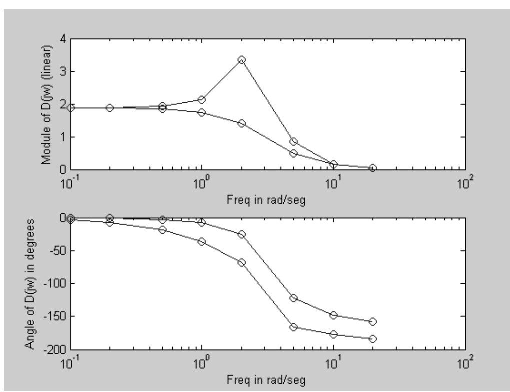

Figure 5. Maximum and minimum module (and angle) of $D\left(j\omega\right)$

Using this available data to adjust the interval parameters of a linear model for the process as if they would be obtained from an empirical environment. The reason why this data has been generated in such a way is because we know, a priori, which are the real intervals for the parameters of the model.
From now on, let us consider that we known the model structure and two out of all its parameters (q_0 and r_0) are unknown and we have to determine them according to that method explained before:

$$D(j\omega) = \frac{q_0 j\omega + 15}{1(j\omega)^2 + r_0 j\omega + 8} \quad (11)$$

The goal is to recover the original parameter subset starting from the available data appeared in Figure 5 with the purposed algorithm.
The optimisation problem (9) adapted to this particular example is solved using the Matlab® Optimization Toolbox that allows to use as a restriction a function that implements the set of rules described in *Section 3*. Since this Toolbox uses SQP algorithm, based on gradient search method, and the optimisation problem is not convex as discussed in *Section 4*, a good seed should be provided in order to avoid local optimum. In this example, the seed has been obtained by applying a parameter bounding-error (consistency) approach based on the use of SIVIA algorithm (Jaulin, 2001) which provides the pairs of q and r that produces realisations in the frequency domain contained within the envelopes of the available data.
Even though the results obtained by applying the purposed algorithm are as far satisfactory, this optimisation approach has to be reformulated into a global optimisation algorithm following Tyler and Morari (1999) as already suggested in *Section 4*.

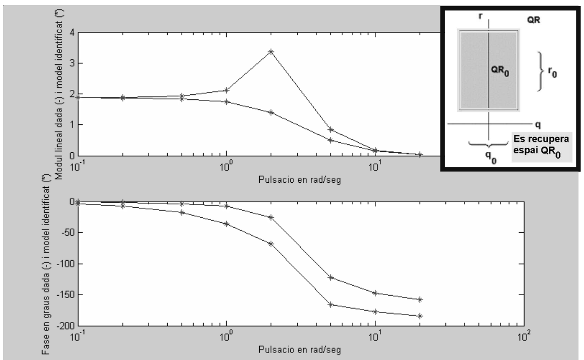

Figure 6. Obtained results. Recovering of the original parameter subspace. Perfect fit by the model with available data

6. CONCLUSION

In this paper, an Optimisation-based Approach to Interval Model Identification in the Frequency Domain has been proposed. As a result of its application, an interval LTI model that satisfies that its worst-case frequency response contains all the measured given data has been obtained. The optimisation problem is formulated using, as constraints, that this inclusion property is explicitly satisfied making use of some auxiliary results used to prove the Kharitonov Theorem. Finally, an example of application of this approach has been proposed to illustrate how the interval identification algorithm works.

APPENDIX

If $\mathrm{Re}\{K_2(j\omega_k)\} > 0$ and $\mathrm{Im}\{K_2(j\omega_k)\} > 0$

$$|P(j\omega_k)|^+ = |K_1(j\omega_k)|$$
$$|P(j\omega_k)|^- = |K_2(j\omega_k)|$$
$$\angle P(j\omega_k)^+ = \angle K_3(j\omega_k)$$
$$\angle P(j\omega_k)^- = \angle K_4(j\omega_k)$$

ElseIf $\mathrm{Re}\{K_2(j\omega_k)\} \le 0$ and $\mathrm{Im}\{K_2(j\omega_k)\} > 0$ and $\mathrm{Re}\{K_4(j\omega_k)\} \ge 0$

$$|P(j\omega_k)|^- = \mathrm{Im}\{K_2(j\omega_k)\}$$
$$\angle P(j\omega_k)^+ = \angle K_2(j\omega_k)$$
$$\angle P(j\omega_k)^- = \angle K_4(j\omega_k)$$

If $\mathrm{Re}\{K_2(j\omega_k)\} + \mathrm{Re}\{K_4(j\omega_k)\} \ge 0$

$$|P(j\omega_k)|^+ = |K_1(j\omega_k)|$$

Else

$$|P(j\omega_k)|^+ = |K_3(j\omega_k)|$$

End

ElseIf $\mathrm{Re}\{K_4(j\omega_k)\} < 0$ and $\mathrm{Im}\{K_4(j\omega_k)\} > 0$

$$|P(j\omega_k)|^+ = |K_3(j\omega_k)|$$
$$|P(j\omega_k)|^- = |K_4(j\omega_k)|$$
$$\angle P(j\omega_k)^+ = \angle K_2(j\omega_k)$$
$$\angle P(j\omega_k)^- = \angle K_4(j\omega_k)$$

ElseIf $\mathrm{Re}\{K_1(j\omega_k)\} < 0$ and $\mathrm{Im}\{K_2(j\omega_k)\} \le 0$ and $\mathrm{Im}\{K_3(j\omega_k)\} \ge 0$

$$|P(j\omega_k)|^- = \mathrm{Re}\{K_1(j\omega_k)\}$$
$$\angle P(j\omega_k)^+ = \angle K_4(j\omega_k)$$
$$\angle P(j\omega_k)^- = \angle K_1(j\omega_k)$$

If $\mathrm{Im}\{K_2(j\omega_k)\} + \mathrm{Im}\{K_3(j\omega_k)\} \ge 0$

$$|P(j\omega_k)|^+ = |K_3(j\omega_k)|$$

Else

$$|P(j\omega_k)|^+ = |K_2(j\omega_k)|$$

End

ElseIf $\mathrm{Re}\{K_1(j\omega_k)\} < 0$ and $\mathrm{Im}\{K_1(j\omega_k)\} < 0$

$$|P(j\omega_k)|^+ = |K_2(j\omega_k)|$$
$$|P(j\omega_k)|^- = |K_1(j\omega_k)|$$
$$\angle P(j\omega_k)^+ = \angle K_4(j\omega_k)$$
$$\angle P(j\omega_k)^- = \angle K_3(j\omega_k)$$

ElseIf $\mathrm{Re}\{K_3(j\omega_k)\} \le 0$ and $\mathrm{Im}\{K_3(j\omega_k)\} < 0$ and $\mathrm{Re}\{K_1(j\omega_k)\} \ge 0$

$$|P(j\omega_k)|^- = \mathrm{Im}\{K_3(j\omega_k)\}$$
$$\angle P(j\omega_k)^+ = \angle K_1(j\omega_k)$$
$$\angle P(j\omega_k)^- = \angle K_3(j\omega_k)$$

If $\mathrm{Re}\{K_2(j\omega_k)\} + \mathrm{Re}\{K_4(j\omega_k)\} \ge 0$

$$|P(j\omega_k)|^+ = |K_4(j\omega_k)|$$

Else

$$|P(j\omega_k)|^+ = |K_2(j\omega_k)|$$

End

ElseIf $\mathrm{Re}\{K_3(j\omega_k)\} > 0$ and $\mathrm{Im}\{K_3(j\omega_k)\} < 0$

$$|P(j\omega_k)|^+ = |K_2(j\omega_k)|$$
$$|P(j\omega_k)|^- = |K_1(j\omega_k)|$$
$$\angle P(j\omega_k)^+ = \angle K_4(j\omega_k)$$
$$\angle P(j\omega_k)^- = \angle K_3(j\omega_k)$$

ElseIf $\mathrm{Re}\{K_2(j\omega_k)\} > 0$ and $\mathrm{Im}\{K_2(j\omega_k)\} \le 0$ and $\mathrm{Im}\{K_3(j\omega_k)\} \ge 0$

$$|P(j\omega_k)|^- = \mathrm{Re}\{K_2(j\omega_k)\}$$
$$\angle P(j\omega_k)^+ = \angle K_4(j\omega_k)$$
$$\angle P(j\omega_k)^- = \angle K_1(j\omega_k)$$

If $\mathrm{Im}\{K_2(j\omega_k)\} + \mathrm{Im}\{K_3(j\omega_k)\} \ge 0$

$$|P(j\omega_k)|^+ = |K_1(j\omega_k)|$$

Else

$$|P(j\omega_k)|^+ = |K_4(j\omega_k)|$$

End

Else

$$|P(j\omega_k)|^- = 0$$
$$\angle P(j\omega_k)^+ = \angle K_4(j\omega_k)$$
$$\angle P(j\omega_k)^- = \angle K_1(j\omega_k)$$

If $\mathrm{Im}\{K_1(j\omega_k)\} + \mathrm{Im}\{K_4(j\omega_k)\} \ge 0$ and $\mathrm{Re}\{K_1(j\omega_k)\} + \mathrm{Re}\{K_2(j\omega_k)\} \ge 0$

$$|P(j\omega_k)|^+ = |K_1(j\omega_k)|$$

ElseIf $\mathrm{Im}\{K_1(j\omega_k)\} + \mathrm{Im}\{K_4(j\omega_k)\} < 0$ and $\mathrm{Re}\{K_1(j\omega_k)\} + \mathrm{Re}\{K_2(j\omega_k)\} < 0$

$$|P(j\omega_k)|^+ = |K_2(j\omega_k)|$$

ElseIf $\mathrm{Im}\{K_1(j\omega_k)\} + \mathrm{Im}\{K_4(j\omega_k)\} > 0$ and $\mathrm{Re}\{K_1(j\omega_k)\} + \mathrm{Re}\{K_2(j\omega_k)\} < 0$

$$|P(j\omega_k)|^+ = |K_3(j\omega_k)|$$

Else

$$|P(j\omega_k)|^+ = |K_4(j\omega_k)|$$

End

End

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