MINIMUM NUMBER OF SAMPLED STREAMS ENSURING CIRCUIT MASS-BALANCE

Fani Kalaitzi and K.G. Tsakalakis

School of Mining and Metallurgical Engineering, National Technical University of Athens, 15780, Zografou, Athens, Greece, e-mail:kostsakg@metal.ntua.gr

Abstract: In the present paper a mathematical procedure is described examining the structure of mineral processing flowsheets in order to determine the minimum sampling requirements and at the same time ensuring a material balance of the circuit. A new equation is proposed and its validation is verified in relevant applications, where a comparison is made with methods previously proposed. The equation proved to be easily applicable and reliable, not requiring in advance rearrangement of the circuit.

Keywords: Mineral Processing, Automatic control (closed-loop), Sampling control, Material balance control, Control algorithms, Flow diagrams, Sensitivity analysis.

1. INTRODUCTION

The mineral industry faced with difficult challenge of improving plant performance is focused on achieving higher plant recoveries and tighter product quality control. The evaluation of plant performance at steady–state conditions by conducting sampling around the plant streams and developing a mass balance flowsheet for the plant, continues to be a powerful tool (Cutting, 1979; Lynch, 1977; Smith and Frew, 1983; Frew, 1983; Smith and Ichiyen, 1973; Wills and Manser, 1985; Wills, 1985; Wills, 1986; Wills, 1992; Kapur et al., 1993a,b; Hodouin, 1982; Hodouin et al., 1984; Hodouin and Manki, 1996; Lyman, 1998; Mirabedini and Hodouin, 1998; Salama, 1999a,b; Kalaitzi and Tsakalakis, 2000; Salama, 2001; Kalaitzi, 2003). The samples collected are analyzed (usually for a metal assay, a size distribution or fraction solids) and the data produced are used to estimate the balanced stream mass splits and balanced stream components by optimizing an objective function. The objective function could be the sum of component closures squares, sum of squares of absolute component errors, sum of squares of absolute value of component errors, minimum/ maximum of absolute value of component errors, and the entropy function (Klimpel, 1979; Jefferson and Scott, 1986; Wills, 1992; Salama, 1999b, Hodouin and Everell, 1980). Although the use of such excess data can be of great value for the accurate assessment of the material flow rates, it is not always suitable for routine plant auditing, where minimum sampling work and sample preparation have to be performed.

Therefore, the objectives of the flowsheet sampling scheme are:

- Determination of the minimum number of sampled streams to produce a flowsheet mass balance,
- Selection of a set of independent stream mass splits.

Flowsheet sampling scheme has been first investigated by Smith and Frew (1983) and Frew (1983). The scheme derived was based on mass balance calculations of a single component (assay) around the flowsheet streams. However, in order to overcome the limitation of the previous case, sampling schemes were developed using multi – component streams (Salama, 1999a).

In the present paper a new equation of a minimum number of sampled streams is proposed. The development of this equation was based on the analysis of only one component per stream sampled. Furthermore, the limitation of the previous techniques is illustrated using a number of examples of plants / circuits on which the material balance is applied.

2. DERIVATION OF THE EQUATION

Define a *node* in a circuit as any point at which process streams either join or separate. Further, define a *simple node* as a node having either two inputs and one output or the converse. It is clear that simple nodes are of two types: *the junction*, at which two input streams are combined to form a single output stream, and *the separator*, at which a single input stream is divided to form two output streams. Nn stands for the number of the total internal nodes. Moreover, Nr designates the number of streams with known flow which are usually input streams (*reference streams*). If Ns is the total number of the streams observed in a flowsheet, then Ns independent equations have to be written to produce a flowsheet mass balance.

Nn equations of total mass balance around each node could be used and also Nr equations of a known mass flow are available. Since the total number of flowrates is Ns, $(Ns - Nn - Nr)$ additional equations should be formed, so that the mass balance system of equations could be solved. Setting Ner as the number of these equations, then:

$$Ner = Ns - Nn - Nr \qquad (1)$$

Generally, it is not always feasible to measure all these unknown flowrates. Thus, a number samples should be collected, analyzed for one or more components in order to produce *partial mass balance* around the nodes these streams are connected with. The number of the additional independent equations for the *partial* mass balance should be Ner.

It should be noticed that there is no need to use the same component around a node. Actually, the best approach relates to the creation of the partial mass balance around a node for that component with the minimal sensitivity to the experimental error. This choice is based on a preliminary sensitivity analysis (Hodouin, 1982; Smith and Frew, 1983; Hodouin et al., 1984; Wills, 1986; Hodouin et al., 1989; Hodouin et al., 1998, Kalaitzi, 2003).

Eq. (1) is of general applicability in any circuit, without any previous need for the flowsheet to be expanded in a simple nodal form (Smith and Frew, 1983).

In the following analysis the actual circuit was assumed to consist of simple nodes (separators or junctions).

Thus, Ner is the number of the additional equations required; an equal number of nodes (Ner) is selected enabling the partial mass balance equation formation.

When the first node is selected, three (3) streams must be sampled, assuming that the node is simple. The next node chosen must have only one common stream with the previous one. Thus, in this case only two streams are necessary to be sampled. Consequently, except for the first node, for the remaining $(Ner - 1)$ nodes, $2(Ner-1)$ streams must be additionally sampled. Ultimately, the minimum number of streams (Nss_{min}), which must be sampled for a mass balance, is:

$$Nss_{min} = 3 + 2(Ner - 1) \qquad (2)$$

Substituting Ner from Eq. (1), the above equation gives:

$$Nss_{min} = 2(Ns - Nn - Nr) + 1 \qquad (3)$$

The total number of streams in a simple nodal representation of a flowsheet containing F feed streams, J simple junctions and S simple separators is:

$$Ns = F + J + 2S \qquad (4)$$

Since, the total number of nodes in terms of separators and junctions is:

$$Nn = J + S \qquad (5)$$

From Eqs. (4) and (5), Eq. (3) becomes:

$$Nss_{min} = 2\,(F + S - Nr) + 1 \qquad (6)$$

3. VERIFICATION

3.1 Application to a simple circuit

Several flowsheets are considered to demonstrate the results reported in this paper. Fig.1a represents a flowsheet, which can be reduced to nodal form in Fig. 1b. As it is evident from the flowsheet, the total number of streams is $Ns = 14$ and the total number of nodes is $Nn = 8$. It is assumed that the known mass flowrates (Nr) are more than one, for example $Nr = 2$, so twelve (12) stream mass flowrates are unknown. Therefore, twelve (12) independent linear equations are required for mass balance calculations and a sampling scheme has to be applied in order to audit the stream mass fractional splits.

Applying Eq. (3) to the above given data, yields that $Nss_{min} = 9$. This finding means that, at least nine (9) streams need to be sampled so that the system of equations has a unique solution.

As an example, we make the assumption that the mass flowrate of streams 1 and 12 are known $(Nr = 2)$, and from the streams 1, 2, 3, 4, 5, 6, 9, 10, 11 (9 streams) samples are received. Thus, $Nn = 8$ equations of total mass balance (around each node) could be written. These equations are:

$$w_1 = w_{12} + w_8 \qquad \text{(Circuit node)}$$
$$w_1 = w_2 + w_3 \qquad \text{(node 1)}$$

$w_2 = w_4 + w_5$ *(node 3)*
$w_5 + w_6 = w_{10}$ *(node 4)*
$w_{10} = w_9 + w_{11}$ *(node 5)*
$w_3 = w_6 + w_7$ *(node 2)*
$w_7 + w_{14} = w_8$ *(node 8)*
$w_9 = w_{14} + w_{13}$ *(node 7)*

where w_i, i = 1,…14 represents the total mass flowrate of the stream i.

Since the mass flowrates of the reference streams (1, 12) have been measured, the streams w_1 and w_{12} are known. In addition the whole plant can be represented as a single node (whole circuit as a node), because the mass of the individual components in the feed remains constant and it is distributed in the products. This equation is used as there is usually very good component separation at this node. The first equation of the above set corresponds to the whole circuit node. Since this equation was included in the system of the equations, one of the material balance equations of the remaining internal nodes must be excluded. In another case the set of $Nn+1$ equations would not be independent. In the example, material balance on the node 6 has been avoided.

From Eq. (1), $Ner = 4$. Thus, using the assays of the sampled streams, four (4) additional independent equations of *partial* mass balance around the nodes can be written.

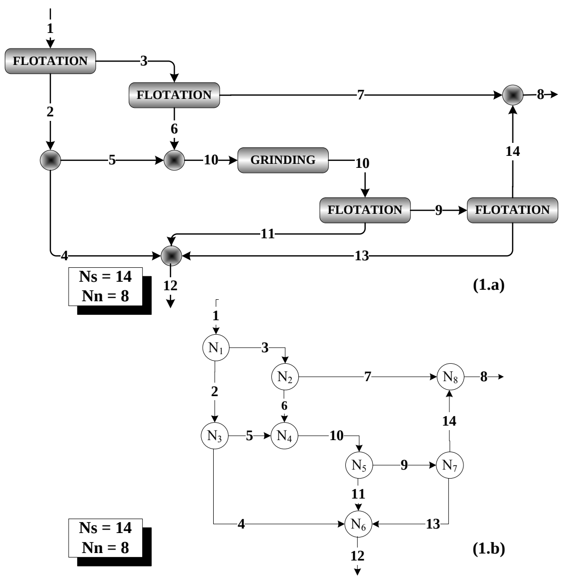

Fig.1: a. Flowsheet, b. Nodal representation

These equations are:

$\alpha_1 w_1 = \alpha_2 w_2 + \alpha_3 w_3$ *(node 5)*
$\alpha_2 w_2 = \alpha_4 w_4 + \alpha_5 w_5$ *(node 4)*
$\alpha_5 w_5 + \alpha_6 w_6 = \alpha_{10} w_{10}$ *(node 3)*
$\alpha_{10} w_{10} = \alpha_9 w_9 + \alpha_{11} w_{11}$ *(node 1)*

where α_i is the component assay of stream i.

Therefore, the whole number of independent equations is: 8+4=12 equations.

3.2 Comparison of the previous and current equation and their application to a complex circuit

In the case where the reference streams are greater than one ($Nr > 1$) and the mass balance of only one component around the flowsheet nodes is applied, two different equations have been proposed by Smith and Frew (1982) and Salama (1999). The equations, giving the minimum number of streams which must be sampled for a mass balance, are given by:

$$Nss_{min} = 2(F+S) - Nr \text{ (Smith and Frew)} \quad (7)$$

$$Nss_{min} = 2(Ns-Nn) - Nr \text{ (Salama)} \quad (8)$$

The two expressions appear different, but in fact they are identical. Substituting for $Ns = F + J + 2S$ and $Nn = J + S$ in Eq. (8), yields Eq. (7).

For $F=1$, $S = 6$, $Nr = 2$ or $Ns = 14$ and $Nn = 8$, the two equations give $Nss_{min} = 10$. This finding means that, from at least ten (10) streams samples must be taken out, while it was previously shown from Eq. (3) that only nine (9) streams are adequate for a mass balance.

The example given below (Fig. 2) verifies that the number of streams given from Eq. (3) is sufficient for mass balance calculations.

In Fig. 2a is shown the pilot plant of the tin concentrator in Camborne School of Mines (Wills, 1986). The plant circuit, reduced to nodal form, is given in Fig. 2b. The concentrates from the sands table (stream 18) and slimes table (stream 17) are accurately predicted (Wills, 1986), so that 16 stream mass flowrates are unknown. Therefore, sixteen (16) independent linear equations are required for mass balance calculations. From Eqs. (7) and (8) and for $F=1$, $S = 7$, $Ns = 18$, $Nn = 16$, $Nr = 2$ results that $Nss_{min} = 14$. This is the minimum number of streams needed for a complete circuit mass balance. On the contrary, Eq. (3) gives a reduced number of streams for the same circuit (Table 1).

Table 1. Number of equations for the prediction of the minimum number of sampled streams

Equation No	**Equation**	Nss_{min}
Eq. (7)	$Nss_{min} = 2(F+S)-Nr$	14
Eq. (8)	$Nss_{min} = 2(Ns-Nn) - Nr$	14
Eq. (3)	$Nss_{min} = 2(Ns-Nn-Nr) +1$	13

The following procedure shows that the number of the samples actually required is only 13, one less than those predicted from Eqs. (7) and (8).

Assuming as known the flowrates of the streams 17 and 18, and if 13 samples have been taken out from the streams 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15 and

18, then $Nn = 10$ equations of total mass balance around each node could be created. These equations are given below:

$$w_6 = w_{18} + w_9 \qquad \textit{(node 6)}$$
$$w_5 = w_6 + w_7 \qquad \textit{(node 4)}$$
$$w_7 + w_9 = w_8 + w_{10} \qquad \textit{(node 5)}$$
$$w_3 = w_4 + w_5 \qquad \textit{(node 2)}$$
$$w_{10} = w_{11} + w_{15} \qquad \textit{(node 7)}$$
$$w_{13} + w_{11} = w_{12} \qquad \textit{(node 8)}$$
$$w_{12} = w_{13} + w_{14} + w_{17} \qquad \textit{(node 9)}$$
$$w_{15} + w_{14} = w_{16} \qquad \textit{(node 10)}$$
$$w_4 + w_8 = w_2 \qquad \textit{(node 3)}$$
$$w_1 + w_2 = w_3 \qquad \textit{(node 1)}$$

where w_i represents the total mass flowrate of the stream i (i = 1,…14).

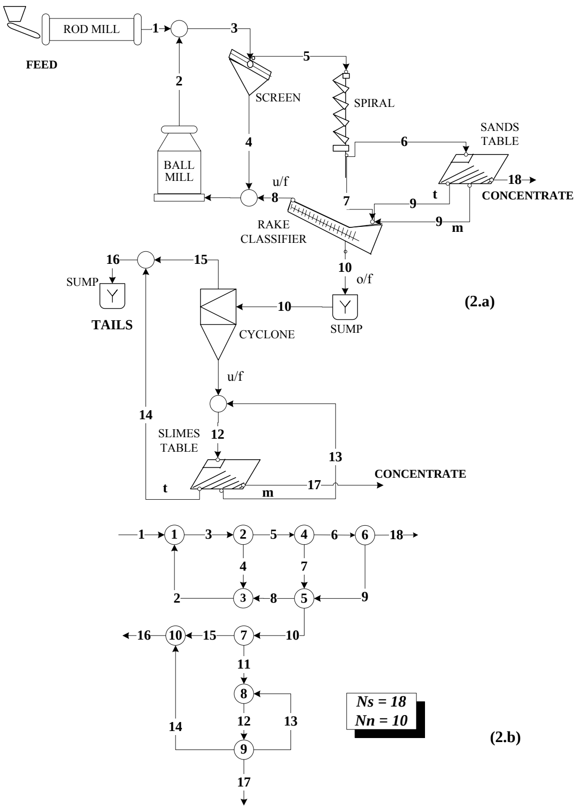

Fig. 2: a. Flowsheet of pilot plant tin concentrator
b. Pilot- plant flowsheet in nodal representation

Furthermore, using the assays (α_i) of the sampled streams, *Ner* additional independent equations of *partial* mass balance around the nodes can be written. These equations are also given below:

$$\alpha_6 w_6 = \alpha_{18} w_{18} + \alpha_9 w_9 \qquad \textit{(node 6)}$$
$$\alpha_5 w_5 = \alpha_6 w_6 + \alpha_7 w_7 \qquad \textit{(node 4)}$$
$$\alpha_7 w_7 + \alpha_9 w_9 = \alpha_8 w_8 + \alpha_{10} w_{10} \qquad \textit{(node 5)}$$
$$\alpha_3 w_3 = \alpha_4 w_4 + \alpha_5 w_5 \qquad \textit{(node 2)}$$
$$\alpha_{10} w_{10} = \alpha_{11} w_{11} + \alpha_{15} w_{15} \qquad \textit{(node 7)}$$
$$\alpha_{13} w_{13} + \alpha_{11} w_{11} = \alpha_{12} w_{12} \qquad \textit{(node 8)}$$

Therefore, the total number of equations formed is: 10+6=16 equations.

Solving the system of the equations with sixteen (16) unknown flowrates it results that, only 13 sampled streams are sufficient and ensure a complete mass balance.

3.3 The problem of choosing the appropriate sampled streams

It should be emphasized that even if the calculation of the minimum number of streams is very helpful in forming a mass balance system, it does not ensure the formation of such a system. The reason for this is that the number of the measurements should be able to produce *Ner* partial mass balance equations around equal in number nodes of the circuit. Moreover, these equations should be *linearly independent* relatively to the rest system. Consequently, except from calculating the number Nss_{min}, it is necessary to choose suitably the sampled streams. This is a problem requiring further discussion.

A very simple circuit has been chosen in order to some exposures be done relatively to this problem. The circuit has been reduced to nodal form shown in Fig 3. It consists of $Ns = 11$ streams and $Nn = 6$ simple nodes. It was also assumed that two flowrates ($Nr = 2$) were known. According to Eq. (1), $Ner = 11\text{-}6\text{-}2 = 3$ partial mass balance equations should be produced, whereas from Eq. (3) $Nss_{min} = 2\ (11\text{-}6\text{-}2) + 1 = 7$ streams should be sampled for the calculation of the total number *Ns* of flowrates. In the following illustration a number of constraints referring to the streams, which must be sampled, are demonstrated.

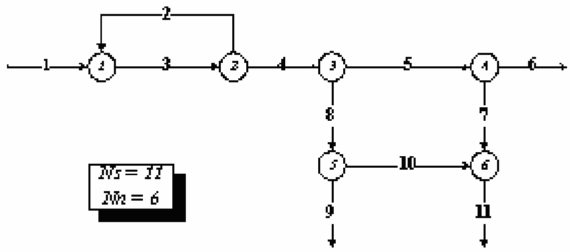

Fig. 3: Flowsheet in nodal representation

a. *The sampled streams should be suitably chosen, so that (using the conducted measurements) the configuration of partial mass balance around the nodes can be performed.*

Let`s consider the circuit shown in Fig. 3. Seven (7) samples (streams 2, 3, 4, 5, 6, 9 and 10), have been collected. Then, only one partial mass balance equation (that around the node 2) could be written. Hence, only the measurements from streams 2, 3 and 4 could be used, while the rest of them (streams 5, 6, 9 and 10) could not form such an equation. On the contrary, if samples were taken out from streams 7 and 8 instead of streams 9 and 10, two additional equations (those around the nodes 3 and 4) could be formed. Finally, the required number of the equations is completed.

b. *If the minimum number of sampled streams is to be used, the nodes selected for forming partial mass equations around them, should be sequentially connected.*

The above condition is necessary, because it ensures that for writing a new equation, it is not required the sampling of three unmarked (not already used) streams. Once again, for the circuit of Fig. 3 if partial mass balance equations are to be formed around the nodes 1, 4 and 6, eight steams should be sampled (1, 2, 3, 5, 6, 7, 10 and 11). However, as it was verified above, samples from only seven (7) streams are sufficient to form the number of equations required. These are the streams 2, 3, 4, 5, 6, 7 and 8 corresponding to the nodes 2, 3 and 4. This reduction in the number of sampled streams is due to the use of stream 7 for the balance equations common to the nodes 4 and 6, so that it is not sampled twice. On the other hand, for the equivalent equation of node 1, three streams, not up to now used, should be sampled.

c. *The set of streams connected to a node that an equation is to be written around, should not have more than one element (stream) in common with the set of streams that have been already used. In the opposite case the current equation will not be independent from the other.*

For example, in the circuit of Fig. 3 if partial mass balance equations have been produced for the nodes 1, 2 and 4, it is necessary to take out samples from the streams 1, 2, 3 and 5, 6, 7 corresponding to the nodes 1 and 4. For the equation of node 2, only one measurement (stream 4) is required because, it includes two common streams with node 1 (streams 2 and 3). Nevertheless, equations performed around the node 1 and 2 are linearly dependent. As a result there is not solution of the system even if it consists of $Ner = 3$ additional equations. Finally, this is reasonable, since the flowrates of the streams 1 and 4 are identical, and consequently the partial mass balances around the nodes 1 and 2 are expressed by the same equation.

4. CONCLUSIONS

Although the use of excess data in plant mass balance calculations can produce accurate results, choosing a single component to assess each node reduces considerably sampling work, preparation and assaying requirements. The attempt of producing a sampling scheme, requiring the minimum number of sampled streams and ensuring that the corresponding system of mass balance equations has a unique solution, leads to some important conclusions:

a. The proposed Eqs. (7) and (8) give exceeding number of sampling streams, when reference streams are greater than one ($Nr > 2$). On the contrary, using Eq. (3), a more accurate result is obtained.

b. Eq. (3) is applicable to any circuit, provided that certain circumstances are valid. However, a system of equations, presenting a unique solution and employing less number of streams than that provided from Eq. (3), could not exist.

c. Additionally, using multi-components streams, equations were also established for the development of partial mass balance for each node. The present method, in order to reduce significantly the number of the sampled streams, increased the number of the components measured on each stream and the number of the corresponding partial equations around the nodes. This is not always feasible, due to the fact that many streams in a circuit consist of only one or two components. Furthermore, the solutions of these equations are not always integers.

d. The number of sampled streams, calculated from Eq. (3), provides the minimum sampling requirements for a material balance. The above findings mean that, there is not any alternative sampling procedure including smaller number of streams. However, as it was previously demonstrated, the circuit topology is very important towards the selection of the sampled streams and there are also some constraints, which should be under consideration in this process. Ultimately, the choice of the streams to be sampled may be a difficult attempt and sometimes without the desirable results.

REFERENCES

Cutting, W.G., (1979), Material balances in metallurgical studies: current use at warren spring laboratory. p.18-22, Presentation at the 1979 AIME Annual Meeting, New Orleans Louisiana.

Frew, J.A., (1983), Computer - aided design of sampling schemes. *International Journal of Mineral Processing*, **11**, pp. 255–265.

Hodouin, D. and Everell M.D., (1980), A hierarchical procedure for adjustment and material balancing of mineral processes data. *International Journal of Mineral Processing*, **7**, pp. 91-116.

Hodouin, D., (1982), Sensitivity analysis of Material Balance Calculations - an Application to a Cement Clinker Grinding Process. *Powder Technology*, **32**, pp. 139–153.

Hodouin, D., Bazin C. and Trasiak A., (1984), Reliability of calculation of mineral proces efficiencies and rate parameters from balanced data. *Control '84 AIME SME/ TMS*, pp. 133-144.

Hodouin, D., F. Flament and C. Bazin (1989), Reliability of Material Balance Calculations, a sensitivity approach. *Minerals Engineering,* **2**, No 2, pp. 157-169, Pergamon Press plc.

Hodouin, D., A. Mirabedini, S. Makni and C. Bazin (1998), Reconciliation of mineral processing data containing correlated measurement errors. *International Journal of Mineral Processing,* **54**, pp.201-215, Elsevier Science Publ.

Jefferson, T.R. and C.H. Scott (1986), The entropy approach to material balancing in mineral processing. *International Journal of Mineral Processing*, **18**, pp. 251–261.

Kalaitzi, F. (2003), Estimation of the operational parameters for optimal design and control of material processing circuits. PhD Thesis, NTUA, Athens (In Greek with English abstract, unpublished).

Kapur, P.C., A. Kumar and D.W. Fuerstenau (1993a), A two tier approach to the mass balance of multilevel mineral processing data. Part 1: Performance index and local balance around a node. *International Journal of Mineral Processing*, **39**, pp. 209-223.

Kapur, P.C., A. Kumar and D.W. Fuerstenau (1993b), A two tier approach to the mass balance of multilevel mineral processing data. Part 2: Splicing of stream flows and global balance over a circuit. *International Journal of Mineral Processing*, **39**, pp. 225-237.

Klimpel, R. (1979), Estimation of weight ratios given component make-up analyses of streams. *SME / AIME Trans*, **266**, pp. 1882–1886.

Lyman, G. (1998), The influence of segregation of particulates on sampling variance the question of distributional heterogeneity. *International Journal of Mineral Processing*, **55**, pp. 95-112.

Lynch, A.J. (1977), Mineral Crushing and Grinding Circuits: Their simulation, Optimisation, Design and Control. Developments in Mineral Processing,. Vol **1**, Chapter 7, Elsevier Science Publ.

Mirabedini, A. and D. Hodouin (1998), Calculation of variance and covariance of sampling errors in complex mineral processing systems, using state - space dynamic models. *International Journal of Mineral Processing*, **55**, pp. 1-20.

Salama, A.I.A. (1999a), Sampling schemes for mass balance of flowsheets with muti-component streams. *International Journal of Mineral Processing*, **55**, pp. 219-229.

Salama, A.I.A., (1999b), The maximum entropy approach and comparison with the least squares approach for flowsheet mass balance. *International Journal of Mineral Processing*, **57**, pp. 219-241.

Salama, A.I.A., (2001), Flowsheet mass balance with partial observation of stream components. *International Journal of Mineral Processing*, **63**, pp.59-77.

Smith, H.W. and N. Ichiyen (1973), Computer adjustment of Metallurgical Balances. *Canadian Institution Metallurgical Bulletin (CIM)*, **66**, pp. 97-100.

Smith, H.W. and Frew J.A., (1983), Design and analysis of sampling experiments - A sensitivity approach*, International Journal of Mineral Processing*, **11**, pp. 267–284.

Tsakalakis, K.G. and F. Kalaitzi, (2000), Software Development for the Calculation and Control of Mineral Processing Circuits. *3rd International Congress of Mineral Wealth*, Vol. **II**, pp. 209-218, Athens Greece (in Greek with English abstract).

Wills, A.B. and Manser E.J., (1985), Reconciliation of simple node excess data. *Transactions of the Institution of Mining and Metallurgy*, **94**, Section C (Dec), p. 209.

Wills, A.B., (1985), Maximizing accuracy of two-product recovery computations. *Transactions of the Institution of Mining and Metallurgy*, **94**, Section C (June), p. 101.

Wills, A.B., (1986), Complex Circuit mass balancing - A simple practical sensitivity analysis method. *International Journal of Mineral Processing*, **16**, pp.245–262.

Wills, A.B., (1992), *Mineral Processing Technology, An introduction to the practical aspects of the treatment and mineral recovery,* Pergamon, Canada, 6th Ed.