IMPLICIT QUADRATIC INTEGRAL METHOD FOR TRANSIENT STABILITY IN POWER SYSTEMS

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Abstract: This paper presents a new approach to perform time simulation of power system transient stability using implicit quadratic integral method. Fast and reliable simulation of the transient stability is required to achieve high reliability based on the on-line stability assessment. The trapezoidal method has been mainly adopted for this purpose. However, the proposed method shows the possibility of improvement in accuracy with no loss of the computation speed. The proposed method is tested with some typical case of 'stiffness problem' and for transient stability analysis in power systems.

Keywords: Differential Equations, Nonlinear System, Numerical Method, Power Systems, Transient Stability Analysis

1. INTRODUCTION

A large research effort has been focused on numerical solutions for nonlinear ordinary differential equations (ODE's). Solution methods can be classified into two large categories; explicit method and implicit method. The former includes Euler methods and Runge-Kutta (RK) methods, and the latter includes the trapezoidal method and the implicit Runge-Kutta method. The explicit methods have been in wide application to most of ODE's due to its simple algorithm and fast computation speed. However, it has been found that the explicit methods may be involved in the instability problem for ODE's including the steepness problem and/or the singularity problems. The algorithm stability has been discussed with the stability functions for various implicit methods in references (Hairer and Norsett, 1993, 1996). A number of numerical methods can be found in the literature for ODE's using neural networks (Logovski, 1992; Braham, 1989; Shelton, *et al.*, 1992). Logovski proposed an iterative method for solving ODE's on Hopfield network (Logovski, 1992).

One of the typical ODE problems arising in science and engineering is the Riccati differential equations (RDE's) given by a nonlinear matrix ODE's, of which the solution algorithms are discussed in (Choi, 1990; Choi and Laub, 1990; Davison and Maik, 1973; Baczynski and Fragoso, 2001; Papavassilopoulos and Cruz, 1979; O' Brien, 1998). The existence of continuous solutions has been discussed for the coupled RDE's arising in closed-loop Nash games with sufficient conditions for existence (Papavassilopoulos and Cruz, 1979). The existence of a stabilizing solution to a RDE has been discussed in terms of a pole set of a related Hamiltonian system (Kundur, 1994).

Regarding transient stability studies of power systems, many papers can be found in the literature (Miki, *et al.*, 2002; Kundur, *et al.*, 2000; Aboreshaid, *et al.*, 1996). Aboresaid *et al.* proposed a method of bisection to reduce the computation time required in the stochastic evaluation of transient stability (Aboreshaid, *et al.*, 1996). A method of transient stability assessment has been presented by use of critical fault clearing time functions generated by simulation of transient phenomena (Miki, *et al.*, 2002). Kundur, Morison and Wang presented an idea of on-line transient stability assessment based on their field experiences from many year off-line applications, which shows the importance of the computation speed of algorithm (Kundur, *et al.*, 2000).

For the time simulation of power system transient stability, the trapezoidal implicit method has been preferred due to its stability-preserved behavior and fast computation speed. Some of the 2-stage Implicit

RK (IRK) methods including the trapezoidal method are A-stable, but do not satisfy the L-stability (Hairer and Norsett, 1996). This may cause some instability problem for ODE's with high stiffness. Besides, the trapezoidal method adopts too rough approximation in the integration. The 3-stage Diagonal Implicit RK (DIRK) method may provide better calculation accuracy and better stability-preserved behaviors. However, higher order IRK's are not appropriate to apply to problems requiring fast computation speed. This study pursues development of a new algorithm to improve the accuracy in the update integration without loss of computation speed compared with the trapezoidal method. This paper proposes a new method adopting an approximated integration technique by approximation the integrand as a quadratic function. It is also shown that improvement in calculation accuracy tends to mitigate the instability problem with sample studies.

2. MATHEMATICAL ANALYSIS

In this section, we introduce the conventional algorithms which are in common use for power system transient stability analysis, giving comments on their features. Finally, a new algorithm will be proposed to overcome the weaknesses of RK and the trapezoidal methods.

Nonlinear system dynamics can be represented by the state equation in the form of

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}(t), t) \tag{1}$$

$$\text{where} \begin{cases} \mathbf{f}(\mathbf{x}(t), t) : \text{nonlinear function } R^n \to R^n \\ \mathbf{x}(t) \in R^n : \text{state variable vector} \\ n : \text{number of state variables} \end{cases}$$

2.1 Conventional Algorithms for Power System Transient Stability Analysis

In this section, we will briefly introduce conventional algorithms to solve (1) in common use for the time simulation of power system transient stability.

Explicit Runge-Kutta method. The algorithm of the 4th order RK method can be characterized by the following update rule :

$$\mathbf{x}(t_0 + h) = \mathbf{x}_0 + a\mathbf{k}_1 + b\mathbf{k}_2 + c\mathbf{k}_3 + d\mathbf{k}_4 \tag{2}$$

$$\begin{aligned} \text{where } \mathbf{k}_1 &= h\mathbf{f}(t_0, \mathbf{x}_0) \\ \mathbf{k}_2 &= h\mathbf{f}(t_0 + ph, \mathbf{x}_0 + q\mathbf{k}_1) \\ \mathbf{k}_3 &= h\mathbf{f}(t_0 + rh, \mathbf{x}_0 + s\mathbf{k}_2) \\ \mathbf{k}_4 &= h\mathbf{f}(t_0 + uh, \mathbf{x}_0 + u\mathbf{k}_3) \\ \text{with } \mathbf{x}_0 &= \mathbf{x}(t_0) \end{aligned}$$

Several 4th order algorithms are possible depending on the choice of the arbitrary constant. Most typical ones of them are as follows :

Scheme 1 :

$$\mathbf{x}_{i+1} = \mathbf{x}_i + \frac{1}{6}(\mathbf{k}_1 + 2\mathbf{k}_2 + 2\mathbf{k}_3 + \mathbf{k}_4) \tag{3}$$

$$\begin{aligned} \text{where } \mathbf{k}_1 &= h\mathbf{f}(t_i, \mathbf{x}_i) \\ \mathbf{k}_2 &= h\mathbf{f}(t_i + \frac{h}{2}, \mathbf{x}_i + \frac{\mathbf{k}_1}{2}) \\ \mathbf{k}_3 &= h\mathbf{f}(t_i + \frac{h}{2}, \mathbf{x}_i + \frac{\mathbf{k}_2}{2}) \\ \mathbf{k}_4 &= h\mathbf{f}(t_i + h, \mathbf{x}_i + \mathbf{k}_3) \end{aligned}$$

Scheme 2 :

$$\mathbf{x}_{i+1} = \mathbf{x}_i + \frac{1}{8}(\mathbf{k}_1 + 3\mathbf{k}_2 + 3\mathbf{k}_3 + \mathbf{k}_4) \tag{4}$$

$$\begin{aligned} \text{where } \mathbf{k}_1 &= h\mathbf{f}(t_i, \mathbf{x}_i) \\ \mathbf{k}_2 &= h\mathbf{f}(t_i + \frac{h}{3}, \mathbf{x}_i + \frac{\mathbf{k}_1}{3}) \\ \mathbf{k}_3 &= h\mathbf{f}(t_i + \frac{2h}{3}, \mathbf{x}_i - \frac{\mathbf{k}_1}{3} + h\mathbf{k}_2) \\ \mathbf{k}_4 &= h\mathbf{f}(t_i + h, \mathbf{x}_i + \mathbf{k}_1 - \mathbf{k}_2 + \mathbf{k}_3) \end{aligned}$$

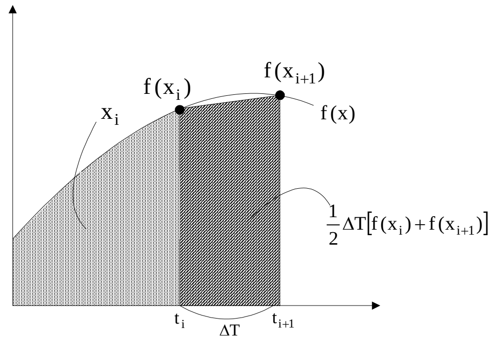


Fig. 1. Trapezoidal Method

In this paper, Scheme 1 is adopted for transient stability time simulation in power systems.

Trapezoidal Method In this section, we shall briefly introduce trapezoidal method. The ODE (1) can be solved by approximated integral with the use of the trapezoidal method.

$$\begin{aligned} \mathbf{x}_{i+1} &= \mathbf{x}_i + \int_{t_i}^{t_{i+1}} \mathbf{f}(\mathbf{x})dt \\ &= \mathbf{x}_i + \frac{1}{2}\Delta T[\mathbf{f}(\mathbf{x}_i) + \mathbf{f}(\mathbf{x}_{i+1})] \end{aligned} \tag{5}$$

Eq. (5) can be rearranged as follows :

$$\mathbf{x}_{i+1} - \frac{1}{2}\Delta T\mathbf{f}(\mathbf{x}_{i+1}) = \mathbf{x}_i + \frac{1}{2}\Delta T\mathbf{f}(\mathbf{x}_i) \tag{6}$$

Implicit Runge-Kutta method (Hairer and Norsett, 1993, 1996) The s-stage implicit RK method has the following update rule

$$k_i = f\left(t_0 + c_i h, x_0 + h\sum_{j=1}^{s} a_{ij}k_i\right) \quad i = 1, \cdots, s \tag{7}$$

$$x_1 = x_0 + h\sum_{i=1}^{s} b_i k_i$$

Where real numbers b_i, a_{ij} (I, j = 1, …, s) can be arbitrary selected and c_i must satisfy the conditions

$$c_i = \sum_{j=1}^{i-1} a_{ij} \tag{8}$$

Varieties of selecting a_{ij} and b_j provide various implicit algorithms, for example, implicit Euler method and midpoint rule for s = 1, the trapezoidal rule and Hammer & Hollingsworth method for s = 2. Here, it is noted that the 2-stage implicit RK method with $a_{11}=a_{12}=a_{21}=0$, $a_{22}=1$, $b_1=b_2=1/2$, and $c_1=0$, $c_2=1$ is just the trapezoidal rule.

2.2 Proposed Method

For the time simulation of power system transient stability, the trapezoidal implicit method has been preferred due to its stability-preserved behavior and fast computation speed. However, the trapezoidal method adopts too rough approximation in the integration. This study pursues development of a new algorithm to improve the accuracy in the update integration without loss of computation speed compared with the trapezoidal method. This paper proposes a new method adopting an approximated integration technique by approximation the integrand as a quadratic function.

The ODE (5) can be integrated by

$$\begin{aligned}\mathbf{x}(t_i + \Delta T) - \mathbf{x}(t_i) &= \int_{t_i}^{t_i+\Delta T} \mathbf{f}(\mathbf{x}(t), t)dt \\ &= \int_{t_i}^{t_i+\Delta T} \mathbf{g}(t)dt\end{aligned} \tag{9}$$

$$\text{where } \mathbf{g}(t) = \mathbf{f}(\mathbf{x}(t),t) \in R^n \tag{10}$$

The integrand $\mathbf{g}(t)$ can be expanded by using Taylor's series :

$$\mathbf{g}(t) = \mathbf{g}(t_i) + \overline{\mathbf{m}}\Delta t + \overline{\mathbf{a}}\Delta t^2 + \text{higher order terms} \tag{11}$$
$$\text{with } \ 0 \le \Delta t \le T$$

The coefficients of the quadratic function in (11) can be easily evaluated as follows :

$$\begin{aligned}\text{i) } \overline{\mathbf{m}} &= \frac{d\mathbf{g}(t)}{dt} = \frac{d}{dt}\mathbf{f}(\mathbf{x}(t),t)\Big|_{t=t_i} \\ &= \frac{\partial \mathbf{f}(\mathbf{x}(t),t)}{\partial \mathbf{x}(t)}\Big|_{t=t_i} \cdot \mathbf{f}(\mathbf{x}(t_i),t_i) + \frac{\partial \mathbf{f}(\mathbf{x}(t),t)}{\partial t}\Big|_{t=t_i}\end{aligned} \tag{12}$$

$$\text{ii) } \mathbf{g}(t_i + \Delta T) \approx \mathbf{g}(t_i) + \overline{\mathbf{m}}\Delta T + \overline{\mathbf{a}}\Delta T^2$$

$$\therefore \overline{\mathbf{a}} = \frac{1}{\Delta T^2}\left(\mathbf{g}(t_{i+1}) - \mathbf{g}(t_i) - \overline{\mathbf{m}}\Delta T\right) \tag{13}$$

It is noted that $\overline{\mathbf{a}}$ is given by a function of update state $\mathbf{x}_{i+1}$. Since

$$\mathbf{g}(t_{i+1}) = \mathbf{f}(\mathbf{x}_{i+1}, t_{i+1}) \tag{14}$$

Now we can obtain (15) by substituting (12) and (13) into (11).

$$\mathbf{g}(t) = \mathbf{g}(t_i) + \overline{\mathbf{m}}\Delta t + \frac{1}{\Delta T^2}\left(\mathbf{g}(t_{i+1}) - \mathbf{g}(t_i) - \overline{\mathbf{m}}\Delta T\right)\Delta t^2 \tag{15}$$

Substituting (15) into the right hand side of (9), we obtain (16).

$$\begin{aligned}&\int_{t_i}^{t_i+\Delta T} \mathbf{g}(t)dt \\ &= \int_0^{\Delta T}\left[\mathbf{g}(t_i) + \overline{\mathbf{m}}\Delta t + \frac{1}{\Delta T^2}\{\mathbf{g}(t_{i+1}) - \mathbf{g}(t_i) - \overline{\mathbf{m}}\Delta T\}\Delta t^2\right]d\Delta t \\ &= \mathbf{g}(t_i)\Delta T + \frac{1}{2}\overline{\mathbf{m}}\Delta T^2 + \frac{1}{3}\Delta T\left[\mathbf{g}(t_{i+1}) - \mathbf{g}(t_i) - \overline{\mathbf{m}}\Delta T\right] \\ &= \frac{1}{3}\left[\mathbf{g}(t_{i+1}) + 2\mathbf{g}(t_i)\right]\Delta T + \frac{1}{6}\overline{\mathbf{m}}\Delta T^2\end{aligned} \tag{16}$$

Substituting (16) into (9) and rewriting in the use of (10), we can obtain

$$\begin{aligned}&\mathbf{x}(t_{i+1}) - \frac{1}{3}\Delta T\mathbf{f}\left(\mathbf{x}(t_{i+1}), t_{i+1}\right) \\ &\quad = \mathbf{x}(t_i) + \frac{2}{3}\Delta T\mathbf{f}\left(\mathbf{x}(t_i), t_i\right) + \frac{1}{6}\overline{\mathbf{m}}(t_i)\Delta T^2\end{aligned}$$

Finally, we have the following nonlinear equation to solve for update.

$$\mathbf{x}^{(k+1)} - \frac{1}{3}\Delta T\mathbf{f}(\mathbf{x}^{(k+1)}, t^{(k+1)}) = \mathbf{b}_i \tag{17.a}$$

$$\text{with } \mathbf{b}_i = \mathbf{x}^{(k)} + \frac{2}{3}\Delta T\mathbf{f}(\mathbf{x}^{(k)}, t^{(k)}) + \frac{1}{6}\overline{\mathbf{m}}(t^{(k)})\Delta T^2 \tag{17.b}$$

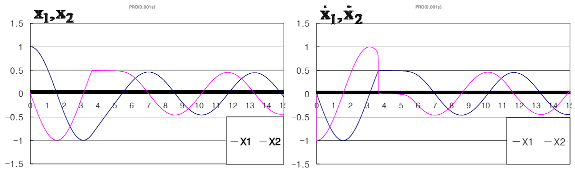
The solution of (17) can be obtained by the Newton-Raphson method through iterative procedures.

3. SAMPLE STUDY

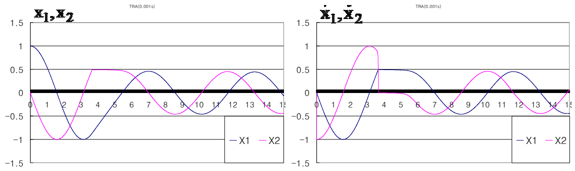
In this section, we have to solve a sample ODE's using the proposed method. The results are compared with those by the conventional methods. Consider a special example as follows :

$$\begin{aligned}\dot{x}_1 &= \varepsilon x_1 + x_2 \\ \dot{x}_2 &= -x_1 - \frac{10^{-2}x_2}{10^2(x_2 - 0.5)^2 + 10^{-4}}\end{aligned} \tag{18}$$
$$\text{where } x_1(0) = 1,\ x_2(0) = 0,\ \varepsilon = -10^{-3}$$

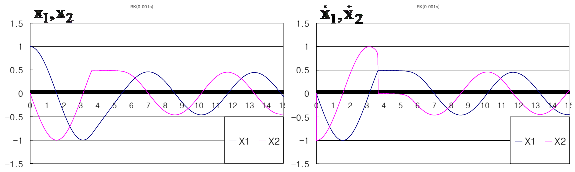
Slope $\overline{\mathbf{m}}$ can be calculated from (12).



(a) Proposed Method

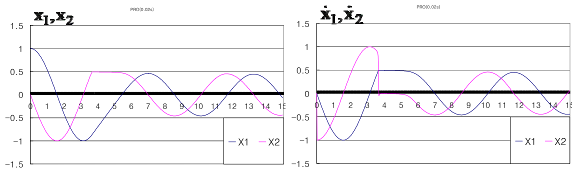


(b) Trapezoidal Method

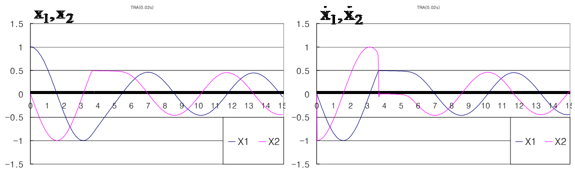


(c) Runge-Kutta Method

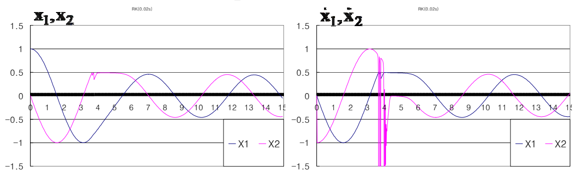
Fig. 2. Time Step : 0.001sec (10^{-3})



(a) Proposed Method



(b) Trapezoidal Method



(c) Runge-Kutta Method

Fig. 3. Time Step : 0.02sec

$$\overline{\mathbf{m}} = \frac{d\mathbf{g}(t)}{dt} = \frac{d}{dt}\mathbf{f}(\mathbf{x}(t),t)\Big|_{t=t_i}$$
$$= \frac{\partial \mathbf{f}(\mathbf{x}(t),t)}{\partial \mathbf{x}}\Big|_{t=t_i} \cdot \mathbf{f}(\mathbf{x}(t_i),t_i) + \frac{\partial \mathbf{f}(\mathbf{x}(t),t)}{\partial t}\Big|_{t=t_i}$$
$$= \begin{bmatrix} \varepsilon & 1 \\ -1 & \dfrac{10^{-2}A - 2x_2(x_2-0.5)}{A^2} \end{bmatrix} \begin{bmatrix} f_1(\mathbf{x}^i, t_i) \\ f_2(\mathbf{x}^i, t_i) \end{bmatrix} \quad (19)$$

where $A = 10^2(x_2-0.5)^2 + 10^{-4}$

In order to solve the nonlinear equation (17.a), we will define new functions $\mathbf{F}(\mathbf{x},t)$ as follows :

$$F_1 = x_1 - \frac{1}{3}\Delta T(\varepsilon x_1 + x_2) - b_1$$
$$F_2 = x_2 - \frac{1}{3}\Delta T\left(-x_1 - \frac{10^{-2}x_2}{10^2(x_2-0.5)^2+10^{-4}}\right) - b_2 \quad (20)$$

Jacobian matrix can be found as follows :

$$\begin{bmatrix} \dfrac{\partial F_1}{\partial x_1} & \dfrac{\partial F_1}{\partial x_2} \\ \dfrac{\partial F_2}{\partial x_1} & \dfrac{\partial F_2}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 1-\dfrac{1}{3}\Delta T\varepsilon & -\dfrac{1}{3}\Delta T \\ \dfrac{1}{3}\Delta T & 1+\dfrac{1}{3}\Delta T\left(\dfrac{10^{-2}A - x_2(x_2-0.5)}{A^2}\right) \end{bmatrix}$$

where $A = 10^2(x_2-0.5)^2 + 10^{-4}$ (21)

Table 1 shows the results of simulation time by the three methods. It shows that the proposed and the trapezoidal methods require roughly same computation time to solve the problems. Explicit RK(4) method takes a little bit shorter time than the other two methods do.

Figs. 2 and 3 show the results in graph. In each figure, the left graphs show the results and the right graphs show the slopes, or $\dot{x}$.

In Figs. 2 and 3, the graphs are obtained by taking simulation time-steps as 0.001 and 0.02 seconds, respectively. The three methods show similar graphs in Fig. 2, however, RK method shows some distorted graph in Fig. 3.

In Table 2, the biggest and the average errors during the simulation are compared. The proposed method shows smaller errors than the trapezoidal method. In case of x_1, the maximum error of the proposed method is only 2.6% of that of the trapezoidal method. Regarding the variable x_2, the maximum error of the proposed method is 35% of that of the trapezoidal.

Table 1 Comparison of Simulation Time

Time	Simulation Time(seconds)		
Step	Proposed	Trapezoidal	R-K
0.0001	47	47	43.3
0.001	4.617	4.670	4.372
0.01	0.503	0.509	0.477
0.02	0.349	0.344	0.335

Table 2. Comparison of Errors

	Proposed		Trapezoidal	
	X_1	X_2	X_1	X_2
Max Error	6×10^{-6}	2.6×10^{-4}	2.3×10^{-4}	7.3×10^{-4}
Average Error	2.1×10^{-7}	1.2×10^{-6}	7.8×10^{-5}	7.8×10^{-5}

The sample study shows that the proposed algorithm considerably improves the calculation accuracy for relatively large time step cases. However, the instability could be a little mitigated compared with the trapezoidal method, and it seems that the proposed algorithm satisfies only the A-stability.

4. TIME-SIMULATION IN POWER SYSTEM

The time simulation of power system transient stability has been performed by using the trapezoidal method and explicit RK(2) and RK(4). The model systems are WSCC 9-bus system and England 39-bus system.

In both WSCC 9-bus system and England 39-bus system, it is assumed that a 3ϕ fault occur at bus 7 and be cleared in 0.1 seconds.

4.1 WSCC 9-bus system

The time simulation has been performed for 5 seconds with two time steps; 0.01sec and 0.05sec. Fig. 4 and Fig. 5 show the results of time simulations with time steps 0.01sec and 0.05sec respectively. The three graphs in the figures represent the rotor angles of the generators.

Table 5 Results of WSCC-9 system(Time step : 0.01sec)

Method	Gen	Max Error	Average Error	Calculate Time
Proposed Method	Gen 1	3.5×10^{-3}	1.5×10^{-4}	0.312
	Gen 2	1.1×10^{-2}	4.4×10^{-4}	
	Gen 3	8.2×10^{-3}	2.8×10^{-4}	
Trapezoidal Method	Gen 1	2.5×10^{-2}	1.1×10^{-2}	0.312
	Gen 2	7.7×10^{-2}	3.2×10^{-2}	
	Gen 3	4.8×10^{-2}	2.0×10^{-2}	
RK Method 4th	Gen 1	2.0×10^{-5}	8.2×10^{-6}	0.732
	Gen 2	6.0×10^{-5}	2.3×10^{-5}	
	Gen 3	5.0×10^{-5}	1.6×10^{-5}	
RK Method 2nd	Gen 1	4.9×10^{-2}	2.2×10^{-2}	0.439
	Gen 2	1.5×10^{-1}	6.2×10^{-2}	
	Gen 3	9.3×10^{-2}	3.8×10^{-2}	

Table 6. Results of WSCC-9 system(Time step : 0.05sec)

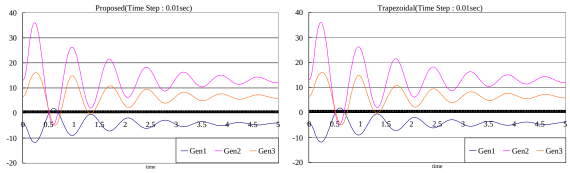
Method	Gen	Max Error	Average Error	Calculate Time
Proposed Method	Gen 1	4.3×10^{-2}	1.9×10^{-2}	0.085
	Gen 2	1.3×10^{-1}	5.5×10^{-2}	
	Gen 3	1.0×10^{-1}	3.4×10^{-2}	
Trapezoidal Method	Gen 1	6.2×10^{-1}	2.8×10^{-1}	0.079
	Gen 2	1.9×10^{-0}	8.0×10^{-1}	
	Gen 3	1.2×10^{-0}	4.9×10^{-1}	
RK Method 4th	Gen 1	1.1×10^{-2}	5.2×10^{-3}	0.169
	Gen 2	3.7×10^{-2}	1.5×10^{-2}	
	Gen 3	3.1×10^{-2}	9.8×10^{-3}	
RK Method 2nd	Gen 1	1.2×10^{-0}	5.5×10^{-1}	0.102
	Gen 2	3.8×10^{-0}	1.6×10^{-0}	
	Gen 3	2.4×10^{-0}	9.6×10^{-1}	

Maximum errors, average errors and calculation time of the 4 methods are compared in Table 5 and 6. The results with time-step 0.001sec are taken as reference to calculate errors.

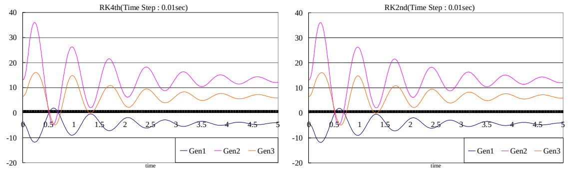
The RK(4) method yields the most accurate results requiring comparatively long computation time. The proposed method gives more accurate results than the trapezoidal and RK(2) methods.

4.2. England 39-bus system

The time simulation has been performed for 5sec with time step 0.01sec. Maximum errors, average errors and calculation time of 4 methods are compared in Table 7. The results with time-step 0.001sec have also been taken as reference to calculate errors.

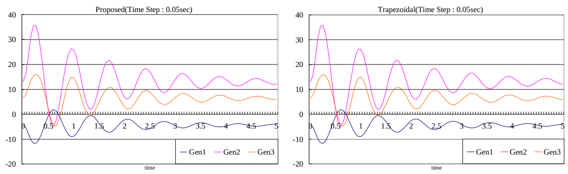


(a) Proposed (b) Trapezoidal

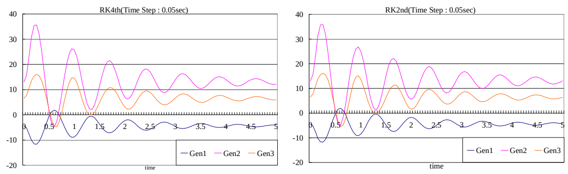


(c) RK 4th (d) RK 2nd

Fig. 4. WSCC-9 Bus System (Time Step : 0.01sec)

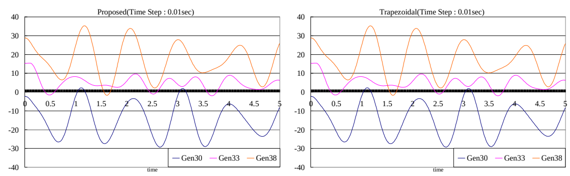


(a) Proposed (b) Trapezoidal

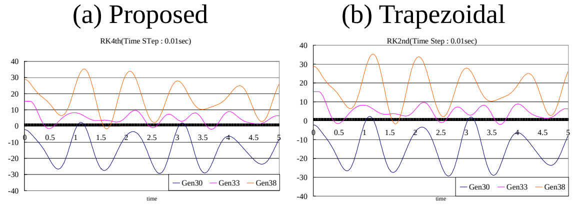


(c) RK 4th (d) RK 2nd

Fig. 5. WSCC-9 Bus System (Time Step : 0.05sec)



(a) Proposed (b) Trapezoidal



(c) RK 4th (d) RK 2nd

Fig. 6. Simulation Results of England 39-bus system

The rotor angles of generators 30, 33, and 38 are shown in Fig. 6 and Table 7.

Examining the results one can find that Method RK(4) gives the most accurate results requiring comparatively long computation time. The proposed method produces more accurate results than the trapezoidal and RK(2) methods. Through the applications to both power systems, we would like to claim that the proposed algorithm have a stability-preserved characteristic good enough for the power system transient stability simulation.

Table 7. Results of Eng 39 System(Time Step : 0.01sec)

Method	Gen	Max Error	Average Error	Calculate Time
Proposed Method	Gen 30	2.1×10^{-3}	5.6×10^{-4}	1.282
	Gen 33	2.3×10^{-3}	6.6×10^{-4}	
	Gen 38	2.5×10^{-3}	6.4×10^{-4}	
Trapezoi-dal Method	Gen 30	1.6×10^{-1}	4.4×10^{-2}	1.297
	Gen 33	1.4×10^{-1}	3.6×10^{-2}	
	Gen 38	2.0×10^{-1}	5.1×10^{-2}	
RK Method 4th	Gen 30	9.0×10^{-5}	2.8×10^{-5}	2.649
	Gen 33	1.5×10^{-4}	4.3×10^{-5}	
	Gen 38	1.3×10^{-4}	3.1×10^{-5}	
RK Method 2nd	Gen 30	3.1×10^{-1}	8.5×10^{-2}	2.065
	Gen 33	2.7×10^{-1}	7.2×10^{-2}	
	Gen 38	4.0×10^{-1}	9.9×10^{-2}	

5. CONCLUSION

This paper presents an approach using a new implicit quadratic integral method to perform the time simulation of power system transient stability. A new implicit quadratic integral method is proposed to solve the 'stiffness problem' without loss of the computation speed in transient stability analysis of power systems by time simulation. The algorithm is developed to improve calculation accuracy by taking quadratic approximation of the integrand in the update integral of implicit methods. The proposed algorithm is tested with a typical sample study and applied to power system transient stability analysis, which shows that the proposed method improves the computation error compared with the trapezoidal method and explicit RK(4) without loss of computation speed and mitigates the instability problem pertinent to the 'stiffness problem'. Further studies should be required to analyze the detailed mathematical characteristics of the proposed algorithm, while this study shows it can be a good substitute of the trapezoidal method for power system transient stability analysis.

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