GENERALIZED STATE SPACE AVERAGING FOR PORT CONTROLLED HAMILTONIAN SYSTEMS [1]

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Abstract: Generalized state space averaging (GSSA) is a powerful way to treat analysis and control problems for variable structure systems (VSS). On the other hand, port-controlled Hamiltonian systems (PCHS) describe, in a modular, network-like way, the interconnection of physical systems using the transfer of energy as the unifying concept. In this paper, a relationship between the PCHS structures of a system and its GSSA expansion is established for a class of Hamiltonians (which includes the quadratic ones), and this is used to design controls from a GSSA truncation which, under certain restrictions, can be used for the full original system. *Copyright ©2005 IFAC*

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1. INTRODUCTION

Variable structure systems (VSS) are piecewise smooth systems, *i.e.* systems evolving under a given set of regular differential equations until an event, determined either by an external clock or by an internal transition, makes the system evolve under another set of equations; in particular, this kind of behavior can occur periodically, and might give rise to very complicated dynamical features (Olivar and Fossas, 1996). VSS appear in a variety of engineering applications (Yu and Xu, 2001), where the non-smoothness is introduced either by physical events, such as impacts or switchings, or by a control action, as in hybrid or sliding mode control. Typical fields of application are rigid body mechanics with impacts or switching circuits in power electronics.

Port controlled Hamiltonian systems (PCHS), with or without dissipation, generalize the Hamiltonian formalism of classical mechanics to physical systems connected in a power-preserving way (van der Schaft and Maschke, 1992). The central mathematical object of the formulation is what is called a Dirac structure, which encodes the detailed connecting network information. A main feature of the formalism is that the interconnection of Hamiltonian subsystems using a Dirac structure yields again a Hamiltonian system (Dalsmo and van der Schaft, 1998). A PCH

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model encodes the detailed energy transfer and storage in the system, and is thus suitable for control schemes based on, and easily interpretable in terms of, the physics of the system (Kugi, 2001) (Ortega *et al.*, 2001).

PCHS are passive in a natural way, and several methods to stabilize them at a desired fixed point have been devised (Ortega *et al.*, 2002). On the other hand, VSS, specially in power electronic applications, can be used to produce a given periodic power signal to feed, for instance, an electric drive or any other power component. In order to use the regulation techniques developed for PCHS, a method to reduce a signal generation or tracking problem to a regulation one is, in general, necessary. One powerful way to do this is averaging (Krein *et al.*, 1990), in particular what is known as Generalized State Space Averaging, or GSSA for short(Sanders *et al.*, 1991). In this method, the state and control variables are expanded in a Fourier-like series with time-dependent coefficients; for periodic behavior, the coefficients will evolve to constants. In many practical applications (Gaviria *et al.*, in press), physical consideration of the task to solve indicates which coefficients to keep, and one obtains a finite-dimensional reduced system to which standard techniques can be applied.

In this paper we present some GSSA results for PCHS. In particular, we show that, under suitable conditions, the GSSA expansions of a PCHS are again PCHS, and that the controls obtained using Hamiltonian passive techniques for the reduced system can be applied to the original system. These results were used in (Gaviria *et al.*, in press) without formal justification.

The paper is organized as follows. Section 2 reviews the basic ideas of GSSA in a way suitable for PCHS. Sections 3 and 4 contain the main results of this paper. Section 3 presents the detailed Hamiltonian structure of a GSSA expansion for a broad class of Hamiltonians, and Section 4 shows under which conditions a control designed from a truncation of the GSSA expansion works as well for the full system. Section 5 illustrates the results using a power converter example. Finally, Section 6 summarizes our results.

2. AVERAGING AND GENERALIZED AVERAGING FOR PORT CONTROLLED HAMILTONIAN SYSTEMS

As explained in the Introduction, this paper presents results which combine the PCHS and GSSA formalisms. Detailed presentations can be found in (Dalsmo and van der Schaft, 1998), (van der Schaft, 2000), (Kugi, 2001) and (Ortega *et al.*, 2002) for PCHS, and in (Caliscan *et al.*, 1999), (Mahdavi *et al.*, 1997), (Sanders *et al.*, 1991) and (Tadmor, 2002) for GSSA.

Assume a VSS system such that the change in the state variables is small over the time length of an structure change, or such that one is not interested about the fine details of the variation. Then one may try to formulate a dynamical system for the time average of the state variables

$$\langle x\rangle(t) = \frac{1}{T}\int_{t-T}^{t} x(\tau)\ \mathrm{d}\tau, \tag{1}$$

where T is the period, assumed constant, of a cycle of structure variations.

Let our VSS system be described in explicit port Hamiltonian form [3]

$$\dot{x} = [\mathcal{J}(S,x) - \mathcal{R}(S,x)]\,\nabla H(x) + g(S,x)u, \tag{2}$$

where S is a (multi)-index, with values on a finite, discrete set, enumerating the different structure topologies. For notational simplicity, we will assume in this Section that we have a single index (corresponding to a single switch, or a set of switches with a single degree of freedom) and that $S \in \{0,1\}$. Hence, we have two possible dynamics, which we denote as

$$\begin{aligned}
&S=0 \Rightarrow\\
&\dot{x} = (\mathcal{J}_0(x) - \mathcal{R}_0(x))\nabla H(x) + g_0(x)u,\\
&S=1 \Rightarrow\\
&\dot{x} = (\mathcal{J}_1(x) - \mathcal{R}_1(x))\nabla H(x) + g_1(x)u.
\end{aligned} \tag{3}$$

Note that controlling the system means choosing the value of S as a function of the state variables, and that u is, in most cases, just a constant external input.

From (1) we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle x\rangle(t) = \frac{x(t)-x(t-T)}{T}. \tag{4}$$

Now the central assumption of the SSA approximation method is that for a given structure we can substitute $x(t)$ by $\langle x\rangle(t)$ in the right-hand side of the dynamical equations, so that (3) become

$$\begin{aligned}
&S=0 \Rightarrow\\
&\dot{x} \approx (\mathcal{J}_0(\langle x\rangle) - \mathcal{R}_0(\langle x\rangle))\nabla H(\langle x\rangle) + g_0(\langle x\rangle)u,\\
&S=1 \Rightarrow\\
&\dot{x} \approx (\mathcal{J}_1(\langle x\rangle) - \mathcal{R}_1(\langle x\rangle))\nabla H(\langle x\rangle) + g_1(\langle x\rangle)u.
\end{aligned} \tag{5}$$

The rationale behind this approximation is that $\langle x\rangle$ does not have time to change too much during

[3] To simplify the notation, gradients are taken as column vectors throughout this paper.

a cycle of structure changes. We assume also that the length of time in a given cycle when the system is in a given topology is determined by a function of the state variables or, in our approximation, a function of the averages, $t_0(\langle x\rangle)$, $t_1(\langle x\rangle)$, with $t_0+t_1=T$. Since we are considering the right-hand sides in (5) constant over the time scale of T, we can integrate the equations to get [4]

$$\begin{aligned} x(t) = x(t-T) &\\ + t_0(\langle x\rangle)[(\mathcal{J}_0(\langle x\rangle) - \mathcal{R}_0(\langle x\rangle))\nabla H(\langle x\rangle) &\\ + g_0(\langle x\rangle)u] &\\ + t_1(\langle x\rangle)[(\mathcal{J}_1(\langle x\rangle) - \mathcal{R}_1(\langle x\rangle))\nabla H(\langle x\rangle) &\\ + g_1(\langle x\rangle)u]. \end{aligned}$$

Using (4) we get the SSA equations for the variable $\langle x\rangle$:

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\langle x\rangle = d_0(\langle x\rangle)[(\mathcal{J}_0(\langle x\rangle) - \mathcal{R}_0(\langle x\rangle))\nabla H(\langle x\rangle) &\\ + g_0(\langle x\rangle)u] &\\ + d_1(\langle x\rangle)[(\mathcal{J}_1(\langle x\rangle) - \mathcal{R}_1(\langle x\rangle))\nabla H(\langle x\rangle) &\\ + g_1(\langle x\rangle)u], \end{aligned} \tag{6}$$

where

$$d_{0,1}(\langle x\rangle) = \frac{t_{0,1}(\langle x\rangle)}{T}, \tag{7}$$

with $d_0+d_1=1$. In the power converter literature d_1 (or d_0, depending on the switch configuration) is referred to as the duty ratio.

One can expect the SSA approximation to give poor results, as compared with the exact VSS model, for cases where T is not small with respect to the time scale of the changes of the state variables that we want to take into account. The GSSA approximation tries to solve this, and capture the fine detail of the state evolution, by considering a full Fourier series, and eventually truncating it, instead of just the "dc" term which appears in (1). Thus, one defines

$$\langle x\rangle_k(t) = \frac{1}{T}\int_{t-T}^{t} x(\tau)e^{-jk\omega\tau}\,\mathrm{d}\tau, \tag{8}$$

with $\omega = 2\pi/T$ and $k\in\mathbb{Z}$. The time functions $\langle x\rangle_k$ are known as index-k averages or k-phasors. Notice that $\langle x\rangle_0$ is just $\langle x\rangle$.

Under standard assumptions about $x(t)$, one gets, for $\tau\in[t-T,t]$ with t fixed,

$$x(\tau) = \sum_{k=-\infty}^{+\infty}\langle x\rangle_k(t)e^{jk\omega\tau}. \tag{9}$$

If the $\langle x\rangle_k(t)$ are computed with (8) for a given t, then (9) just reproduces $x(\tau)$ periodically outside $[t-T,t]$, so it does not yield x outside of $[t-T,t]$ if x is not T-periodic. However, the idea of GSSA is to let t vary in (8) so that we really have a kind of "moving" Fourier series:

$$x(\tau) = \sum_{k=-\infty}^{+\infty}\langle x\rangle_k(t)e^{jk\omega\tau}, \quad \forall\tau. \tag{10}$$

A more mathematically advanced discussion is presented in (Tadmor, 2002).

In order to obtain a dynamical GSSA model we need the following two essential properties:

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle x\rangle_k(t) = \left\langle\frac{\mathrm{d}x}{\mathrm{d}t}\right\rangle_k(t) - jk\omega\langle x\rangle_k(t), \tag{11}$$

$$\langle xy\rangle_k = \sum_{l=-\infty}^{+\infty}\langle x\rangle_{k-l}\langle y\rangle_l. \tag{12}$$

Using (11) and (2) one gets

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\langle x\rangle_k &= \left\langle\frac{\mathrm{d}x}{\mathrm{d}t}\right\rangle_k - jk\omega\langle x\rangle_k \\ &= \langle[\mathcal{J}(S,x) - \mathcal{R}(S,x)]\,\nabla H(x) \\ &\quad + g(S,x)u\rangle_k - jk\omega\langle x\rangle_k. \end{aligned} \tag{13}$$

Assuming that the structure matrices $\mathcal{J}$ and $\mathcal{R}$, the Hamiltonian H, and the interconnection matrix g have a series expansion in their variables, the convolution formula (12) can be used and an (infinite) dimensional system for the $\langle x\rangle_k$ can be obtained. Notice that, if we restrict ourselves to the dc terms (and without taking into consideration the contributions of the higher order harmonics to the dc averages), then (13) boils down to (6) since, under these assumptions, the zero-order average of a product is the product of the zero-order averages.

Notice that $\langle x\rangle_k$ is in general complex and that, for x real,

$$\langle x\rangle_{-k} = \overline{\langle x\rangle_k}. \tag{14}$$

We will use the notation $\langle x\rangle_k = x_k^R + jx_k^I$, where the averaging notation has been suppressed. In terms of these real and imaginary parts, the convolution property (12) becomes (notice that $x_0^I = 0$ for x real)

$$\begin{aligned} \langle xy\rangle_k^R &= x_k^R y_0^R \\ &+\sum_{l=1}^{\infty}\left\{(x_{k-l}^R + x_{k+l}^R)y_l^R - (x_{k-l}^I - x_{k+l}^I)y_l^I\right\} \\ \langle xy\rangle_k^I &= x_k^I y_0^R \\ &+\sum_{l=1}^{\infty}\left\{(x_{k-l}^I + x_{k+l}^I)y_l^R + (x_{k-l}^R - x_{k+l}^R)y_l^I\right\} \end{aligned} \tag{15}$$

[4] We also assume that u varies slowly over this time scale; in fact u is constant in many applications.

3. PCHS STRUCTURE OF THE GSSA APPROXIMATION

In this Section the detailed form of the Hamiltonian function, the structure and dissipation matrices and the interconnection term for the GSSA expansion of a class of PCH systems will be worked out.

Proposition 1. Let Σ be the PCH system defined by

$$\dot{x} = (A(x,S))\,\nabla H + f(x,S) \qquad (16)$$

where $A(x,S) = J(x,S) - R(x,S)$ and $f(x,S) = g(x,S)u$, $x \in \mathbb{R}^n$, $S \in \mathbb{R}^m$, $u \in \mathbb{R}^p$ is a constant input and $H \in \mathcal{C}^\infty(\mathbb{R}^n, \mathbb{R})$ is a Hamiltonian function. Let Σ_{PH} be the phasor system associated to Σ:

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle x\rangle_k = -jk\omega\langle x\rangle_k + \langle A\nabla H\rangle_k + \langle f\rangle_k, \; k \in \mathbb{Z} \qquad (17)$$

Let $\xi \equiv \langle \nabla H\rangle$. Assume that there exists a phasor Hamiltonian $H_{PH}(x_0, x_k^R, x_k^I)$ such that

$$2\frac{\partial H_{PH}}{\partial x_0} = \xi_0, \; \frac{\partial H_{PH}}{\partial x_k^R} = \xi_k^R, \; \frac{\partial H_{PH}}{\partial x_k^I} = \xi_k^I, \; k > 0, \qquad (18)$$

and symmetric matrices F_k for each $k > 0$ such that

$$F_k\frac{\partial H_{PH}}{\partial x_k^R} = x_k^R, \quad F_k\frac{\partial H_{PH}}{\partial x_k^I} = x_k^I. \qquad (19)$$

Then the phasor system can be written as an infinite dimensional Hamiltonian system for $\mathcal{X} = (x_0, x_k^R, x_k^I)$:

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{X} = A_{PH}\nabla H_{PH} + f_{PH} \qquad (20)$$

with $A_{PH} = J_{PH} - R_{PH}$ for some matrices J_{PH} skew-symmetric and R_{PH} symmetric and satisfying $(\nabla H_{PH})^T R_{PH} \nabla H_{PH} \geq 0$.

Proof. Splitting the phasors into real and imaginary parts, ordering the terms as

$$\mathcal{X} = (x_0, x_1^R, x_1^I, x_2^R, x_2^I, \ldots)$$

and using (13) and (15), it is immediat to obtain

$$\begin{aligned}\dot{x}_0 &= \tilde{A}_{00}\partial_{x_0}H_{PH} \\ &\quad + \sum_{l=1}^{\infty}\tilde{A}_{0l}\begin{pmatrix}\partial_{x_l^R}H_{PH}\\ \partial_{x_l^I}H_{PH}\end{pmatrix} + f_0, \\ \frac{\partial}{\mathrm{d}t}\begin{pmatrix}x_k^R\\ x_k^I\end{pmatrix} &= \tilde{A}_{k0}\partial_{x_0}H_{PH} \\ &\quad + \sum_{l=1}^{\infty}\tilde{A}_{kl}\begin{pmatrix}\partial_{x_l^R}H_{PH}\\ \partial_{x_l^I}H_{PH}\end{pmatrix} + \begin{pmatrix}f_k^R\\ f_k^I\end{pmatrix},\end{aligned}$$

where, using also the above notation for the averaged elements of A, $\tilde{A}_{00} = 2A_0$ and

$$\tilde{A}_{0l} = \begin{pmatrix}2A_l^R & 2A_l^I\end{pmatrix}, \; \tilde{A}_{k0} = \begin{pmatrix}2A_k^R\\ 2A_k^I\end{pmatrix},$$

$$\tilde{A}_{kl} = \begin{pmatrix} A_{k-l}^R + A_{k+l}^R & -A_{k-l}^I + A_{k+l}^I + \delta_{kl}k\omega F_k \\ A_{k-l}^I + A_{k+l}^I - \delta_{kl}k\omega F_k & A_{k-l}^R - A_{k+l}^R \end{pmatrix},$$

with the F_k terms coming from the $-jk\omega\langle x\rangle_k$ parts in (17) and contributing to J_{PH}. Using $A_k = \langle J\rangle_k - \langle R\rangle_k$ and $A_{-k}^R = A_k^R$, $A_{-k}^I = -A_k^I$, it is immediate to check the skew-symmetry of the structure matrix and the symmetry of the dissipation matrix of the phasor system. Notice that R_{PH} is not, in general, semi-positive definite. However, it can be proved that $R_{PH} \geq 0$ on the subspace formed by gradients of phasor Hamiltonians (18). Since for passivity-based control R_{PH} is used only in this setting, this is not a problem.□

As an example, if terms up to the second harmonic are kept, the structure+dissipation matrix becomes $(5 \times n)$-dimensional and is given by

$$\begin{pmatrix} 2A_0 & 2A_1^R & 2A_1^I & 2A_2^R & 2A_2^I \\ 2A_1^R & A_0 + A_2^R & A_2^I + \omega F_1 & A_1^R + A_3^R & A_1^I + A_3^I \\ 2A_1^I & A_2^I - \omega F_1 & A_0 - A_3^R & -A_1^I + A_3^I & A_1^R - A_3^R \\ 2A_2^R & A_1^R + A_3^R & -A_1^I + A_3^I & A_0 + A_4^R & A_4^I + 2\omega F_2 \\ 2A_2^I & A_1^I + A_3^I & A_1^R - A_3^R & A_4^I - 2\omega F_2 & A_0 - A_4^R \end{pmatrix} \qquad (21)$$

where each entry is $n \times n$.

Notice that generalized quadratic Hamiltonians defined as

$$H(x) = \frac{1}{2}x^T W x + Dx \qquad (22)$$

satisfy the hypothesis of Proposition 1, with $F_k = W^{-1}, \forall\, k$. Even if W is singular, matrices J_{PH} and R_{PH} can still be found (Gaviria *et al.*, in press).

4. CONTROL DESIGN BASED ON A GSSA TRUNCATION

Let us assume that we have the PCH phasor system Σ_{PH} obtained from the PCH system Σ as explained in Section 3. Assume that it is known that the specifications of a control problem for the VSS Σ yield a steady state zero dynamics and an input S with a finite number of harmonics. According to this, let us split the phasor states and the control inputs into two sets, $\langle x\rangle_k = (z_1, z_2)$ and $\langle S\rangle_k = (v_1, v_2)$, such that $\lim_{t\to\infty} z_2(t) = 0$ and $v_2(t) \equiv 0$. We also assume that f produces terms only for the z_1 components and that H_{PH} can be written additively as $H_{PH} = H_1(z_1) + H_2(z_2)$. Then the phasor system is given by

$$\begin{aligned}\dot{z}_1 &= (J_{11} - R_{11})\nabla H_1 + (J_{12} - R_{12})\nabla H_2 + f_1, \\ \dot{z}_2 &= (J_{21} - R_{21})\nabla H_1 + (J_{22} - R_{22})\nabla H_2, \qquad (23)\end{aligned}$$

where, except for H_1 and H_2, everything depends on (z_1, z_2, v_1, v_2), and all the matrices have the appropriate symmetry and definiteness so that the system is Hamiltonian dissipative.

Proposition 2. For the PCH system given by (23), assume that

(1) There exists $\hat{v} = (\hat{v}_1, 0)$ such that, in closed loop,
$$\dot{z}_1 = (J_{11}^d - R_{11}^d)\nabla H_1^d,$$
where $J_{11}^d(z_1)$, $R_{11}^d(z_1)$ and $H_1^d(z_1)$ constitute the desired Hamiltonian system (Ortega *et al.*, 2002) for z_1, $J_{11}^d = -J_{11}^{dT}$, $R_{11}^d = R_{11}^{dT} \geq 0$, and H_1^d has a minimum at the desired regulation point $\hat{z}_1$.
(2) $J_{12}\nabla H_2 = 0$.
(3) $R_{12} = R_{21} = 0$.
(4) $\partial_{z_2}(J_{11} - R_{11}) = 0$.
(5) $R_{22} > 0$.
(6) H_2 has a global minimum at $z_2 = 0$.

Then the control action $\hat{v} = (\hat{v}_1, 0)$ renders the equilibrium point $(\hat{z}_1, 0)$ asymptotically stable.

Proof. Using the first four hypothesis in the Proposition, the closed loop dynamics of (23) becomes [5]

$$\begin{aligned}\dot{z}_1 &= (J_{11}^d - R_{11}^d)\nabla H_1^d,\\ \dot{z}_2 &= J_{21}\nabla H_1 + (J_{22} - R_{22})\nabla H_2.\end{aligned}$$

and we see that the closed-loop dynamics of z_1 is decoupled from that of z_2, although the later depends on the former. To prove asymptotic stability of $(\hat{z}_1, 0)$, consider the Lyapunov function $H_p(z_1, z_2) = H_1^d(z_1) + H_2(z_2)$. One has

$$\begin{aligned}\frac{\mathrm{d}H_p}{\mathrm{d}t} &= (\nabla H_1^d)^T(J_{11}^d - R_{11}^d)\nabla H_1^d\\ &\quad + (\nabla H_2)^T(J_{21}\nabla H_1 + (J_{22} - R_{22})\nabla H_2)\\ &= -(\nabla H_1^d)^T R_{11}^d \nabla H_1^d - (\nabla H_2)^T R_{22}\nabla H_2)\\ &\leq 0,\end{aligned}$$

where the skew-symmetry of J_{11}^d and J_{22} has been used, together with $(\nabla H_2)^T J_{21} = (J_{21}^T\nabla H_2)^T = -(J_{12}\nabla H_2)^T = 0$ and $R_{11}^d \geq 0$, $R_{22} > 0$. Since H_p has a minimum at $(\hat{z}_1, 0)$, the above computation shows, by invoking Lyapunov's first theorem, that $(\hat{z}_1, 0)$ is indeed an asymptotically stable point of the closed-loop dynamics. □

5. EXAMPLE: A FULL-BRIDGE RECTIFIER

Although the hypothesis of Proposition 2 may seem somewhat restrictive, they are encountered in practical cases, since one has the freedom to choose the splitting into the interesting modes z_1 and the rest. We present a full-bridge boost-like rectifier in this formalism; more details can be found in (Gaviria *et al.*, in press).

The rectifier is shown in Fig. 1 and the state space equations are given by

$$\frac{d\phi(t)}{dt} = \frac{-\tilde{S}(t)}{C}\, q(t) - \frac{r}{L}\phi(t) + v_i(t) \quad (24)$$
$$\frac{dq(t)}{dt} = \frac{\tilde{S}(t)}{L}\, \phi(t) - i_l(t) \quad (25)$$

with $\tilde{S}(t) \in \{-1, 1\}\ \forall\ t$, i_l the load current and $v_i(t) = E\sin\omega t$.

As in (Escobar *et al.*, 2001), the control objectives for this rectifier are [6]

- The DC value of the output voltage $\frac{q(t)}{C}$, $\frac{\langle q(t)\rangle_0}{C}$ should be equal to a desired constant value $V_d > E$:
$$\langle q(t)\rangle_0{}^* = CV_d \quad (26)$$
- The power factor of the converter should be equal to one. This means that, in steady-state, the inductor current $\frac{\phi(t)}{L}$ follows a sinusoidal signal with the same frequency and phase as the AC-line voltage source:
$$\phi^*(t) = LI_d\sin(\omega_o t), \quad (27)$$
where I_d is the appropriate constant value fulfilling the aforementioned objective.

Note that the second control objective does not correspond to a tracking problem because the amplitude I_d depends on $i_l(t)$.

It turns out that the change of variables $\tilde{x}_1 = \phi$, $\tilde{x}_2 = q^2/2$, together with the control redefinition $S = -q\tilde{S}$, plays a fundamental role in fulfilling the conditions of Proposition 2. Using these, the system can be written as a PCHS

$$\dot{\tilde{x}} = \left(\begin{pmatrix} 0 & S\\ -S & 0\end{pmatrix} - \begin{pmatrix} r & 0\\ 0 & Ci_l\sqrt{2\tilde{x}_2}\end{pmatrix}\right)\partial_x H + \begin{pmatrix} v_i\\ 0\end{pmatrix}, \quad (28)$$

with Hamiltonian

$$H = \frac{1}{2L}\tilde{x}_1^2 + \frac{1}{C}\tilde{x}_2. \quad (29)$$

Notice that $\tilde{x}_2 \geq 0$ and that $i_l \geq 0$ because the load voltage is never negative.

Taking into account the control objectives, it is sensible to choose as truncated GSSA variables the dc mode of $\tilde{x}_2$ and the first harmonic of $\tilde{x}_1$, yielding a 3-dimensional GSSA truncated PCH system in the variables $(x_1, x_2, x_3) = (\tilde{x}_{20}, \tilde{x}_{11}^R, \tilde{x}_{11}^I)$, with structure and dissipation matrices [7]

[5] The fourth condition is necessary since the desired Hamiltonian dynamics for z_1 was designed with $z_2 = 0$.

[6] We denote the value in steady-state with a *.

[7] A small modification (several factors of 2) respect to the general result must be introduced due to the non-invertibility of the quadratic part of H.

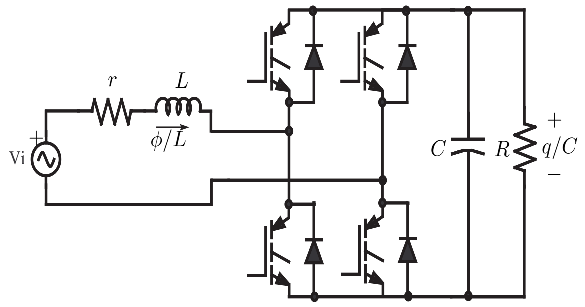

Fig. 1. Full-bridge boost-like rectifier

$$J_{PH} = \begin{pmatrix} 0 & -S_1^R & -S_1^I \\ S_1^R & 0 & \frac{\omega}{2}L \\ S_1^I & -\frac{\omega}{2}L & 0 \end{pmatrix}, \qquad (30)$$

$$R_{PH} = \begin{pmatrix} C\langle i_l \rangle_0 \sqrt{2x_1} & 0 & 0 \\ 0 & \frac{r}{2} & 0 \\ 0 & 0 & \frac{r}{2} \end{pmatrix}, \qquad (31)$$

and with phasor Hamiltonian

$$H_{PH} = \frac{1}{C}x_1 + \frac{1}{L}\left(x_2^2 + x_3^2\right). \qquad (32)$$

The dc component of the load current, $\langle i_l \rangle_0$, is suposed to be measurable. For the purpose of designing the control on a truncated space, we choose $z_1 = (x_1, x_2, x_3)$, and put the rest in z_2. It can be then seen that the hypothesis of Proposition 2 are fulfilled; the control obtained then by IDA-PBC techniques can be used on the full system, with good results both in simulation and experiment (Gaviria *et al.*, in press).

6. CONCLUSIONS

We have shown that systems obtained from a port controlled Hamiltonian system using a GSSA expansion are, under mild conditions for the Hamiltonian, again PCHS. If the control objectives have a finite harmonic content, the GSSA expansion allows to convert a tracking problem into a regulation one for the phasor coefficients. Truncation of the phasor system allows the design of a controller, using Hamiltonian passive techniques, which, if certain structural conditions are met, can be used in the full phasor system to meet the regulation objectives. Application of this technique to power electronic converters has been reported elsewhere (Gaviria *et al.*, in press).

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