ROBUST DISTURBANCE DECOUPLING IN LINEAR MULTIVARIABLE SYSTEMS VIA UNKNOWN-INPUT OBSERVERS

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Abstract. The method of unmodelled disturbance decoupling compensator (DDC) design for multivariable systems is proposed using the unknown-input observer (UIO) technique. The inverse model-based DDC equations were founded in explicit form; at that, it has been shown that the disturbance estimation may be eliminated from the control law, if founded system structure non-singularity condition takes place. For the case, when such a condition is violated, the realizable form of the DDC with internal small time-constant dynamic filter is proposed. Because the slow motion in obtained closed-loop two-time-scale system coincides with the processes in the system with ideal disturbance compensator, if the fast motion is stable, the DDC design problem was reduced to robust stabilization of singularly perturbed system.

Keywords: compensators, disturbance rejection, inverse systems, model-based control, observers, robust stability.

1. INTRODUCTION

The problem of unknown and unmeasurable disturbance decoupling (DDP) in multivariable systems along with reference signal tracking is one of the most important in control theory. Because uncertainties of the plant may be treated as a parametric disturbance of nominal plant model, the DDP is closely connected with general problem of robust control. The conditions of the DDP solvability were stated by Basile and Marro (1992). Nevertheless, in spite of the existence of general solution of the in term of invariant subspaces, the DDC state-space realization is of a great interest.

Recently in such a way a number of model-based control methods have been developed for disturbance rejection in multivariable systems taking into account the requirements of accuracy, dynamic performance, stability and robustness. Most of them are based on the utilization of current information about disturbances, obtained by the direct or indirect measurements. Such an approach is realized in control structures known as "two-degree-of-freedom controllers" (TDF) (Wolovich, 1995) and may be treated as combined feedback and feedforward control. The corresponding design methods using the various types of plant and disturbance models, so called Internal Model Control (Morari and Zafirov, 1989, Tsypkin and Holmberg, 1995) are very popular in robust control theory. However, in most practical applications the typical situation is characterized by the lack of *a priory* information, which is necessary for disturbance modeling and identification.

In this paper the disturbance decoupling compensator (DDC) design method for multivariable systems with incomplete measurements is proposed using the UIO technique. The design procedure includes two steps: disturbance observer design and disturbance compensator design. It has been shown, that if certain type of system structure non-singularity conditions takes place, the disturbance estimation may be eliminated from the control law and DDC equations are obtained in the explicit form. For the case, when such a conditions are violated, the proposed realizable form of the DDC includes additional internal dynamic filter with small time constant.

2. PROBLEM STATEMENT

Consider a linear multivariable system with unknown state-dependent disturbance, described by the state-space model

$$\begin{aligned} &\dot{x}(t) = Ax(t) + Bu(t) + Nf(x(t),t), \\ &y_c(t) = Cx(t),\ y_m(t) = Mx(t), \end{aligned} \tag{1}$$

where $x(t) \in \boldsymbol{R}^n$ - state vector , $u(t) \in \boldsymbol{R}^m$ - control action, $f(x(t),t) \in \boldsymbol{R}^q$, $\|f(t)\| \le c_f < \infty$ - unknown restricted disturbance, $y_c(t) \in \boldsymbol{R}^r$, $y_m(t) \in \boldsymbol{R}^p$ - output controlled and measured variables respectively. It is assumed, that $\operatorname{rank} B = m$, $\operatorname{rank} C = r$, $\operatorname{rank} N = q$, $\operatorname{rank} M = p$.

Matrices $S_{CB}(\alpha_1) = CA^{\alpha_1 - 1}B$, $S_{MN}(\alpha_2) = MA^{\alpha_2 - 1}N$ are known as Markov parameters of system (1), and the minimal integers α_1, α_2 , so that $S_{CB}(\alpha_1) \ne 0$, $S_{MN}(\alpha_2) \ne 0$, are known as a relative orders of control and disturbance transfer functions.

Let the following assumptions take place:

$$\begin{aligned} &\text{(a)} \quad \operatorname{rank} B = \operatorname{rank} S_{CB}(\alpha_1) = r, \\ &\text{(b)} \quad \operatorname{rank} N \le \operatorname{rank} S_{MN}(\alpha_2) = p. \end{aligned} \tag{2}$$

Without loss of generality for simplicity reason it will be assumed that $\alpha_1 = \alpha_2 = 1$ and notation $S_{CB}(1) = S_{CB}, S_{MN}(1) = S_{MN}$ is used.

The problem under consideration is to find the control low $u(t)$, so that the controlled output $y_c^*(t)$ satisfies the reference model equation $\dot{y}_c^*(t) = A^* y_c^*(t) + y_{ref}(t)$ along with disturbance $f((x),t)$ decoupling. Formally, the control goal is $\overline{\lim} \| e_c(t) \| \le \varepsilon^*,\ \ t \to \infty$, where ε^* - pre-established sufficiently small constant, $e_c(t) = y_c^*(t) - y_c(t)$ - control error.

3. DISTURBANCE OBSERVER DESIGN

The first step of the proposed DDC design procedure is the state and disturbance reduced-order observer design using UIO approach (Hou and Mueller, 1992). Let $z(t) = Rx(t) \in \boldsymbol{R}^{n-p}$ be the aggregated auxiliary variables, where R is the suitable appropriate aggregate matrix, such as $\operatorname{rank}\left(M^{\mathrm{T}} \mid R^{\mathrm{T}}\right) = n$. Then the state vector estimation may be obtained as follows:

$$\hat{x}(t) = Py_m(t) + Q\overline{x}(t), \tag{3}$$

where matrices $P \in \boldsymbol{R}^{n \times p}$, $Q \in \boldsymbol{R}^{n \times n-p}$ are defined as

$$\begin{aligned} &MP = I_p,\ RQ = I_{n-p},\ PM + QR = I_n, \\ &MQ = 0_{p,n-p},\ RP = 0_{n-p,p}. \end{aligned} \tag{4}$$

The estimation $\overline{x}(t)$ of aggregated vector $z(t)$ is given by minimal-order UIO

$$\dot{\overline{x}}(t) = \overline{F}\overline{x}(t) + \overline{G}_1 y_m(t) + \overline{H}\dot{y}_m(t) + \overline{G}_0 u(t). \tag{5}$$

The UIO parameters are determined from generalized “disturbance invariance conditions” (Hou and Mueller, 1992)

$$\begin{aligned} &(R - \overline{H}M)A - F(R - \overline{H}M) = (\overline{G}_1 + \overline{F}\overline{H})M, \\ &RN - \overline{H}NM = 0,\ \overline{G}_0 - RB + \overline{H}MB = 0. \end{aligned} \tag{6}$$

If assumption (2b) takes place, a solution of (6) may be obtained as

$$\begin{aligned} &\overline{F} = R\Pi_N AQ,\ \overline{G}_0 = (R - \overline{H}M)B,\ \overline{G}_1 = R\Pi_N AP, \\ &\overline{H} = RNS_{MN}^+, \Pi_N = I_n - NS_{MN}^+ M, \end{aligned} \tag{7}$$

where "+" denotes Moore-Penrouze generalized inversion, and matrices P, Q are uniquely determined by R . Using the disturbance estimation

$$\hat{f}(t) = N^+ \left(\dot{\hat{x}}(t) - A\hat{x}(t) - Bu(t) \right), \tag{8}$$

one can obtaine the minimal-order state and disturbance observer (SDO) equation in the form of system (1) inverse model (Lyubchik, 1995)

$$\begin{aligned} &\dot{\overline{x}}(t) = R\Pi_N AQ\overline{x}(t) + R\Pi_N APy_m(t) + \\ &+ RNS_{MN}^+ \dot{y}_m(t) + R\Pi_N Bu(t), \end{aligned}$$

$$\begin{aligned} &\hat{f}(t) = \overline{C}_N(\dot{y}_m(t) - MAQ\overline{x}(t) - MAPy_m(t) - \\ &- S_{MB}u(t)), \\ &C_N = S_{MN}^+ + N^+ P\Omega_N,\quad \Omega_N = I_p - S_{MN}S_{MN}^+. \end{aligned} \tag{9}$$

The estimation errors $e_x(t) = x(t) - \hat{x}(t)$, $e_f(t) = f(x,t) - \hat{f}(t)$ are given by the equations:

$$\dot{\bar{e}}_x(t) = \bar{F}(R)\bar{e}_x(t),\ e_x(t) = Q\bar{e}_x(t),$$
$$\bar{e}_f(t) = -C_N MAQ\bar{e}_x(t). \quad (10)$$

and its dynamic properties is determined by tuning matrix R selection.

Concretely define the matrices P, Q choice

$$(P \mid Q) = \begin{pmatrix} P_1 & Q_1 \\ P_2 & Q_2 \end{pmatrix},\ P_1 = I_p,\ Q_1 = 0_{p,n-p}, \quad (11)$$

then $R = Q_2^{-1}\left(-P_2 \mid I_{n-p}\right)$ and P_1, Q_2 are arbitrary matrices, so that $\det\ Q_2 \neq 0$.

For system (1) block matrix representation

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix},\ M = \begin{pmatrix} I_p & 0_{n-p,p} \end{pmatrix},\ N = \begin{pmatrix} N_1 \\ N_2 \end{pmatrix}_{n-p}^{p}$$

the observer dynamics matrix has the form:

$$\bar{F}(R) = Q_2^{-1}\left(\bar{A}_{22} - P_2\bar{A}_{12}\right)Q_2,$$
$$\bar{A}_{12} = \Omega_{N_{11}} A_{12},\quad \tilde{A}_{22} = A_{22} - N_2 N_1^{+} A_{12} \quad (12)$$
$$\Omega_{N_1} = I_q - N_1 N_1^{+}.$$

Thus the matrix Q_2 defines the similarity transformation and doesn't change the spectrum of $\bar{F}_1(R_1)$, which determined by arbitrary matrix $P_2 \in \boldsymbol{R}^{n-p \times p}$. The last one may be chosen by pole-placement method if pair $(\bar{A}_{22},\ \bar{A}_{12})$ is observable. Such a condition is equivalent to observability of matrix pair (Π_N, M) (Hou and Mueller, 1992). Therefore, the aggregate matrix R is determined up to an arbitrary nonsingular matrix Q_2.

The observability condition is obviously violated in the case when $p = q$. At that, $\Omega_{N_1} = 0$ and $\bar{F}(R)$ doesn't depend from P_2. In such a case for the tuning properties guarantee it is expediently to use the so-called „regularized" UIO (Kostenko and Lyubchik, 1996), which ensure the approximate observer invariance with respect to the unknown disturbance:

$$\left\| RN - \bar{H}CN \right\|^2 + \nu \left\| \bar{H} \right\|^2 \to \min_{H}, \quad (13)$$

where $\nu > 0$ is a small regularization parameter. In such a case

$$\bar{H}(\nu) = RNS^{\mathrm{T}}\left(\nu I_q + S_{MN} S_{MN}{}^{\mathrm{T}}\right)^{-1},$$
$$\Pi_N(\nu) = I_n - H(\nu)M \quad (14)$$

and regularized SDO design problem solution may be obtained in the following form:

$$\bar{F}(\nu) = \tilde{A}_{22}(\nu) - P_2\Omega_{N1}(\nu)A_{12},$$
$$\tilde{A}_{22}(\nu) = A_{22} - N_2\Psi_{N1}(\nu)A_{12},$$
$$\Psi_{N1}(\nu) = N_1{}^{\mathrm{T}}\left(\nu I_q + N_1 N_1{}^{\mathrm{T}}\right), \quad (15)$$
$$\Omega_{N1}(\nu) = I_q - N_1 N_1{}^{\mathrm{T}}\left(\nu I_q + N_1 N_1{}^{\mathrm{T}}\right)^{-1} =$$
$$= \nu\left(\nu I_q + N_1 N_1{}^{\mathrm{T}}\right)^{-1}.$$

The estimation errors for the regularized SDO are given by the equation:

$$\dot{\bar{e}}_x(t) = \bar{F}(\nu)\bar{e}_x(t) + \nu RN\left(\nu I_q + S_{MN}{}^{\mathrm{T}} S_{MN}\right)^{-1} f(x,t),$$
$$e_f(t) = -N^{+}\left(P\Omega_N(\nu) + H_N(\nu)\right)MAQ\bar{e}_x(t) + \quad (16)$$
$$+\nu N^{+}\left(I_n - PM\right)\left(\nu I_q + S_{MN}{}^{\mathrm{T}} S_{MN}\right)^{-1} f(x,t)$$

4. DISTURBANCE COMPENSATOR DESIGN

The disturbance compensative control law may be obtained using reference signal and disturbance estimation in the form of TDF controller. In the case of "square plant" ($r = m$) under the assumption (2a)

$$u^{*}(t) = S_{CB}^{-1}(y_{ref}(t) + C_A\hat{x}(t) - S_{CN}\hat{f}(t)),$$
$$C_A = A^{*}C - CA. \quad (17)$$

where A^{*} is a reference model dynamic matrix.

It is easy to show, that if the following system structure non-singularity condition takes place

$$\operatorname{rank} \bar{S} = m + q,\quad \bar{S} = \begin{pmatrix} I_m & S_{CB}^{-1}S_{CN} \\ C_N S_{MB} & I_q \end{pmatrix}, \quad (18)$$

or equivalently, $\det \Phi \neq 0,\ \Phi = I_q - C_N S_{MB} S_{CB}^{-1} S_{CN}$, disturbance estimation may be eliminated from the controller equation and DDC will be obtained in the form of TDF controller. For example, in particular case, when $S_{CN} = 0$, the DDC equations are:

$$\dot{\bar{x}}(t) = F^{0}\bar{x}(t) + R\Pi_N A^{0}(P\Omega_N + H_N)y_m(t) +$$
$$+ R\Pi_N H_B y_{ref}(t), \quad (17)$$

$$u^*(t) = S_{CB}^{-1}(y_{ref}(t) + C_A Q\overline{x}(t)) +$$
$$+ S_{CB}^{-1} C_A (P\Omega_N + H_N) y_m(t)),$$
$$F^0 = R\Pi_N A^0 Q, \quad A^0 = A + H_B C_A,$$
$$H_B = BS_{CB}^{-1}, \quad H_N = NS_{MN}^{+}. \tag{19}$$

In many practical applications conditions (18) are usually violated. In such a case the realizable control law may be obtained using the disturbance estimations, dynamically transformed by the internal "fast" filter with small parameters:

$$u^*(t) = S_{CB}^{-1}(y^*(t) + C_A \hat{x}(t) - S_{CN}\tilde{f}(t))$$
$$\varepsilon \dot{\tilde{f}}(t) = -\tilde{f}(t) + (1-\mu)\hat{f}(t), \tag{20}$$

where $0 < \varepsilon << 1, \quad 0 < \mu << 1$ are the filter parameters. Taking into account, that $S_{CB}^{-1} S_{CN} C_N S_{MB} = I_m$ if $\Phi = 0$, it easy to obtain the resulting equations of DDC with internal "fast" filter:

$$\varepsilon \dot{\tilde{u}}(t) = -\mu \tilde{u}(t) + (1-\mu)(\varphi_1(t) + S_{CB}^{-1} S_{CN} \varphi_2(t)),$$
$$u^*(t) = \tilde{u}(t) + \varphi_1(t), \tag{21}$$
$$\varphi_1(t) = S_{CB}^{-1}(y_{ref}(t) + C_A \hat{x}(t)),$$
$$\varphi_2(t) = C_N(\dot{y}_m(t) - MAQ\overline{x}(t) - MAPy_m(t)).$$

If system structural matrix $\overline{S}$ is nonsingular, the control law (17) may be directly applied and closed-loop system equation is:

$$\dot{x}(t) = A^0 x(t) + \Pi_B N f(t) + H_B y_{ref}(t) + Le_x(t),$$
$$A^0 = A + H_B C_A = \Pi_B A + H_B A^* C, \tag{22}$$

where L is a certain matrix. Taking into account that $CA^0 = A^* C$, it is evident, that for $p > q$ control goal is achieved, if closed-loop system (22) is stable.

For nonminimum-phase transfer function of systems (1) control channel, the closed-loop system matrix A^0 is unstable and problem of closed-loop system stabilizing arises. The usual state feedback $u(t) = u^*(t) - K\hat{x}(t)$ doesn't change the spectrum of A^0, because $\Pi_B(A + BK) = 0$. In such a case in accordance with local optimal control (LOC) method (Kelmans, *et al.*, 1981) the control signal should be found by the local control criteria minimization

$$\left\| y_{ref}(t) + C_A A\hat{x}(t) - S_{CB} u(t) - S_{CN}\hat{f}(t) \right\|^2 +$$
$$+ \beta \left\| u(t) \right\|^2 \to \min_u \tag{23}$$

where $\beta > 0$ is a weight coefficient. The corresponding local optimal control law is given by

$$u_\beta^*(t) = D_1(\beta)\left(y_{ref}(t) + C_A A\hat{x}(t) - S_{CN}\hat{f}(t)\right),$$
$$D_1(\beta) = \left(\beta I_m + S_{CB}^{\mathrm{T}} S_{CB}\right)^{-1} S_{CB}^{\mathrm{T}}, \tag{24}$$

or $u_\beta^*(t) = D(\beta) S_{CB} u^*(t)$.

From (23) the equation of closed-loop system follows

$$\dot{x}(t) = A_0(\beta)x(t) + BD(\beta)y_{ref}(t) +$$
$$+ \Pi_B(\beta) Nf(x,t) + L_\beta e_x(t),$$
$$A_0(\beta) = A + BD(\beta)C_A = \Pi_B(\beta)A + BD(\beta)A^* C,$$
$$\Pi_B(\beta) = I_n - BD(\beta)C. \tag{25}$$

Using the control $u(t) = u^*(t) - K\hat{x}(t)$, one can find $A_0(\beta, K) = A_0(\beta) - B_\beta K, B_\beta = \beta B\left(\beta I_m + S_{CB}^{\mathrm{T}} S_{CB}\right)^{-1}$, and system (22) may be stabilized, if the matrix pair $\left(A_0(\beta), B_\beta\right)$ is controllable. Finally,

$$\dot{e}_c(t) = A^* e_c(t) - \beta S_{CB}\left(\beta I_m + S_{CB}^{\mathrm{T}} S_{CB}\right)^{-1} u^*(t), \tag{26}$$

and control goal is achieved with $\varepsilon^*(\beta)$.

For the system with structural singularity, when DDC with "fast" internal filter is used (20), the closed-loop system equations are given by

$$\dot{x}(t) = A^0 x(t) + Nf(x(t),t) - H_B S_{CN}\tilde{f}(t) +$$
$$+ H_B y_{ref}(t) + Le_x(t),$$
$$\varepsilon \dot{\tilde{f}}(t) = -\tilde{f}(t) + (1-\mu) f(x(t),t) -$$
$$-(1-\mu)e_f(t). \tag{27}$$

The closed-loop system (27) has the structure of two-time-scale system. The slow motion under $\varepsilon = 0$ coincides with the process in system with ideal DDC (22), and the fast one satisfied the dynamic equation:

$$E(\varepsilon)\dot{\tilde{x}}(t) = \tilde{A}^0 \tilde{x}(t) + \tilde{B}^0 f(\tilde{x}(t)). \tag{28}$$
$$E(\varepsilon) = \begin{pmatrix} I_n & 0 \\ 0 & \varepsilon I_q \end{pmatrix}, \quad \tilde{A}^0 = \begin{pmatrix} A^0 & -H_B S_{CN} \\ 0_{q,n} & -I_q \end{pmatrix},$$
$$\tilde{B}^0 = \begin{pmatrix} N \\ (1-\mu)I_q \end{pmatrix}.$$

So the fast motion stability problem is reduced to the robust stability analysis of system (27), (28), which may be performed by known methods (Shiljak, 1978; Lunze, 1988). For the particular case of linear state-dependent uncertain parametric disturbance

$f(x(t),t) = \Delta_A x(t)$, where $\Delta_A, \|\Delta_A\| \le c_A$ is the system (1) dynamic matrix perturbation, the closed-loop system matrix is

$$\tilde{A}^0_\varepsilon(\Delta_A) = \begin{pmatrix} A^0 + N\Delta_A & -H_B S_{CN} \\ \varepsilon^{-1}(1-\mu)\Delta_A & -\varepsilon^{-1} I_q \end{pmatrix}, \quad (29)$$

and fast motion stability analysis is reduced to the linear robust stability problem:

$$\operatorname{Re} \lambda(\tilde{A}^0_\varepsilon(\Delta_A)) \le -\eta, \quad \|\Delta_A\| \le c_A, \quad (30)$$

which may be solved by suitable technique (Morari and Zafirov, 1989).

5. EXAMPLE. SUSPENSION CONTROL OF MAGNETICALLY LEVITATED VEHICLE

As an example of proposed approach consider the robust suspension control of magnetically levitated system. The simple linearized mathematical model of electromechanical system was taken in the form:

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{f}_1(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & h \\ -a_0 & -a_1 & -a_2 & 0 \\ 0 & 0 & 0 & v \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ f_1(t) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ b \\ 0 \end{pmatrix} u(t) + \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} f_2(t),$$
$$y_c(t) = x_1(t), \quad y^1_m(t) = x_1(t), \quad y^2_m(t) = x_3(t) \quad (31)$$

where $x_1(t)$ - levitated body deviation, $x_2(t)$ - body velocity, $x_3(t)$ - current in electromagnetic control device, $f_1(t) = \varphi(t)$ - "slow" input disturbance, $f_2(t) = f(x(t), u(t))$ - internal "fast" disturbance,

$$\dot{z}(t) = \nu z(t), \quad \varphi(t) = h z(t), \quad \nu \le 0,$$
$$f(t) = \Delta^{\mathrm{T}}_a(t) x(t) + \Delta_b(t) u(t), \quad (32)$$

where Δ_a, $\Delta_b(t)$ - an unknown functions, which characterize the system's non-stationary parameter variations.

The control problem under consideration is the following: using the measurements $y^1_m(t) = x_1(t)$, $y^2_m(t) = x_3(t)$, find the control function $u(t)$, so that the controlled output $y_c(t) = x_1(t)$ tracks the signal, generated by the 3-d order reference model

$$\dddot{y}^*(t) + \alpha_2 \ddot{y}^*(t) + \alpha_1 \dot{y}^*(t) + \alpha_0 y^*(t) = 0 . \quad (33)$$

In accordance with the proposed technique, the disturbance compensating control law as a function of disturbance estimations was determined as:

$$u(t) = -b^{-1}(\tilde{C}_A \hat{x}(t) + \tilde{\varphi}(t) + \hat{f}(t)),$$
$$\tilde{\varphi}(t) = \alpha_2 \hat{\varphi}(t) + \dot{\hat{\varphi}}(t), \quad (34)$$
$$\tilde{C}_A = \alpha_0 C_0 + \alpha_1 C_0 A_0 + \alpha_2 C_0 A_0^2 + C_0 A_0^3 =$$
$$= \begin{pmatrix} -a_0 + \alpha_0 & -a_1 + \alpha_1 & -a_2 + \alpha_2 \end{pmatrix}$$

The realizable form of controller with "fast" filter is:

$$\varepsilon \dot{\tilde{u}}(t) = -\mu \tilde{u}(t) - b^{-1}(1-\mu)(r_1(t) + r_2(t)),$$
$$u(t) = \tilde{u}(t) - b^{-1} r_1(t), \quad (35)$$
$$r_1(t) = \tilde{C}_A \hat{x}(t) + \tilde{\varphi}(t), \quad r_2(t) = \hat{f}_0(t)$$

For the augmented system with state vector $(x_{1-3}(t), x_4(t) = \varphi(t))$, corresponding state vector estimaties are obtained by the reduced order UIO:

$$\dot{\bar{x}}_1(t) = -\pi_1 \bar{x}_1(t) + h \bar{x}_2(t) + (\pi_2 h - \pi_1^2) y^1_m(t) + y^2_m,$$
$$\dot{\bar{x}}_2(t) = -\pi_2 \bar{x}_1(t) + v \bar{x}_2(t) + (\pi_2 v - \pi_1 \pi_2) y^1_m(t),$$
$$\hat{x}_1(t) = y^1_m(t),$$
$$\hat{x}_2(t) = \bar{x}_1(t) + \pi_1 y^1_m(t),$$
$$\hat{x}_3(t) = y^2_m(t),$$
$$\hat{x}_4(t) = \bar{x}_2(t) + \pi_2 y^1_m(t). \quad (36)$$

where π_1 π_2 - observer tuning parameters.

Disturbances estimations obtained by the combination of PI and UI observers are:

$$\hat{\varphi}(t) = \hat{x}_4(t) = \bar{x}_2(t) + \pi_2 y^1_m(t),$$
$$\dot{\hat{\varphi}}(t) = -\pi_2 \bar{x}_1(t) + v \bar{x}_2(t) + (\pi_2 v - \pi_1 \pi_2) y^1_m(t) + \pi_2 \dot{y}^1_m(t),$$
$$\hat{f}(t) = \hat{f}_0(t) - b u(t), \quad \hat{f}_0(t) = a_1 \bar{x}_1(t) + (a_0 + a_1 \pi_1) y^1_m + a_2 y^2_m + \dot{y}^2_m(t) \quad (37)$$

The final DDC equation with internal filter are:

$$\dot{\bar{x}}_1(t) = -\pi_1 \bar{x}_1(t) + \bar{x}_2(t) + (\pi_2 - \pi_1^2) y^1_m(t) + y^2_m,$$
$$\dot{\bar{x}}_2(t) = -\pi_2 \bar{x}_1(t) - \pi_1 \pi_2 y^1_m(t),$$
$$\varepsilon \dot{\tilde{u}}(t) = -\mu \tilde{u}(t) - b^{-1}(1-\mu)[(\alpha_1 - \pi_2) \bar{x}_1(t) + \alpha_2 \bar{x}_2(t) +$$
$$+ (\alpha_1 \pi_1 + \alpha_2 \pi_2 - \pi_1 \pi_2 - \alpha_0 - \mu \pi_2) y^1_m(t) + (\alpha_2 - \mu) y^2_m(t)],$$
$$u(t) = \tilde{u}(t) - b^{-1}[(\alpha_1 - a_1) \bar{x}_1(t) + \alpha_2 \bar{x}_2(t) + \gamma_1 y^1_m(t) + \gamma_2 y^2_m(t)]$$
$$\gamma_1 = \alpha_1 \pi_1 + \alpha_2 \pi_2 - \alpha_0 - a_0 - a_1 \pi_1 - \varepsilon^{-1} b^{-1} (1-\mu) \pi_2,$$
$$\gamma_2 = \alpha_2 - a_2 - \varepsilon^{-1} b^{-1} (1-\mu) \quad (38)$$

Simulation results, presented in Fig. 1-3, were obtained for $a_0 = 1, a_1 = 2, a_2 = 2, b = 1, h = 1, \nu = 0$ plant parameters and $\alpha_0 = 6, \alpha_1 = 11, \alpha_2 = 6$ reference model parameters. Disturbance $f_1(t)$ was simulated by step wave function, and disturbance

$f_2(t) = \theta(t)(e^T x(t) + u(t)), \theta(t) = \sin(0.5t)$. Tuning parameters of DDC are $\pi_1 = -1, \pi_2 = -2$, and internal filter parameters are $\varepsilon = 0.1$, $\mu = 0.01$.

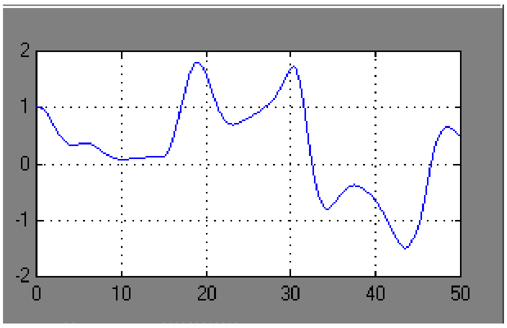


Fig.1. Controlled output $y_c(t)$ (feedback control).

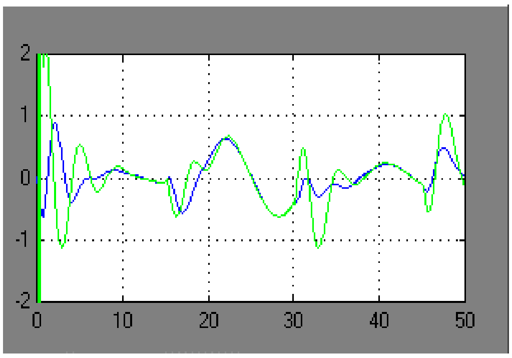
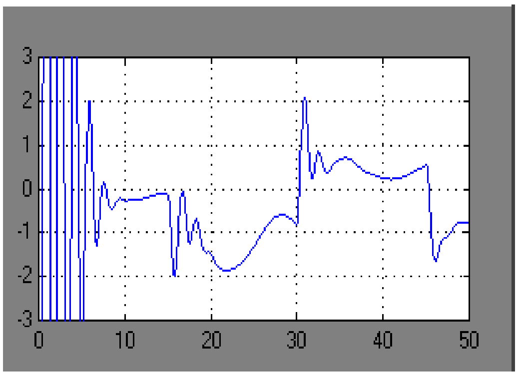
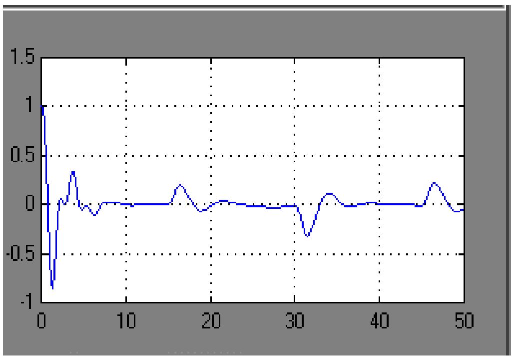


Fig. 2. Disturbance $f_2(t)$ estimation $\hat{f}_2(t)$.



a)



b)

Fig. 3. Control function $u(t)$ (a) and output controlled variable $y_c(t)$ (b) under the DDC.

Simulation results demonstrated high accuracy unmeasured disturbances rejection in closed-loop system with DDC.

CONCLUSION

The new type of control structure for unknown state-dependent disturbance decoupling in multivariable systems was obtained. It is shown, that DDC design using UIO approach is reduced to the problem of robust stabilization of singularly perturbated system. It has been shown that if the fast motion in the closed-loop system is stable the slow one coincides with the processes in the system with ideal compensator. The resulting controller's equations don't include the disturbances estimations and the designed DDC has the structure of multivariable PI-controller with small parameters. Designed DDC ensures unknown disturbance rejection with high accuracy and has a good robust properties concern the plant's model and parameters.

ACKNOWLEDGMENT

L.M. Lyubchyk would like to thank German Academic Exchange Service (DAAD) for supported his research visit to Wuppertal University.

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