MULTI-ROBOT FORMATION CONTROL AND TERRAIN SERVOING WITH LIMITED SENSOR INFORMATION

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Abstract: In this paper mobile multi-agent systems with limited sensor information are studied. Two control algorithms are proposed that do not require global information, and are easy to implement. The control problems are motivated by robotic applications such as cleaning, grass mowing and land mines detection, where a common control problem is the complete coverage path planning, for which it is known that parallel formation is optimal. The proposed control algorithms provide terrain servoing for the leading robot and parallel formation keeping for the followers, both of which are only based on measurements from range sensors.

Keywords: Mobile robots, robot navigation, robot control, feedback, sensors.

1. INTRODUCTION

For mobile robots two of the basic functionalities are to navigate and to follow. Naturally both of them are well studied for either single agent systems or multi-agent systems (Egerstedt *et al.*, 2001; Das *et al.*, 2002; Lawton *et al.*, 2003; Egersted and Hu, 2001; Kang *et al.*, 2002; Jadbabaie *et al.*, 2003; Vidal *et al.*, 2003; Bicho and Monteiro, 2003). Many methods in the literature, however, are focused on the problem of designing full state feedback controllers for goal reaching, target tracking or maintaining a preassigned formation. The issue of sensor and actuator limitations seems to be more or less overlooked, even though it has been a major issue in research areas such as SLAM (Dissanayake *et al.*, 2001; Wang *et al.*, 2003). Obviously it needs to be addressed already in designing the control algorithms.

A good example of this is a team of mobile robots with low-resolution range sensors that move in an unknown environment. By cooperating and exchanging information, the robots are able to get a good overlook of the surroundings in terms of angles and distances, even though one robot alone has a limited and maybe even incorrect view of the environment. However, this situation poses new requirements on the motion controls, since the robots must always stay within communication range of each other.

In this paper we will focus on mobile multi-agent systems with limited information from range sensors. We will provide some control algorithms that do not require global information, and are easy to implement. Our approach differs from the other approaches in the literature such as (Das *et al.*, 2002; Kang *et al.*, 2002; Vidal *et al.*, 2003) in that different sensing information and actuator constraints are assumed. Therefore a new control strategy is needed in order to have a robust formation control. The particular control problems we study are motivated by robotic applications such as cleaning, grass mowing and land mines

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detection, where a common control problem is the complete coverage path planning (CCPP) problem for multiple agents. It is known that parallel formation is optimal for CCPP. The main contribution of the paper is two new control algorithms that provide terrain servoing for the leading robot and parallel formation keeping for the followers, both of which are only based on measurements from range sensors.

The outline of the paper is as follows. In Section 2 we propose a control strategy for parallel formation keeping and study stability behavior of the controller. In Section 3 an on-line terrain servoing algorithm, based on a semi-local coordinate frame, is provided. With this coordinate frame we can avoid possible singularities in path parameterization that we would get in a global coordinate frame. In Section 4 some simulation is done to illustrate the results.

2. PARALLEL FORMATION KEEPING

In this section we design a control algorithm for a group of mobile robots following another mobile agent, which we from now on will refer to as the "leader". The goal of the control is to make all the robots move side by side at some specified distance from each other, while adjusting the speed and orientation to the leader.

Through out this paper we use the following nonholonomic model for each robot:

$$\begin{aligned} \dot{x} &= v\cos\phi \\ \dot{y} &= v\sin\phi \\ \dot{\phi} &= \omega \end{aligned} \tag{1}$$

where v, ϕ and ω denote the speed, rotation and angular velocity of the robot with respect to some fixed coordinate system.

As we mentioned, the objective of the parallel tracking control is to align all the robots with another moving object so that they move in parallel with the same orientation (Figure 1). Ideally, the angle from the leader's axis of orientation to the following robot should be $\frac{\pi}{2}$. Eventually we get a "chain" of robots, all moving in parallel in the same direction. Since our eventual control objective is to align the middle points of two robots, the linearization technique used widely in the literature can not be applied here. One might argue that two off-the-axis points with small distance to the axes can be aligned instead (Das *et al.*, 2002), so feedback linearization can be applied. However, in this case the control action could be very big and ill conditioned.

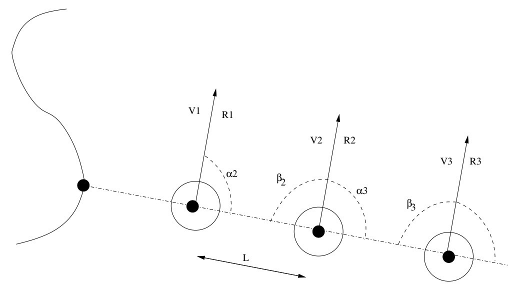

Fig. 1. Horizontal tracking and formation keeping.

We first explain briefly why we can not apply the usual linearization for parallel formation.

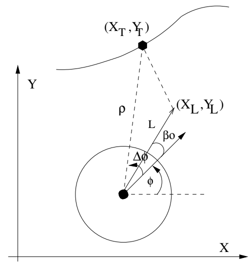

Fig. 2. Tracking with a sharp angle.

Now let the point (x_L, y_L) be defined as in Figure 2, i.e., located on the line that has an angle β_0 from the robot's axis of orientation, and having a distance L from the center of the robot. If the robot is centered in (x, y), then

$$\begin{aligned} x_L &= x + L\cos(\phi+\beta_0) \\ y_L &= y + L\sin(\phi+\beta_0). \end{aligned} \tag{2}$$

Derivation of equation (2) in combination with the unicycle model (1) gives the connection between the velocity and angular velocity of the robot, v and ω, and the motion of the point (x_L, y_L).

$$\begin{bmatrix} \dot{x}_L \\ \dot{y}_L \end{bmatrix} = \begin{bmatrix} \cos\phi & -L\sin(\phi+\beta_0) \\ \sin\phi & L\cos(\phi+\beta_0) \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{3}$$

It is obvious that by this way one can invert the matrix in (3) as long as $|\beta_0| < \frac{\pi}{2}$.

If $\dot{x}_L$ and $\dot{y}_L$ are chosen for some positive k as

$$\begin{aligned} \dot{x}_L &= -k(x_L - x_T) + \dot{x}_T \\ \dot{y}_L &= -k(y_L - y_T) + \dot{y}_T, \end{aligned} \tag{4}$$

which includes both a proportional and a derivative part, x_L and y_L are driven towards x_T and y_T. Then we can obtain the corresponding controls for the robot, i.e., v and ω.

With this idea, having parallel formation would amount to having $|\beta_0| = \frac{\pi}{2}$. This is exactly the situation where we would have singularity. What we propose here is instead a nonlinear control strategy. As we will show, the control is guaranteed to stay stable for some very mild assumptions on the motion of the leader.

Now we assume that there are n mobile robots, $R_1, \cdots, R_n$, where R_1 is the leader. R_1 will navigate by servoing through the terrain, which will be discussed in the next section. The rest of the robots should line up with the leader in parallel. Thus for each follower, we have

$$\begin{aligned} \dot{x}_i &= v_i \cos\phi_i \\ \dot{y}_i &= v_i \sin\phi_i, \quad i = 2, \cdots, n \\ \dot{\phi}_i &= \omega_i. \end{aligned} \tag{5}$$

Denote that the actual distance between R_i and R_{i-1} is d_i while L is the desired distance, $\gamma_i = \phi_i - \phi_{i-1}$. α_i is the actual relative angle from R_i to the orientation of R_{i-1} and β_i is the actual relative angle from the orientation of R_i to R_{i-1} (see Figure 1), both of which are desired to be $\frac{\pi}{2}$.

Then we can rewrite system (5) as

$$\begin{aligned} \dot{d}_i &= -v_i \cos\beta_i - v_{i-1}\cos\alpha_i \\ \dot{\gamma}_i &= \omega_i - \omega_{i-1}, \quad i = 2, \cdots, n \\ \dot{\beta}_i &= -\omega_i + \frac{v_i}{d_i}\sin\beta_i - \frac{v_{i-1}}{d_i}\sin\alpha_i. \end{aligned} \tag{6}$$

We note that $\alpha_i = \pi - \gamma_i - \beta_i$. This is apparently a cascaded system due to the appearance of v_{i-1} and ω_{i-1}. Now our control objective can be defined as:

Given $v_1(t)$ and $\omega_1(t)$, find control $v_i(t)$ and $\omega_i(t)$ $i = 2, \cdots, n$ such that for $i = 2, \cdots, n$

$$d_i \to L, \quad \gamma_i \to 0, \quad \beta_i \to \frac{\pi}{2} \quad \text{as } t \to \infty. \tag{7}$$

Theorem 1. Suppose the reference velocities from the lead robot R_1 satisfy the following condition: $|v_1(t)|, |\omega_1(t)|$ and $|\dot{v}_1(t)|, |\dot{\omega}_1(t)|$ are bounded, and

$$\begin{aligned} \int_{t_0}^{t} (v_1(s) \ +iL\omega_1(s))^2 ds > \delta_i(t - t_0) \\ \forall t > t_0 \geq 0 \quad i \leq n, \end{aligned} \tag{8}$$

where δ_i is a positive constant. Then the cascaded system (6) is locally exponentially stabilized around the equilibrium ($d_2 = L$, $\gamma_2 = 0$, $\beta_2 = \frac{\pi}{2}, \cdots, d_n = L$, $\gamma_n = 0$, $\beta_n = \frac{\pi}{2}$) by the following control

$$\begin{aligned} v_i &= a_i L\Delta\beta_i + L\omega_i + v_{i-1} \\ \omega_i &= b_i v_{i-1}\Delta d_i - c_i\gamma_i + \omega_{i-1}, \quad i = 2, \cdots, n \end{aligned} \tag{9}$$

where a_i, b_i, c_i are positive constants that can be designed.

Remark: It is well known that for such nonholonomic systems, stabilization to an equilibrium point by any C^1 state feedback control is not possible. Thus when v_1 and ω_1 are set to zero, it is not possible to stabilize the system.

Proof: Denote $\Delta d_i = d_i - L$, $\Delta\beta_i = \frac{\pi}{2} - \beta_i$, then

$$\begin{aligned} \Delta\dot{d}_i &= -v_i \sin\Delta\beta_i + v_{i-1}\sin(\Delta\beta_i - \gamma_i) \\ \dot{\gamma}_i &= \omega_i - \omega_{i-1} \\ \Delta\dot{\beta}_i &= \omega_i - \frac{(v_i\cos\Delta\beta_i - v_{i-1}\cos(\Delta\beta_i - \gamma_i)}{L + \Delta d_i}. \end{aligned} \tag{10}$$

Plug in the control (9) and linearize the system:

$$\begin{aligned} \Delta\dot{d}_i &= -L\omega_1(t)\Delta\beta_i - (v_1(t) + iL\omega_1(t))\gamma_i \\ \dot{\gamma}_i &= b_i(v_1(t) + iL\omega_1(t))\Delta d_i - c_i\gamma_i \\ \Delta\dot{\beta}_i &= -a_i\Delta\beta_i. \end{aligned} \tag{11}$$

Obviously, to show the closed-loop system of (10) is exponentially stable is to show (11) is exponentially stable. And in order to show (11) is exponentially stable, we only need to show the following system is exponentially stable for $i = 2, \cdots, n$:

$$\begin{aligned} \Delta\dot{d}_i &= -\lambda_i(t)\gamma_i \\ \dot{\gamma}_i &= b_i\lambda_i(t)\Delta d_i - c_i\gamma_i, \end{aligned} \tag{12}$$

where $\lambda_i(t) = v_1(t) + iL\omega_1(t)$. With a quadratic positive definite and decrescent candidate Lyapunov function for each $1 < i \leq n$

$$V = \frac{1}{2}b_i(\Delta d_i - \epsilon\lambda_i(t)\gamma_i)^2 + \frac{1}{2}\gamma_i^2,$$

where $\epsilon > 0$, we have

$$\dot{V} = -\epsilon\lambda_i^2(t)b_i^2\Delta d_i^2 - (c_i + \epsilon f(t))\gamma_i^2 + \epsilon g(t)\Delta_i\gamma_i,$$

where $f(t)$ and $g(t)$ are bounded. When ϵ is sufficiently small, we can easily show that

$$\dot{V} \leq -\frac{1}{2}\epsilon b_i^2\lambda_i^2(t)(\Delta d_i^2 + \gamma_i^2).$$

Having the condition (8) in mind, one can use Gronwall's inequality to show that (12) is exponentially stable for each $1 < i \leq n$. $\square$

3. TERRAIN SERVOING

3.1 Terrain servoing of the lead robot

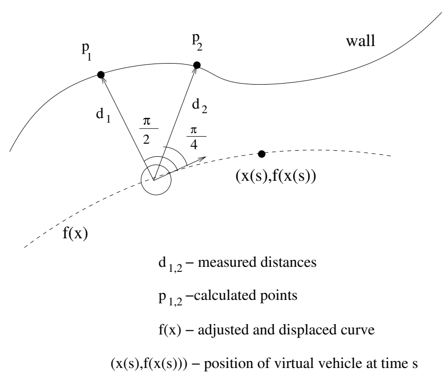

Fig. 3. Robot 1 following the wall using terrain servoing.

Terrain servoing is performed by Robot 1 to navigate along the wall (Figure 3). The sensors

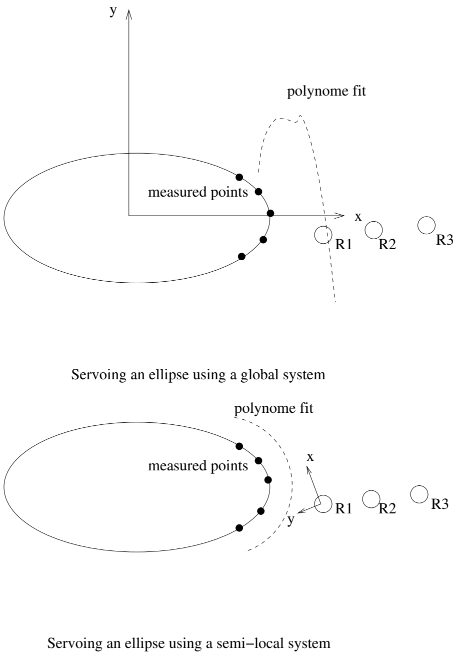

Fig. 4. Advantages of a semi-local frame.

making angle $\pi/4$ and $\pi/2$ with the orientation of the robot are used to measure distances to the wall. With each measurement two distances are found, d_1 and d_2, corresponding to points on the wall. These points are collected in a vector (x_i, y_i) and used to approximate the shape of the wall. At every few time steps a second order polynomial $f(x)$ is fit to the m last elements of (x_i, y_i). A higher order polynomial might give a better fit to the data but will for some data give unnecessarily complicated curves. For non-convex walls it is necessary to employ sensors at other angles as well to find the closest wall, but the principle is the same.

Orthogonally displacing $f(x)$ a distance d_w from the wall gives the path that the Robot 1 will follow. Not following the measurements directly gives the advantage of never making a decision based upon a single measurement. In addition, if all sensors loose contact with the wall, the robot has the option to backtrack along the path until contact is established and then rotate to find a new measurement.

3.2 A semi-local coordinate system

Using a global coordinate system, approximating the contour of the wall with a polynomial will be difficult for some shapes. For instance, when servoing an elliptic shape, no matter how you place your frame you will always end up with two singular points. On the other hand, a coordinate system fixed in one of the robots would give us trouble expressing the dynamics.

Here, a semi-local frame is used instead. At $t = t_0$ Robot 1 is placed in the origin. While Robot 1 moves the frame remains fixed until $t = t_1$. At this time a coordinate transform is made so that the origin is placed in the present coordinates for Robot 1 and the new frame is oriented so that the x-axis is aligned with the orientation of Robot 1. This new system is kept until $t = t_2$ when another transform is made. The changing times t_i vary with the shape of the wall; when the curvature is high the frame needs to move more often. If the polynomial at time t_i is

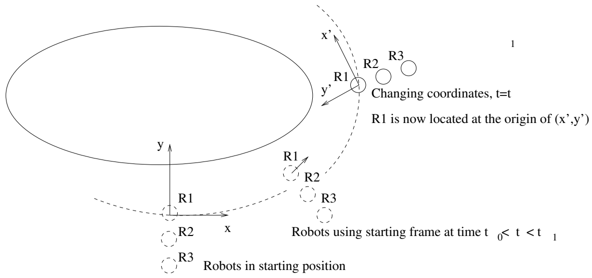

Fig. 5. The semi-local frame.

$$f(x) = ax^2 + bx + c,$$

the next switching time is

$$t_{i+1} = t_i + \tau_1 + \tau_2 a$$

where $\tau_{1,2}$ are constants. Transforming the system is done using translation and rotation:

$$\begin{cases} \begin{bmatrix} x' \\ y' \end{bmatrix} = R(\psi_{R1}) \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} x_{R1} \\ y_{R1} \end{bmatrix} \\ \psi' = \psi - \psi_{R1} \end{cases}$$

where (x', y', ψ') is the new system, $(x_{R1}, y_{R1}, \psi_{R1})$ are the old coordinates for Robot 1 and $R(\psi_{R1})$ is a rotation matrix:

$$R(\psi_{R1}) = \begin{bmatrix} \cos\psi_{R1} & \sin\psi_{R1} \\ -\sin\psi_{R1} & \cos\psi_{R1} \end{bmatrix}.$$

It is clear that with a semi-local frame, we can avoid the singular points that might appear in a global frame. Besides, at each local frame, localization of the robot can be done, for example, by the encoders. Thus from a practical point of view, this method is more implementable than a global frame since localization would be more difficult in that case. The advantages of a moving frame will be exemplified in Section 4. In the next subsection we discuss how the lead robot should move along the path we have interpolated.

3.3 A virtual vehicle approach to path following

The task of Robot 1 is to keep track of the wall and move along it. Our control here is developed from the virtual vehicle approach in (Egerstedt *et al.*, 2001).

The control in (Egerstedt *et al.*, 2001) was developed to steer a mobile robot along a pre-defined

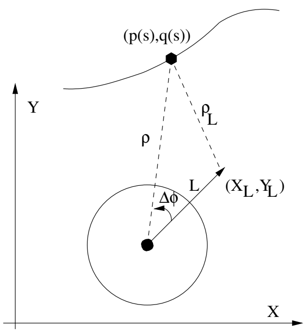

Fig. 6. The virtual vehicle approach.

trajectory. A "look-ahead" distance L, from the center of the robot to the reference point on the path, had to be given due to the necessity to avoid singularities. Otherwise, as the distance to target decreases, the angle to target ($\Delta\phi$ in Figure 6), measured from the robot's axis of orientation, will eventually be undefined. However, from which direction or orientation this look-ahead distance should be kept was not specified.

Here we want the mobile robot at all instants to be oriented towards its leader and therefore we choose a point (x_L, y_L) on the robot's axis of orientation (see Figure 6) as the reference for the robot, instead of the center. This particular choice of reference point not only can be driven arbitrarily close to the leader without causing the relative angle to be undefined, but also enable us to obtain the desired control of the orientation.

Using the linearization arguments in the beginning of Section 2, we can easily obtain the following control:

$$\begin{cases} \dot{s} = \dfrac{v_0}{\sqrt{p'^2 + q'^2}}\, e^{-\alpha\rho_L} \\ v = k(\rho\cos\Delta\phi - L) + v_0 e^{-\alpha\rho_L}\cos(\theta_r - \phi) \\ \omega = \dfrac{k\rho}{L}\,\sin\Delta\phi + \dfrac{v_0}{L}\sin(\theta_r - \phi) \end{cases}, \tag{13}$$

where we assume the reference path is parameterized by $(p(s), q(s))$, thus the first equation governs the evolution of the "virtual vehicle" on the path, s. Here $v_0 > 0$ can be shown to be the desired tracking speed, $\theta_r = atan(\frac{q'(s)}{p'(s)})$, $\alpha > 0$, and ρ_L is defined in Figure 6.

4. SIMULATIONS

Simulations have been carried out for both a convex and a non-convex case for the different approaches. As a convex wall an ellipse is chosen, and for the non-convex case we use a sinusoidal curve. The robot formation moves along the wall. The deviations from the desired parallel formation are measured with respect to both angles and distances. This control strategy should be applicable to various types of robots although our simulations are only modeled on the Khepera mobile robot platform - a circular platform with a diameter of 55 mm with two wheels and sensors at angles $\pm\pi/2$, $\pm\pi/4$, 0 and π. Due to the page limitation, we unfortunately can not list the parameters used in the simulation.

4.1 Convex Case - Planned and reactive approach

Three robots move along the ellipse in a parallel formation. The trajectories are plotted and the deviations from the desired positions for all robots are measured. The equation of the curve representing the wall is $y = 15+10\sin(t)$, $x = 20\cos(t)$, the desired distance to it is $d_w = 5$ and desired distance between robots is $L = 4$.

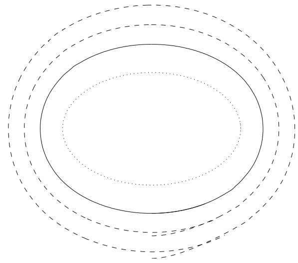

Fig. 7. Convex case: The paths for all robots. The wall is dotted, the leader trajectory is a solid line and the follower trajectories are dashed.

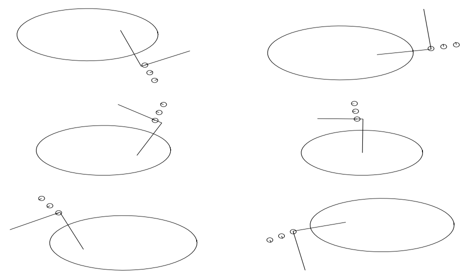

Fig. 8. Convex case: The robots servoing in parallel formation with the moving frame plotted.

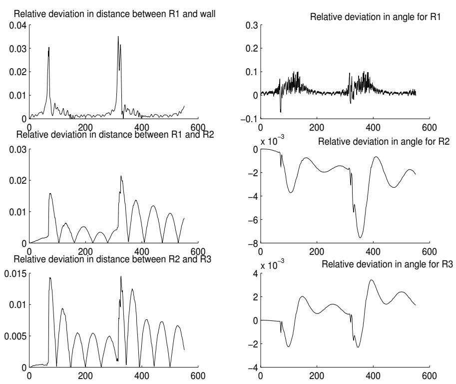

Fig. 9. Convex case: The formation errors in distance and angle.

Although Figure 7 shows that the robots follow their trajectories well, we can note that the deviation from desired angle (Figure 9, right column) has two peaks. These correspond to where the curvature of the ellipse is the highest.

4.2 Non-convex Case

Following a non-convex wall while keeping a parallel formation is a bit more challenging, but our simulations show that the control works also in non-convex situations. The equation of the curve representing the wall is $y = 5+4\sin(0.1x)$, the desired distance to it is $d_w = 8$ and desired distance between robots is $L = 6$. The peaks in the plots of Figure 12 correspond to the non-convex parts of the wall.

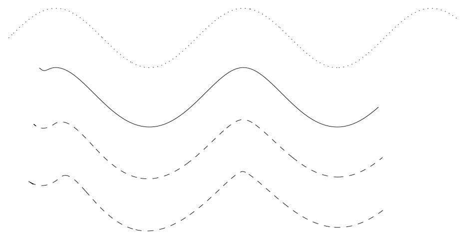

Fig. 10. Non-convex case: The paths for all robots. The wall is dotted, the leader trajectory is a solid line and the follower trajectories are dashed.

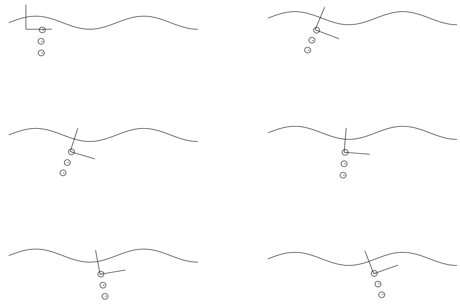

Fig. 11. Non-convex case: The robots servoing in parallel with the moving frame also plotted.

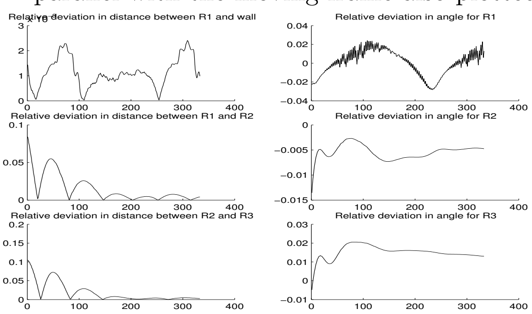

Fig. 12. The formation errors in distance and angle.

5. CONCLUSIONS

Formation control is well studied in the literature. In this paper we have proposed some new control algorithms for parallel formation keeping and terrain servoing that are only based on local information and easy to implement.

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