HYBRID CONTROL SCHEME WITH DISCRETE ESTIMATOR FOR EFFICIENT DISTURBANCE REJECTION

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Abstract: The variable structure control system includes a set of controllers that have been prepared and tuned in advance. The output of the most suitable controller from this set is switched to the activity according to the changes of the static and dynamic properties of the controlled system and according to the input disturbances during the operation. Many classes of controllers can be included in the hybrid control strategy and it can be applied to a fairly general class of controlled systems. Thus, this hybrid controller is characterised by considerable flexibility and wide application range. The discrete time incremental estimator, convenient for the switching and for the control, is described and tested.

Keywords: hybrid system, variable structure control, multivariable system, estimator, design, simulation.

1. INTRODUCTION

The structures of the multi input and multi output [MIMO] systems are very different. On the one hand, there are systems with several dynamic blocks, which have single manipulated variable input and the outputs of the whole system are weighted sums of the outputs of the individual blocks. There is no interaction between state variables of the different blocks. Such a structure can be called a block system (Fig.1). On the other hand, there are systems, where each state variable of the MIMO system can be influenced by another arbitrary state variable from the system and vice versa. They can by called global systems (Fig.2).

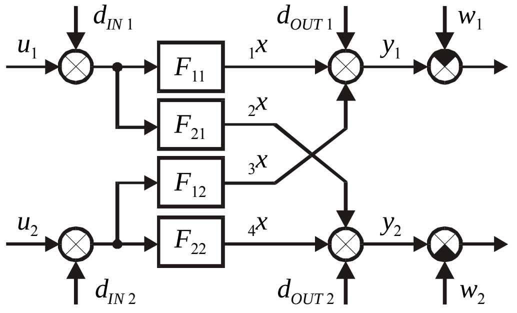


Fig. 1. The block structure of the controlled system.

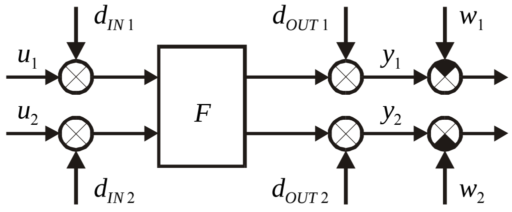


Fig. 2. The global structure of the controlled system.

2. THE BLOCK SYSTEM STRUCTURE

The fourth order linear system in Fig.1 can serve as an example of the block structure. The system has two control inputs u and two controlled variables y. The discrete time description of the system is

$$\boldsymbol{x} = z^{-1}\mathbf{A}\,\boldsymbol{x} + z^{-1}\mathbf{B}\,\boldsymbol{u}\,,\quad \boldsymbol{y} = \mathbf{C}\,\boldsymbol{x}\,, \tag{1}$$

where

$$\mathbf{A} = \begin{bmatrix} a_{11} & 0 & 0 & 0 \\ 0 & a_{22} & 0 & 0 \\ 0 & 0 & a_{33} & 0 \\ 0 & 0 & 0 & a_{44} \end{bmatrix},\quad \mathbf{B} = \begin{bmatrix} b_{11} & 0 \\ b_{21} & 0 \\ 0 & b_{32} \\ 0 & b_{42} \end{bmatrix},$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}.$$

This description can be manipulated to the form

$$\begin{aligned} &\left(1 - z^{-1}a_{11}\right)\left(1 - z^{-1}a_{33}\right)y_1 = \\ &= z^{-1}\left(1 - z^{-1}a_{33}\right)b_{11}u_1 + z^{-1}\left(1 - z^{-1}a_{11}\right)b_{32}u_2\,, \\ &\left(1 - z^{-1}a_{22}\right)\left(1 - z^{-1}a_{44}\right)y_2 = \\ &= z^{-1}\left(1 - z^{-1}a_{44}\right)b_{21}u_1 + z^{-1}\left(1 - z^{-1}a_{22}\right)b_{42}u_2 \end{aligned} \tag{2}$$

and the transfer function of the system can be written as

$$\begin{aligned} y_1 &= \frac{z^{-1}}{1 - z^{-1}a_{11}} b_{11}u_1 + \frac{z^{-1}}{1 - z^{-1}a_{33}} b_{32}u_2\,, \\ y_2 &= \frac{z^{-1}}{1 - z^{-1}a_{22}} b_{21}u_1 + \frac{z^{-1}}{1 - z^{-1}a_{44}} b_{42}u_2\,. \end{aligned} \tag{3}$$

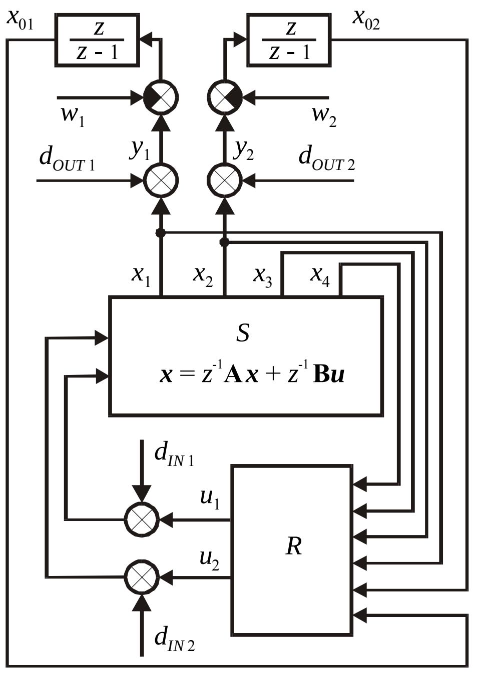


Fig. 3. The scheme of the multivariable control system for design of the state controller and for the comparison of the control results.

The system description (2) and (3) is suitable for the system identification from measurement data. The model with the description (2) is in Fig.3. It is augmented with a control system with the state controller R.

The controller is designed for this model. A discrete time integrator is connected to the outputs y_1 and y_2 of the model for the controller R to be able to reject all of the input disturbances of the controlled system with zero steady state error. All of the state variables of the system are supposed to be measurable. If this is not possible in the practice, the same controller R with the same tuning is used in the control scheme with the incremental discrete time estimator E in Fig.4. All of the coefficients in Fig.4 are the same as in Fig.3.

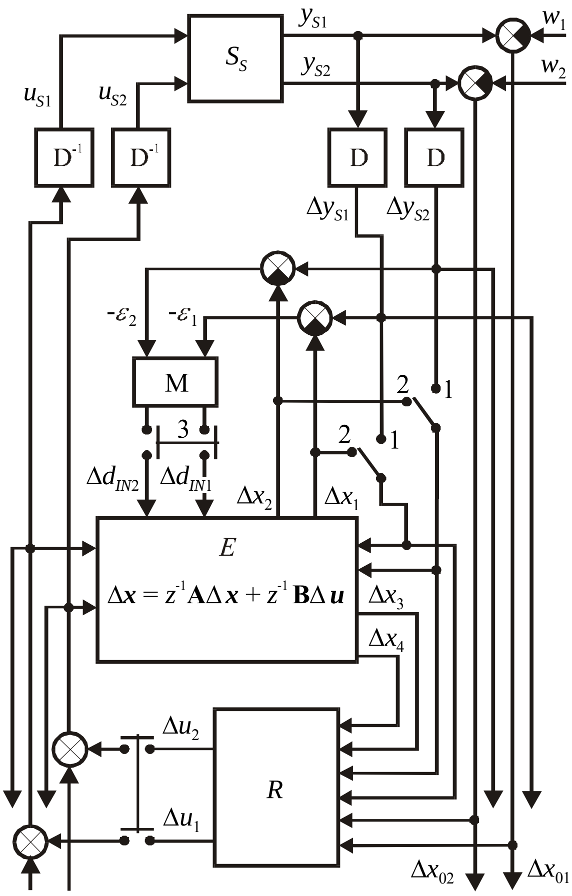


Fig. 4. The scheme of the variable structure control with the estimator E. $\mathbf{M} = \begin{bmatrix} b_{11} & b_{32} \\ b_{21} & b_{42} \end{bmatrix}^{-1}$, $D = 1 - z^{-1}$, $D^{-1} = 1/(1 - z^{-1})$.

The incremental estimator in Fig.4 (see Hanuš *et al.*, 2000, 2001, 2004b) can be tuned for the expected disturbances d_{IN}, d_{OUT} and d_{ARX} (only the first element is in the transfer function numerator) and it is able to achieve their best rejection with the prepared controller beginning with the first sampling period. Other disturbances are rejected optimal after N periods (N≤ n, n is the system order). The choice of the estimator is carried out by means of the switches in the Table 1.

Table 1 The estimator tuning for different disturbances

disturbance input	switch position		estimator type
d_{IN}	1	3 – on	E_{IN}
d_{OUT}	2	3 – off	E_{OUT}
d_{ARX}	1	3 – off	E_{ARX}

The size if the d_{IN} input is estimated by means of the matrix **M**.

2.1 *Illustrative example.*

The control schemes in Fig.3 and Fig.4 were tested by means of the computer simulation. The transfer functions of the continuous controlled system in Fig.1 were

$$F_{11} = \frac{1}{0.5s+1},\quad F_{21} = -\frac{0.5}{s+1},$$
$$F_{12} = -\frac{0.5}{1.5s+1},\quad F_{22} = \frac{1}{2s+1}. \tag{4}$$

Fig.5 shows the comparison of the responses of two control schemes. First, the response of the control scheme where the controlled system S_S is continuous and the state variables of its discrete time model are estimated using a discrete time estimator of the type that is shown in Fig.4. Second, the controlled system S is discrete and its state vector is directly accessible to the state feedback controller. The results are demonstrated in Fig.5. The sampling interval is 0.2 s.

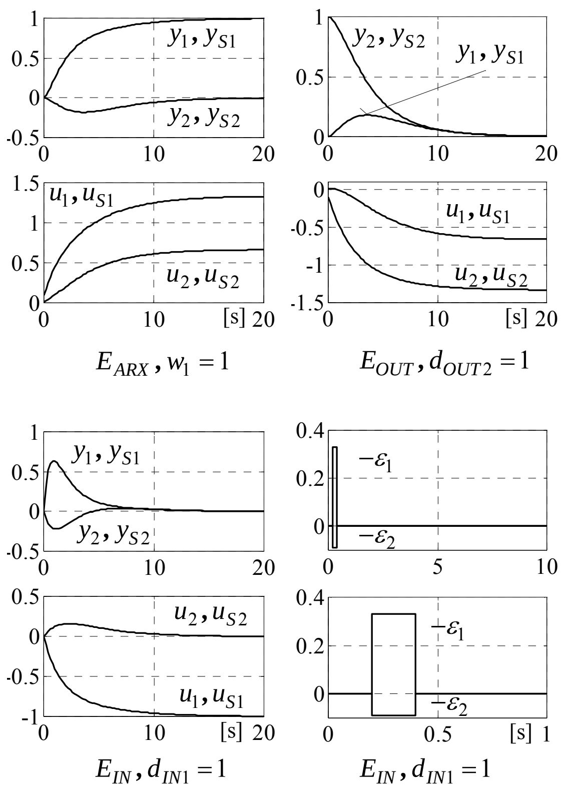


Fig. 5. Responses with block structure system.

3. GLOBAL STRUCTURE

An example of a two input two output global structure system in Fig.2 of the third order in observer canonical form may by described with (1), where

$$\mathbf{A} = \begin{bmatrix} a_{11} & 1 & 0 \\ a_{21} & 0 & 1 \\ a_{31} & 0 & 0 \end{bmatrix},\ \mathbf{B} = [\mathbf{B}_1 \quad \mathbf{B}_2] = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix},$$
$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 \\ c_1 & c_2 & c_3 \end{bmatrix}. \tag{5}$$

The relation between the inputs u_1 or u_2 and the output y_1 or y_2 of the controlled system (1),(5) obtained from the identification in the form of difference equations is

$$Ay_1 = B_1u_1 + B_2u_2 = \left(z^{-k}\right)_1\mathbf{B}\boldsymbol{u}\,, \tag{6}$$
$$Ay_2 = B_3u_1 + B_4u_2 = \left(z^{-k}\right)_2\mathbf{B}\boldsymbol{u}\,, \tag{7}$$

where $_1\mathbf{B} = [_1\mathbf{B}_1 \quad _1\mathbf{B}_2]$, $_2\mathbf{B} = [_2\mathbf{B}_1 \quad _2\mathbf{B}_2]$,
$_i\mathbf{B}_j^{\mathrm{T}} = [_ib_{1j} \quad _ib_{2j} \quad _ib_{3j}]$,
$\left(z^{-k}\right)$ - one row matrix with the elements z^{-k}, $k = 1$ to 3.

The polynomials A on the left side of the equations are identical, they correspond to the characteristic polynomial of the controlled system (the determinant of the matrix $(\mathbf{I} - z^{-1}\mathbf{A})$ in (1)).

The solution of the system (1),(5) where the elements of the matrix **C** are unknown yet, is

$$\det\left(\mathbf{I} - z^{-1}\mathbf{A}\right)\boldsymbol{y} = z^{-1}\mathbf{C}\left(\mathbf{I} - z^{-1}\mathbf{A}\right)_{\mathrm{adj}}\mathbf{B}\boldsymbol{u} = = z^{-1}\begin{bmatrix} 1 & z^{-1} & z^{-2} \\ C_1 & C_2 & C_3 \end{bmatrix}\mathbf{B}\boldsymbol{u}\,, \tag{8}$$

where
$C_1 = c_1 + c_2\left(z^{-1}a_{21} + z^{-2}a_{31}\right) + c_3z^{-1}a_{31}$,
$C_2 = c_1z^{-1} + c_2\left(1 - z^{-1}a_{11}\right) + c_3z^{-2}a_{31}$,
$C_3 = c_1z^{-2} + c_2z^{-1}\left(1 - z^{-1}a_{11}\right) + c_3\left(1 - z^{-1}a_{11} - z^{-2}a_{21}\right)$.

It follows after the comparison with the relation (6) and (8)

$$\mathbf{B} = {_1\mathbf{B}} \tag{9}$$

and after the comparison (7) and (8) it follows

$$\det\left(\mathbf{I} - z^{-1}\mathbf{A}\right)y_2 = z^{-1}[C_1 \quad C_2 \quad C_3][\mathbf{B}_1 \quad \mathbf{B}_2]\boldsymbol{u} = = \left(z^{-k}\right)[_2\mathbf{B}_1 \quad _2\mathbf{B}_2]\boldsymbol{u}\,. \tag{10}$$

After arranging the matrix in the relation (10) (the polynomials elements multiplied by $z^{-k}c_i$ (10) are placed in the k-th row and i-th column of the matrix $^1\mathbf{B}$ (11) or $^2\mathbf{B}$ (12)) and in the view of the fact, that the same coefficients c_i should be convenient for the arbitrary input of the manipulated variable u_1 and u_2, the following equations hold

$$\left(z^{-k}\right)\,{}^{1}\mathbf{B}\begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix}u_1=\left(z^{-k}\right)\,{}_{2}\mathbf{B}_1\,u_1, \qquad (11)$$

$$\left(z^{-k}\right)\,{}^{2}\mathbf{B}\begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix}u_2=\left(z^{-k}\right)\,{}_{2}\mathbf{B}_2\,u_2, \qquad (12)$$

$$\begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix}={}^{1}\mathbf{B}^{-1}\,{}_{2}\mathbf{B}_1={}^{2}\mathbf{B}^{-1}\,{}_{2}\mathbf{B}_2\,. \qquad (13)$$

In the end both of the controlled variables y_1 and y_2 are included in the state vector of the controlled system model using the linear transformation

$$\begin{bmatrix}y_1\\y_2\\x_3\end{bmatrix}=\mathbf{T}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}1&0&0\\c_1&c_2&c_3\\0&0&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}. \qquad (14)$$

The final form of the state description of the system is

$$\begin{bmatrix}y_1\\y_2\\x_3\end{bmatrix}=z^{-1}\mathbf{TAT}^{-1}\begin{bmatrix}y_1\\y_2\\x_3\end{bmatrix}+z^{-1}\mathbf{TB}\begin{bmatrix}u_1\\u_2\end{bmatrix}. \qquad (15)$$

The structure of the estimator (Fig.4) corresponds to the description (15). However it is possible to use instead of (15) directly the identification difference equations (6) and (7) or the mixed difference equations

$$y_1={}_{1}A_1y_1+{}_{1}A_2y_2+{}_{1}B_1u_1+{}_{1}B_2u_2\,,$$

$$y_2={}_{2}A_1y_1+{}_{2}A_2y_2+{}_{2}B_1u_1+{}_{2}B_2u_2\,.$$

The control results correspond with the results of the estimator designed according to the description (15) only, if the total orders of the estimators are equal or if the real input disturbance corresponds to that (w, d_{OUT} in Fig.7), for which the estimator are tuned.

3.1 *Illustrative example.*

The system of the fourth order with multiple time constants T=1s and with the transfer function (16) from the control input u_1 was chosen for testing

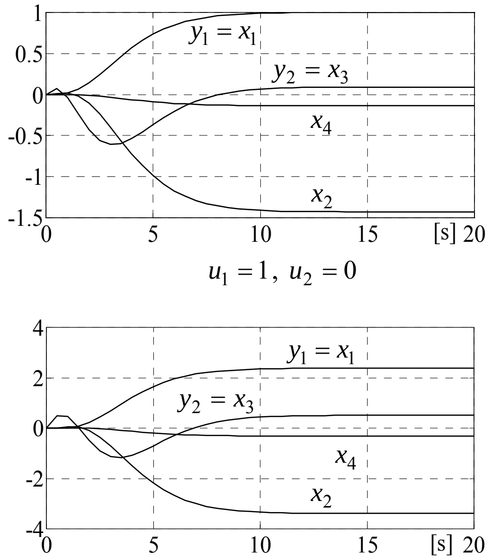
$$F_1=\frac{1}{(s+1)^4}\,. \qquad (16)$$

The corresponding discrete time state descriptions in observer canonical form with two control inputs u_1 and u_2 and two controlled variables y_1 and y_2 is then (1), where the first column $\mathbf{B}_1$ in the matrix $\mathbf{B}$ corresponds to the transfer function (16) and the second column $\mathbf{B}_2$ and the matrix $\mathbf{C}$ are chosen. The relation $c_1=-c_3(a_{31}+a_{41})$ was respected by the choice. The linear transformation matrix is then

$$\mathbf{T}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\c_1&0&c_3&0\\0&0&0&1\end{bmatrix}. \qquad (17)$$

The relation for c_1 helps the closed system matrix to be regular and well conditioned. Otherwise slight changes of reference inputs w_1 and w_2 could result in huge changes of manipulated variables u_1 and u_2. This effect is typical of MIMO systems and does not exist in SISO systems.

The step responses of the controlled system are in Fig.6. The demonstrations of the control processes are in Fig.7. The model S_S in Fig.4 used for the simulation is discrete, the sampling interval is 0.5s.



$u_1 = 0,\ u_2 = 1$

Fig. 6. Step responses of the global structure system.

4. DECOUPLING CONTROL

This feature is often demanded in the practice. It doesn't mean however, that the decoupling control is better in the sense of the performance criterion (quadratic), for which the control is designed.

The design of the decoupling control is based on the description (1) and (2) and on its solution in the form

$$\det\left(\mathbf{I}-z^{-1}\mathbf{A}\right)\boldsymbol{y}=z^{-1}\mathbf{C}\left(\mathbf{I}-z^{-1}\mathbf{A}\right)_{\mathrm{adj}}\mathbf{B}\boldsymbol{u}=z^{-1}\mathbf{D}\boldsymbol{u}\,. \qquad (18)$$

After the reduction in the rows, the following relation holds

$$\left[\det_i y_i\right]=z^{-1}\mathbf{D}_0\,\boldsymbol{u}\,, \qquad (19)$$

where

$\left[\det_i y_i\right]$ - the vector $(r,1)$ after the reduction in the rows,

$\mathbf{D}_0$ - the matrix $\mathbf{D}(r,r)$ after the reduction.

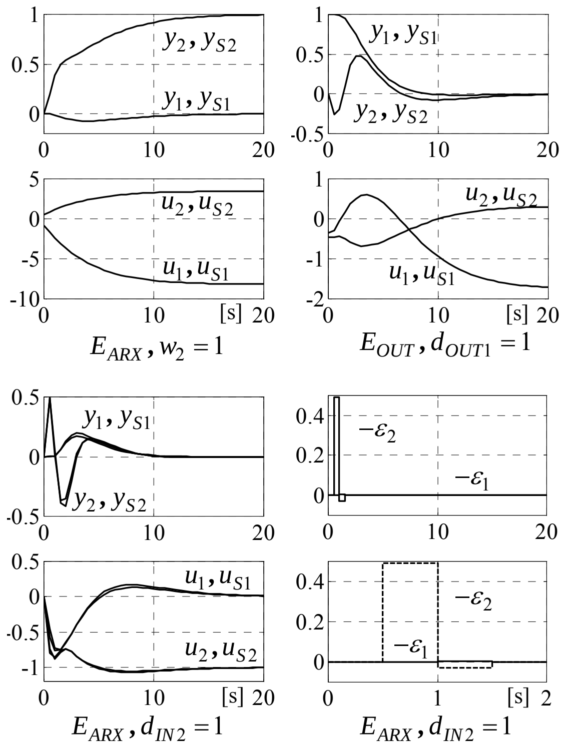


Fig. 7. Responses with global structure system.

After introducing the decoupling condition, the following equation must be satisfied

$$z^{-1}\mathbf{D}_0\,\boldsymbol{u} = z^{-1}\mathbf{F}_0\,\boldsymbol{v}\ . \qquad (20)$$

where

$\boldsymbol{v}$ - the vector $(r,1)$ of the virtual manipulated variables,

$\mathbf{F}_0$ - the optional diagonal matrix (r,r).

The values of the real manipulated variables

$$\det\mathbf{D}_0\,\boldsymbol{u} = \mathbf{D}_{0\mathrm{adj}}\mathbf{F}_0\,\boldsymbol{v} \qquad (21)$$

are determined directly in the control system from the outputs v_i of the SISO discrete time controllers R_i of arbitrary type. They are designed for the system

$$\det{}_i y_i = z^{-1}F_{0i}v_i,\ \ i = 1\ldots r\ . \qquad (22)$$

The manual operation is possible by means of the variable v_i or by means of the real manipulated variable u_i.

4.1 *Illustrative example.*

The responses of the decoupling control system were simulated with controlled system (1),(2) and with condition (20) satisfied. The elements of the matrix $\mathbf{F}_0$ are: $F_{01} = b_{11}\left(1 - z^{-1}a_{33}\right)$, $F_{02} = b_{42}\left(1 - z^{-1}a_{22}\right)$. The decoupling control is simulated by means of the discrete model S on the scheme on Fig.3. The responses are compared with the result obtained in the chapter 2.1. Full decoupling is achieved if the control coefficients corresponding with x_3 and x_4 are zero. The demonstrations of the control responses are in Fig.8,9,10.

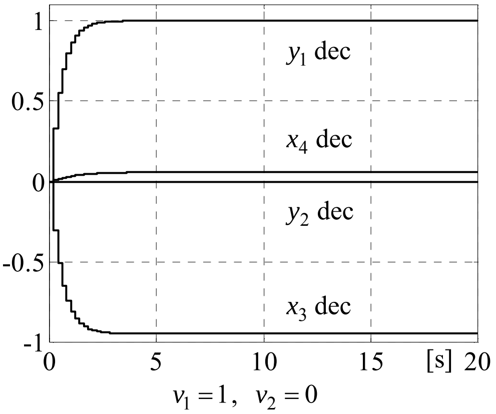


Fig. 8. Manual operation with decoupling control process ($y_1\text{dec} = F_{11}v_1 = \dfrac{z^{-1}b_{11}}{1 - z^{-1}a_{11}}v_1$ from (2)).

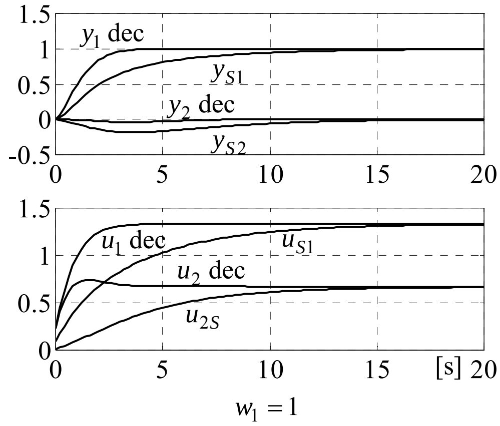


Fig. 9. Decoupling control responses with block structure system.

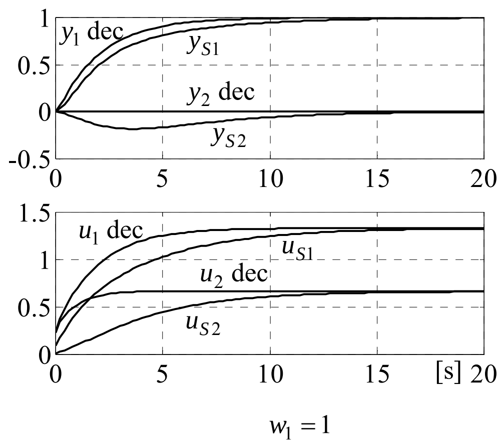


Fig. 10. Full decoupling control responses with block structure system.

The speed control of the steam turbine with the generator and the pressure control of the extraction steam from the turbine is a typical example of decoupling control. The system of the second order with the block structure is used for the simplified description of the controlled system

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = z^{-1} \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + z^{-1} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & -b_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (23)$$

where

u_1 - the flow volume through the high pressure turbine part valves,

u_2 - the flow volume through the low pressure turbine part valves,

y_1 - the turbine speed,

y_2 - the pressure of the extraction steam.

The relation (21) between the virtual and real manipulated variables is (choosing $\mathbf{F}_0 = \mathbf{I}$) static in this case (the number of the controlled variables is equal to the total order of the controlled system). It is solved in the practice by means of the rocking lever, which connects the action points of the valves and the sensors. The method of the alignment chart (nomogram) design is used. Both of the controllers are continuous and proportional (Fig.11).

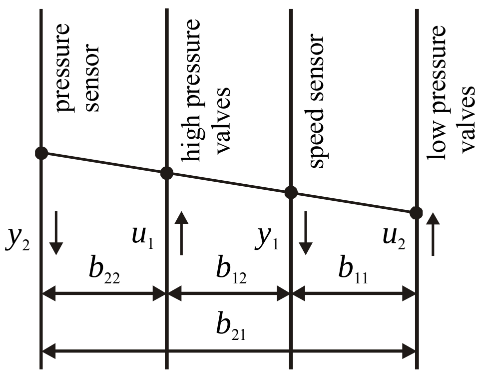


Fig. 11. The rocking lever for the decoupling control. The distances b_{ij} on the figure correspond to the coefficients in the system description (23) and to the scales of the controlled y_i and manipulated u_i variables on the figure.

5. CONCLUSION

The discrete time incremental estimator was tested in laboratory in connection with the continuous and discrete time controller, with the fuzzy controller (see Hanuš *et al*., 2002), with a non-linear controlled system, with a system with delay (see Hanuš *et al*., 2003), in SISO or MIMO system. The interpolation between the nominal points of the set and the iteration is possible (see Herajn and Janeček, 2003). It is possible to change the transfer functions of the whole set by changing the sampling frequency (see Hanuš and Tůma, 2004a). The results of the testing confirm that the proposed hybrid control strategy is able to achieve optimal or nearly optimal control performance under a variety of conditions and with a general class of controlled systems. Further research directions will be focused on application of this hybrid strategy to systems that are itself of hybrid nature (see e.g. Hlava and Šulc, 2002 for introduction to hybrid systems).

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