NICELY NONLINEAR ENGINE TORQUE ESTIMATOR

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Abstract: In automotive powertrain control strategy, engine torque estimators are preferred to expensive sensors for obvious economic reasons. In the literature several works have been presented proposing both instantaneous engine torque and mean engine torque estimators. The first ones are based on complex models and tipically are suitable for diagnosis purposes, while the latter estimate the torque by means of static maps. In this work, a nonlinear torque estimator for a spark ignition engine has been developed. The total torque acting at the engine shaft has been obtained solving a tracking problem by means of an LQ control strategy. The novel idea is to reduce the nonlinear engine model to a second order Taylor approximation, named *a nicely nonlinear model.* It is then linearized via feedback and an LQ controller is designed. This approach has been tested on Mid-Size real time dSPACE simulator showing excellent results both for performance and for computational load. *Copyright© 2005 IFAC.*

Keywords: Automotive Control, Engine Torque Estimation, Nicely Nonlinear Estimators, LQ Methods.

1. INTRODUCTION

In automotive control strategies and in particular in vehicle dynamic applications, in order to achieve specific goals, an amount of torque generated by the internal combustion engine is requested at the crankshaft. Among others, in traction control applications a net torque regulation is required during the wheel slipping or to satisfy the driver's request. Similarly, for emissions reduction and fuel consumption strategies, a suitable torque profile is actuated guaranteeing the requested performance to the driver.

It is apparent that a good engine torque controller is critical to reach high performances. To this aim, since cheap and non-invasive on-line engine torque sensors are not available for commercial reasons, see Schagerberg and McKelvey (2002), an observer can improve substantially the performances of a feedback torque control scheme.

In literature, several works are presented on this topic. In Kiencke and Nielsen (2000) a Kalman Filter is proposed to estimate the indicate torque for misfire detection purpose; in I. Haskara and L. Mianzo (2001) a real-time cylinder pressure and indicated torque estimator via a second order sliding mode technique is presented. In Wang et al. (1997) the indicated torque has been estimated by means of a sliding-mode observer, in which the gain of the switching function varies during the four strokes. In Azzoni et al. (1998) the indicated torque has been estimated via a frequency response technique; in Lee et al. (2001) a stochastic method has been employed for the engine torque estimator while in Wang et al. (1995) the unknown input observer technique has been used.

In this work an innovative estimation technique, partially based on the introduction of nicely nonlinear models, see Guardabassi (1992), is proposed. The estimation problem has been treated as a tracking problem, solved by means of the LQ control technique. This has been performed reducing the engine model to the second order Taylor approximation, called nicely nonlinear model. The nonlinearities are then compensated via an input-output feedback action, and the LQ controller is designed. The second order approximation has been preferred to the classic linearized system in order to achieve higher performance in the neighborhood of the approximation point. The proposed technique, tested with a dSPACE rapid prototyping system, shows very encouraging results due to the very small errors in torque estimation, in spite of model uncertainties and intentional raw model calibration. Finally an integration step suitable for a real time implementation has been selected.

2. PROBLEM FORMULATION

The goal of this work is to estimate the mean value of the total torque acting on the crankshaft starting from the injected fuel, the air-fuel ratio, the spark angle and the measured shaft speed. To this aim, the estimation problem has been treated as a tracking problem, as shown in Figure 1.

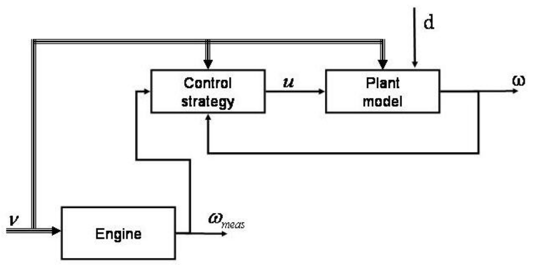

Fig. 1. Tracking control scheme. d is the unknown disturbance, which groups all external load torques, and ν is the known disturbance, a vector formed by fuel mass flow rate ($\dot{m}_f$), air-fuel ratio (λ) and spark advance (θ)

Here, the system to be controlled (called *Plant model* in figure) is a simplified nonlinear model of the combustion process. It models the engine torque production and the shaft speed dynamics. In order to track the reference (ω_{meas}), the measured shaft speed, a model based control strategy is designed. In particular, the control input u represents the amount of extra-torque that the system needs to reach the desired shaft speed compensating both model uncertainties and the disturbance d which takes into account all unknown external torques.

In the following sections, the plant model and the control strategy are respectively described in details. Simulation results and conclusions close the paper.

3. PLANT MODEL

In Figure 2 the model used to design the control strategy is shown. A similar version was presented in Fiengo et al. (2002) and in the following is briefly illustrated. This model is based on the assumption that measurements of air and fuel entering the cylinders are available. It describes the combustion process and computes the mean values of engine torque (T_{comb}) starting from the injected fuel ($\dot{m}_{\text{f}}$), the air-fuel ratio (λ) and the spark advance (θ). The output of the model is the engine speed (ω), obtained by the single inertia crankshaft model, according to

$$\dot{\omega} = f(\omega, u, \nu) - d \tag{1}$$

with

$$f(\omega, u, \nu) \triangleq \frac{1}{J}\left[T_{\text{comb}}(\omega, \nu) - T_{\text{fric}}(\omega) + u\right] \tag{2a}$$

$$d \triangleq \frac{1}{J} T_{\text{load}} \tag{2b}$$

where: $\nu = \left[\, \dot{m}_f \; \lambda \; \theta \,\right]^T$ are the inputs of the model that cannot be handled (i.e. known disturbances); J is the total inertia; T_{load} is the unknown load torque; T_{fric} is the friction torque, computed as a function of the engine speed through the following black box model (block *Friction Torque* in Figure 2)

$$T_{\text{fric}} = k_1 + k_2\omega + k_3\omega^2 \tag{3}$$

in which the constants k_1, k_2, k_3 are to be identified.

The block *Combustion* models the effective torque generated by the combustion (T_{comb}). Firstly,

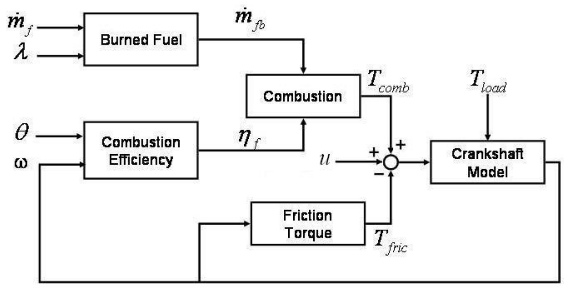

Fig. 2. Overall model scheme: $\dot{m}_{\text{f}}$ is the injected fuel, $\dot{m}_{\text{fb}}$ is the burned fuel, λ is the normalized air-fuel ratio, θ is the spark advance, η_{f} is the combustion efficiency, T_{comb} is the combustion torque, T_{fric} is the friction torque, T_{load} is the load torque, ω is the crankshaft speed.

the mechanical power is computed multiplying the combustion efficiency (η_f) by the quantity of chemical energy generated by the combustion, obtained by the lower heating value of the fuel (Q_{LHV}) by the amount of the injected fuel supposed to burn during the combustion process ($\dot{m}_{\mathrm{fb}}$).

The combustion efficiency measures the capacity of the engine to transform the fuel chemical energy into mechanical energy and here is modeled as a static map, function of spark advance and engine speed.

The burned fuel ($\dot{m}_{\mathrm{fb}}$) is computed multiplying the injected fuel $\dot{m}_{\mathrm{f}}$ and an efficiency term (η_λ), depending on air-fuel ratio and estimating the fraction of the injected fuel burning during the combustion phase

$$\dot{m}_{\mathrm{fb}} = \dot{m}_{\mathrm{f}} \cdot \eta_\lambda(\lambda). \tag{4}$$

This relationship is based on the assumption that only the air-fuel mixture that is in a stoichiometric ratio participates actively to the combustion process: if the air-fuel mixture is lean ($\lambda \geq 1$) all the fuel in the cylinder takes part at the combustion; conversely, in rich condition ($\lambda < 1$), only the amount of fuel in stoichiometric ratio with the air in the cylinder ($\dot{m}_{\mathrm{f}}\lambda$) participates to the combustion. Hence, it is computed according to

$$\eta_\lambda(\lambda) = \begin{cases} 1 \text{ when } \lambda \geq 1 \\ \lambda \text{ when } \lambda < 1 \end{cases} \tag{5}$$

Finally, the effective torque is obtained dividing the mechanical power and the engine speed, according to

$$T_{\mathrm{comb}} = \frac{\eta_{\mathrm{f}} \dot{m}_{\mathrm{fb}} Q_{\mathrm{LHV}}}{\omega} \tag{6}$$

4. CONTROL STRATEGY

The control problem was treated as an LQ optimal problem aimed to minimize the objective function

$$V = \frac{1}{2} \int_0^\infty \left[q \left(\omega - \omega_{\mathrm{meas}} \right)^2 + \rho u^2 \right] dt. \tag{7}$$

Letting $(\bar{\omega}, \bar{u}, \bar{\nu})$ be a generic point in the state and input space and $\bar{t}$ a generic time instant, a nicely nonlinear approximation (see Guardabassi (1992)) of the system (1) is obtained as follows

$$\begin{aligned} \delta\dot{\omega} =& A_0 \delta\omega + B\delta u + \Gamma_0 \delta\nu + f\left(\bar{\omega}, \bar{u}, \bar{\nu}\right) + \\ &+ \frac{1}{2} \left[\delta\omega \quad \delta\nu\right] A_1 \left[\delta\omega \quad \delta\nu\right]^T \end{aligned} \tag{8}$$

where: A_0, B and Γ_0 are the Jacobian matrices of the function f, A_1 is the Hessian matrix and $\delta\omega = \omega - \bar{\omega}$, $\delta\nu = \nu - \bar{\nu}$, $\delta u = u - \bar{u}$ are the variations from the generic point $(\bar{\omega}, \bar{u}, \bar{\nu})$.

To compute the system (8), a second order Taylor approximation has been used. It was preferred to

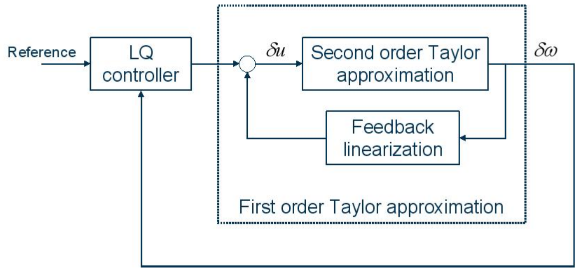

Fig. 3. The scheme of the feedback linerization of a nicely nonlinear model.

the linear first order Taylor approximation in order to obtained a simplified system able to better represent the full model in the neighborhood of the approximation point. Let us now define the control input

$$u \triangleq v_0 - v_1. \tag{9}$$

The signal v_1 is used to feedback linearize the system (8), as shows the Figure 3, according to (notice B is a scalar)

$$\begin{aligned} v_1 =& B^{-1} \left[f\left(\bar{\omega}, \bar{u}, \bar{\nu}\right) + \Gamma_0 \delta\nu + \right. \\ &\left. + \frac{1}{2} \left[\delta\omega \quad \delta\nu\right] A_1 \left[\delta\omega \quad \delta\nu\right]^T \right] \end{aligned} \tag{10}$$

Hence, substituting (9) and (10) into (7) and (8), the optimal control problem becomes

$$\begin{aligned} \delta\dot{\omega} =& A_0 \delta\omega + B v_0 - B\bar{u} \\ V =& \frac{1}{2} \int_0^\infty \left[q \left(\omega - \omega_{\mathrm{meas}} \right)^2 + \rho \left(v_0 - v_1 \right)^2 \right] dt \end{aligned}$$

where v_1 is considered the reference for the control input v_0. To reduce the computational complexity, the suboptimal control algorithm is obtained (see Appendix for mathematical details and Fiengo (2001); Anderson and Moore (1979)) according to

$$v_0 = -\rho^{-1} B^T P \delta\omega - \rho^{-1} B^T b + v_1 \tag{11}$$

where P is the solution of the algebraic Riccati equation

$$P A_0 + A_0^T P - P B \rho^{-1} B^T P + q = 0 \tag{12}$$

and b is the solution of

$$\begin{aligned} \left(A_0 - B\rho^{-1} B^T P\right) b + P B (v_1 - \bar{u}) + \\ - q \left(\omega_{\mathrm{meas}} - \bar{\omega}\right) = 0. \end{aligned} \tag{13}$$

Finally, substituting (11) into (9), the control law is obtained

$$u = -\rho^{-1} B^T P \delta\omega - \rho^{-1} B^T b \tag{14}$$

It is interesting to note that the feedback linearization term (10) is cancelled in (14) since it is already considered in the feedforward control action (13).

Once the control input is computed, the estimation of the mean value of the torque acting on the crankshaft (T_{shaft}) is obtained by adding the signal u (14) to the torque terms computed by the model (see equations (3) and (6)) as follows

$$T_{\text{shaft}} = T_{\text{comb}}\,(\omega, \nu) - T_{\text{fric}}(\omega) + u. \qquad (15)$$

5. RESULTS

In this section the simulation results are shown. The presented estimator has been tested on a dSPACE Mid-Size Simulator running a 4-cylinder engine model able to reproduce the instantaneous engine speed and torque, simulating the combustion during the entire engine cycle.

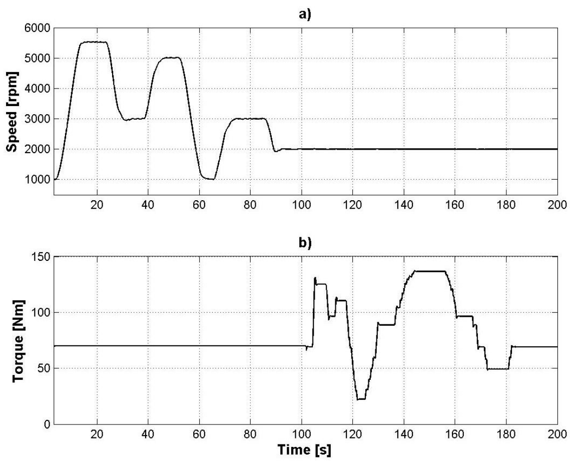

Fig. 4. Acquisition data. In the first 100 seconds, speed transients at constant load torque are shown, while in the remaining load torque transients at constant engine speed are performed.

The estimator has been tested both in speed transients and load torque transients. As regards speed transients, the load torque has been fixed and the pedal has been varied in order to obtain the speed profiles shown in the firsts 100 seconds of the Figure 4. During the remaining 100 seconds, an engine speed regulation has been performed by a test-bed controller while the pedal is varied. The approximation of the system and the solution of the LQ problem are performed from time to time, e.g. when the variations of input and state exceed selected thresholds. For the results presented here, the thresholds are selected as 10% of the engine speed and injected fuel. The solver integration step size has been fixed to 20 milliseconds. Obviously the step size and the linearization threshold have to be selected to reach the best compromise between the performances and the required computational power.

The simulation results are excellent. In particular, the mean absolute errors on the shaft torque is 0.1712 Nm and 0.0685 Nm respectively during the speed transients and during the load torque transients. Since it is not possible to distinguish the measured and the estimated signals, in the following only the zooms of the worst cases are reported shown in separated plots. In particular,

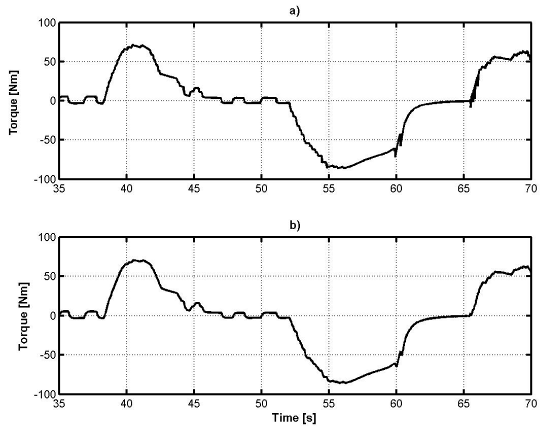

Fig. 5. Speed transient at a constant load torque of 20 Nm. In a) and b) the measured and the estimated torque are presented respectively.

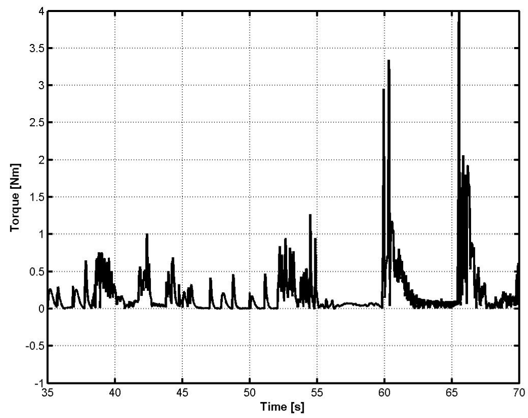

Fig. 6. The absolute torque error in a speed transient at a constant load torque of 20 Nm.

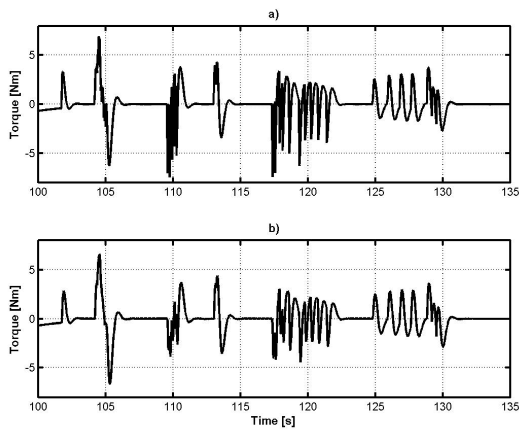

Fig. 7. Speed transient at a constant load torque of 20 Nm. In a) and b) the measured and the estimated torque are presented respectively.

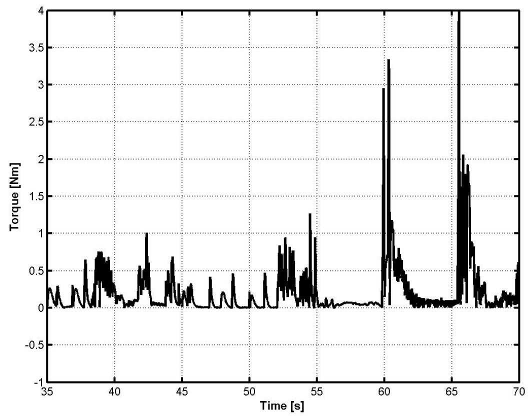

Fig. 8. The absolute torque error in a speed transient at a constant load torque of 20 Nm.

in Figure 5 and 6 the zoom of the simulation results during a speed transient are shown. In Figure 5, a transient speed at constant load torque of 20 Nm is shown while in 6 the related absolute torque error is reported. It is interesting to note the peaks in the absolute error, around 60 s and 65 s, due to the variation of the controller gains during the approximation task; this problem has been partially solved adopting a bumpless transfer strategy. In the remaining part of the time interval the error is very low, attaining a mean absolute error of 0.21 Nm. Conversely, Figures 7 and 8 show the simulations results during a load torque transient at 2000 rpm.

Finally, a robustness analysis has been performed by inserting uncertainty in the combustion efficiency. To this aim, a multiplicative term α has been considered in $\bar{\eta}_{\mathrm{f}} = \alpha\eta_{\mathrm{f}}$. The simulations revealed that the estimator shows a small sensitivity to the inaccuracy of the combustion efficiency map, as confirmed by the relationship:

$$\frac{\Delta e}{e} \simeq 4 \cdot 10^{-1} \frac{\Delta\alpha}{\alpha} \tag{16}$$

where e is the absolute error and α is the nominal value of the parameter, equal to 1. Moreover the uncertainty in the combustion efficiency has been increase reducing the number of points (N) of the map η_{f}. In Figure 9 the variation of absolute error $\Delta e/e$ is plotted. Once again, the performance of the estimator shows high robustness to this kind of uncertainty. It is interesting to observe the non monotonic trend of the plotted data.

6. CONCLUSIONS

An engine torque estimator, based on feedback linearization of nicely nonlinear models, has been presented. The estimator computes the total torque available to the engine shaft with a small error and guarantees great robustness to uncertainties in the engine model. This feature is due to the statement of the estimation problem as a tracking problem, that allows to compensate both external load torques and model uncertainties. A drawback of this approach is the sensitivity to the uncertainty on the crankshaft inertia. Finally, the estimator allows a real time implementation, due to the low computational load, and reduces the calibration effort.

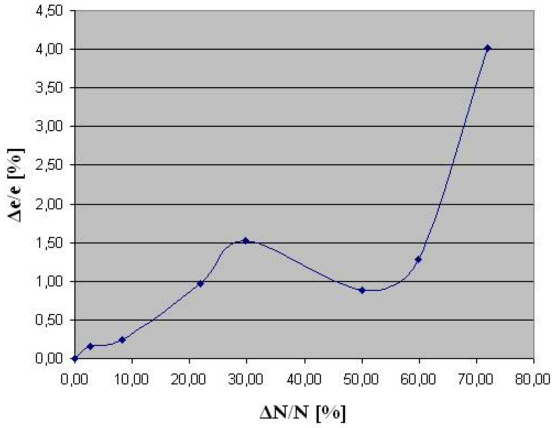

Fig. 9. The increment error due to a reduction of the point in the map of the combustion efficiency.

7. APPENDIX

Let consider the following optimal control problem for nonlinear systems

$$\dot{x} = f(x,u), \qquad x(0) = x_0, \tag{17a}$$

$$\min_{u(.)} V = \min_{u(.)} \frac{1}{2} \int_0^T \left[(x-\tilde{x})^T Q (x-\tilde{x}) + (u-\tilde{u})^T R (u-\tilde{u})\right], \tag{17b}$$

where x is the variable state, u is the controllable input and $\tilde{x}$ and $\tilde{u}$ are respectively the state and input reference. Since, on many occasions, finding the exact solution to this nonlinear optimal control problem is practically unfeasible, it is illustrated a suboptimal procedure based on successive linearization of the problem. This approach resembles receding horizon techniques, such as in Mayne and Michalska (1990) and in Chen and Allgöwer (1998).

Let $\overline{x}$ and $\overline{u}$ be a generic point in the state and input space, and compute the linearized system at this point

$$\delta\dot{x} = A\delta x + B\delta u + f(\overline{x}, \overline{u}) \tag{18}$$

where: δx and δu are the deviations from the chosen fixed point ($\overline{x}$, $\overline{u}$); A and B are the Jacobian matrices.

The objective function (17b) becomes

$$V = \frac{1}{2}\int_0^T \left[(\delta x - (\tilde{x} - \overline{x}))^T Q(\delta x + \right. \\ \left. - (\tilde{x} - \overline{x})) + (u - \tilde{u})^T R(u - \tilde{u})\right] dt, \tag{19}$$

where $(\tilde{x} - \overline{x})$ is the new signal to be tracked. Finally, substituting $\delta u = u - \overline{u}$ into (18), we obtain:

$$\delta\dot{x} = A\delta x + Bu + c \tag{20}$$

where c is a constant equal to $[f(\overline{x}, \overline{u}) - B\overline{u}]$. Then, the suboptimal control problem is solved according to

$$PA + A^T P - PBR^{-1}B^T P + Q = 0 \tag{21}$$

$$(A - BR^{-1}B^T P)^T b + Pc + PB\tilde{u} + \\ - Q(\tilde{x} - \overline{x}) = 0 \tag{22}$$

$$u_{\text{subopt}} = -R^{-1}B^T P\delta x - R^{-1}B^T b + \tilde{u}. \tag{23}$$

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