APPLICATIONS OF MIXED H_2 AND H_∞ INPUT DESIGN IN IDENTIFICATION

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Abstract: The objective of this contribution is to quantify benefits of optimal input design compared to the use of standard identification input signals, *e.g.* PRBS signals for some common, and important, application areas of system identification. Two benchmark problems taken from process control and control of flexible mechanical structures are considered. We present results both when the design is based on knowledge of the true system (in general the optimal design depends on the system itself) and for a practical two step procedure when an initial model estimate is used in the design instead of the true system. The results show that there is a substantial reduction in experiment time and input excitation level. A discussion on the sensitivity of the optimal input design to model estimates is provided.

Keywords: input design, identification, process control, mechanical systems

1. INTRODUCTION

Many industrial processes have (very) slow responses which leads to long and expensive identification experiments (Ogunnaike, 1996). It is thus important to design the experiments carefully as to maximize the information contents. Another area where input design can be crucial is when identifying flexible mechanical structures. Here, time is not crucial but the experiments are usually severely constrained in order to not damage equipment.

Input design has a long history and (Zhu, 2001; Rivera *et al.*, 2003; Lee, 2003; Jacobsen, 1994) are some recent contributions related to process control. Typical design problems correspond to non-convex programs and, hence, computational aspects have limited the applicability of optimal input design. One way of avoiding this has been to rely on high-order expressions for the model accuracy (Gevers and Ljung, 1986; Forssell and Ljung, 2000). Recently, another interesting approach to input design has been opened up. It has been shown that a wide range of input design problems are equivalent to convex programs (Hildebrand and Gevers, 2003; Jansson and Hjalmarsson, 2004*a*; Bombois *et al.*, 2004*b*). The purpose of this contribution is to examine more closely what these new frameworks have to offer for the aforementioned application areas. More precisely, the objective is twofold:

(I) The first aspect is to quantify possible benefits of optimal input design for the two applications. The use of input signals with optimal frequency distribution will be compared to the use of standard identification input signals *e.g.* PRBS signals, see for example (Tan and Godfrey, 2002). The benefits will be quantified in terms of saved experiment time and in possible reduction of input excitation.

Since process modelling may be very time consuming we will in this paper illustrate possible time savings by using an optimal strategy for the considered process model. Here the time it takes to obtain a certain quality of the model is measured and compared for different inputs when the input energy is held constant.

For the mechanical system, the experiment time is in many cases not an issue. Instead, we will study possible savings in the level of input excitation with an optimal strategy.

(II) The second aspect is to enlighten some robustness issues regarding the input design. Optimal input designs in general depend on the unknown true system. This is typically circumvented by replacing the true system with some estimate in

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the design procedure. But there exist very few hard results on the sensitivity and the robustness of these optimal designs with respect to uncertainties in the model estimate that is used in the design.

Here we will illustrate situations where input designs are very sensitive to model errors included in the design. Furthermore, we will redo the comparison in (I), but in a more realistic setting where the optimal design philosophy is replaced by a two-step procedure. The approach taken is inspired by the work in (Lindqvist and Hjalmarsson, 2001) and (Lindqvist, 2001). In the first step an initial model is estimated based on a PRBS input. An input design based on this estimate is then applied to the process. This adaptive approach is compared to an approach which only uses PRBS as input. The comparison is based on Monte-Carlo simulations in order to study the average gain in performance.

Usually the comparison between different input signals is in terms of confidence bounds, see *e.g.* (Gevers and Ljung, 1986), (Forssell and Ljung, 2000) and (Shirt *et al.*, 1994). For industrial applications however perhaps more relevant measures are excitation level and experiment time, treated *e.g.* in (Bombois *et al.*, 2004*b*) and (Rivera *et al.*, 2003). This paper is organized as follows. Section 2 gives some background on the considered identification problem including the optimal input design setup. The optimal input design is applied on a process plant in Section 3 and on a resonant system in Section 4. The paper is concluded in Section 5.

2. BASIC ASSUMPTIONS AND CONSTRAINTS

We will assume that the systems obey the discrete-time linear relation

$$y(t) = G_o(q)u(t) + H_o(q)e(t) \tag{1}$$

where G_o and H_o represents the system and the noise dynamics, respectively, with q being the delay operator. Furthermore, y is the output, u is the input and e is white noise. For the modelling of (1) we will consider identification within the prediction error framework, (Ljung, 1999). Thus the starting point is a parameterized model structure represented by

$$y(t) = G(q,\theta)u(t) + H(q,\theta)e(t). \tag{2}$$

where the transfer operators $G(q,\theta)$ and $H(q,\theta)$ are parameterized by a vector $\theta \in \mathbb{R}^n$. Based on (2), the one-step ahead predictor of the output becomes $\hat{y}(t,\theta) = H^{-1}(q,\theta)G(q,\theta)u(t) + [1 - H^{-1}(q,\theta)]y(t)$. A common way to determine the parameter estimate, $\hat{\theta}_N$, based on N observations of input/output data, is to pick the minimizer

$$\hat{\theta}_N = \arg\min_{\theta} \frac{1}{2N} \sum_{t=1}^{N} (y(t) - \hat{y}(t,\theta))^2$$

This has become a quite standard approach. One of the reasons for this is that there exist a large amount of statistical results that support the method. For example, when the model is flexible enough to capture the true dynamics, it is well known that the estimate $\hat{\theta}_N$ will converge, under mild assumptions, to the parameters of the true system. Furthermore, it is possible to exactly characterize the asymptotic covariance matrix P of the parameters, see *e.g.* (Ljung, 1999). It can be shown that the only quantity in open-loop operation that can be used to shape P, is actually the spectrum of the input. This fact has been very important from an input design perspective and it has been widely applied, see *e.g.* (Goodwin and Payne, 1977; Cooley and Lee, 2001; Hildebrand and Gevers, 2003; Bombois *et al.*, 2004*a*; Jansson and Hjalmarsson, 2004*a*).

We will assume that we have a full-order model structure and hence only variance errors occurs. Therefore, for large data lengths, the model error can be characterized by some function of the parameter covariance P. To define a proper quality function one has to take the intended use of the model into account. Here we will consider control design. In control applications, it is common to have frequency by frequency conditions of the error on the frequency function estimate. One example is

$$\Delta(\theta) = T\,\frac{G_o - G(\theta)}{G(\theta)} \tag{3}$$

where T is a weighting function. When T is equal to the designed complementary sensitivity function, the $\mathscr{H}_\infty$−norm of (3) has been considered as a relevant measure of both robust stability and robust performance (Morari and Zafiriou, 1989; Zhou *et al.*, 1996; Hjalmarsson and Jansson, 2003); *e.g.* , $\|\Delta(\theta)\|_\infty < 1$ is a classical robust stability condition. When the model $G(\theta)$ is obtained from an identification experiment it will lie in an uncertainty set. Hence a reasonable objective is therefore to design the identification experiment such that $\Delta(\theta)$ becomes small for all models in such an uncertainty set. We will consider the uncertainty set

$$U_\theta = \{\theta : N(\theta-\theta_o)^T P^{-1}(\Phi_u)(\theta-\theta_o) \le \chi\} \tag{4}$$

which defines a confidence region for the estimated parameters and where χ specifies the size of this confidence region [2]. This gives the following quality measure

$$|\Delta(\theta)| \le \gamma \,\forall\, \omega,\ \forall \theta \in U_\theta \tag{5}$$

This constraint implies that $\|\Delta\|_\infty \le \gamma$ for all models in the confidence region (4). Based on the quality measure (5), we will pose the input design problem as

$$\begin{aligned} \underset{\Phi_u}{\text{minimize}}\quad & \alpha \\ \text{subject to}\quad & |\Delta(\theta)| \le \gamma \quad \forall\, \omega \\ & N(\theta-\theta_o)^T P^{-1}(\Phi_u)(\theta-\theta_o) \le \chi \\ & \frac{1}{2\pi}\int_{-\pi}^{\pi} \Phi_u(\omega)d\omega \le \alpha \\ & 0 \le \Phi_u(\omega) \le \beta(\omega) \end{aligned} \tag{6}$$

The objective of this input design problem is to find the input spectrum with the least energy that satisfies (5). There may also exist a frequency by frequency constraint on the input spectrum here represented by $\beta(\omega)$. The input design problem is a non-convex and non-trivial optimization problem. However, by suitable parameterizations of the input spectrum it is possible to obtain a finite-dimensional convex program, see (Jansson and Hjalmarsson, 2004*b*) and (Jansson

[2] For more details on confidence regions, we refer to (Ljung, 1999).

and Hjalmarsson, 2004*a*). Here we will use one of these parameterizations given by

$$\Phi_u = \sum_{k=0}^{M-1} c_k \cos(k\omega) \tag{7}$$

This parametrization is rather flexible. For example, both power and frequency by frequency constraints on the input spectrum can be handled, which will be shown in the examples. Furthermore, any spectrum can be approximated to any demanded accuracy provided that the order M is sufficiently large. However, when M becomes too large, computational complexity becomes an issue. This parametrization was originally introduced in (Lindqvist and Hjalmarsson, 2001). Notice that (7) corresponds to white noise when $M = 1$.

The parameters in (6) are specified such that $|\Delta|$ has to be less than 0.1 for at least 95% of the estimated models. Hence $\gamma = 0.1$ and the size of the confidence region χ is determined such that $Pr(\chi^2(n) \leq \chi) = 0.95$ where n denotes the number of parameters in the model [3]. For example, $\chi = 9.49$ when $n = 4$. The frequency weighting T in (3) will be a discretization of $\tilde{T}$ or $\tilde{T}^2$ where

$$\tilde{T}(s) = \frac{\omega_0^2}{(s^2 + 2\xi\omega_0 s + \omega_0^2)} \tag{8}$$

with the damping $\xi = 0.7$ and where ω_0 will be used to change the bandwidth of $\tilde{T}$.

3. A PROCESS CONTROL APPLICATION

The main objective of this section is to give a flavor of the usefulness of using optimal input design for identification of models for process control design. The process plant is defined by (1) with the ARX structure

$$G_o(q) = \frac{B(q)}{A(q)} \quad H_o(q) = \frac{1}{A(q)} \tag{9}$$

where $A(q) = 1 - 1.511q^{-1} + 0.5488q^{-2}$ and $B(q) = 0.02059q^{-1} + 0.01686q^{-2}$. The sampling time is 10 seconds and $e(t)$ has variance 0.01. This is a a slight modification of a typical process control application considered in (Skogestad, 2003). The process has a rise time of 227 seconds, and consequently, collecting data samples for the identification takes long time. Therefore the objective of using optimal input design for this plant is to keep the experiment time to a minimum.

3.1 Optimal design compared to white input signals

Here we will compare the experiment time required for an optimal input design to achieve a certain quality constraint with the corresponding time required for a white noise input. The optimal input design will be based on knowledge of the true system. In reality, of course, this is not a feasible solution since the true system is unknown. However, the motivation for this analysis is to investigate what could *in the best case* be achieved with optimal input design.

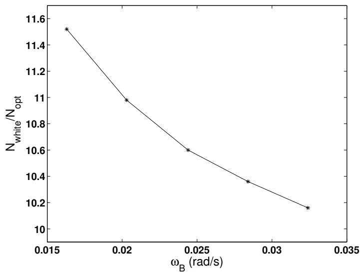

Fig. 1. The ratio $\frac{N_{white}}{N_{opt}}$ as function of the bandwidth of T for the process plant.

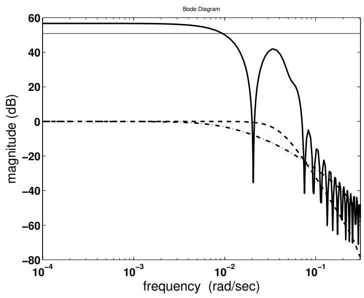

Fig. 2. The process plant. Thick solid line: optimal input spectrum. Dashed line: transfer function T. Dash-dotted line: open loop system. Thin solid line: white input spectrum.

The optimal design is based on a data length of $N_{opt} = 500$. The order of the input spectrum is $M = 30$, see (7). Furthermore, T is a discretization of $\tilde{T}^2$ in (8) and there is no frequency by frequency bound $\beta(\omega)$ on Φ_u. In the comparison we have normalized the power of the white input to be equal to the power of the obtained optimal input. The data length of the white input has then been increased until 95% of the obtained models satisfy $|\Delta| \leq 0.1$. This data length is denoted N_{white} and the ratio N_{white}/N_{opt} is plotted in Figure 1 for different bandwidths of T. This example shows that the white input requires about 10 times more data to satisfy the quality constraint, which is a quite substantial amount of data. In other words, the optimal experiment takes less than one and a half hour compared to almost 14 hours with a white input. The input spectra corresponding to high bandwidth of T are shown in Figure 2.

3.2 Optimal input design in practice

To handle the more realistic situation where the true system is unknown, we will replace the optimal design strategy by a two-step procedure. In the first step an initial model is estimated based on a PRBS input [4]. This model estimate is used as a replacement for the true system in the design problem (6). The obtained sub-optimal solution is then applied to the process in the second step. This adaptive approach is compared to an approach which only uses PRBS as input. The two strategies are illustrated in Figure 3. The main objective is to investigate whether there are any benefits of using a sub-optimal design approach or not. The comparison is based on Monte-Carlo simulations in order to study the average gain in performance.

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[3] $\chi^2(n)$ denotes the χ^2-distribution with n degrees of freedom.

[4] PRBS is a periodic, deterministic signal with white-noise-like properties.

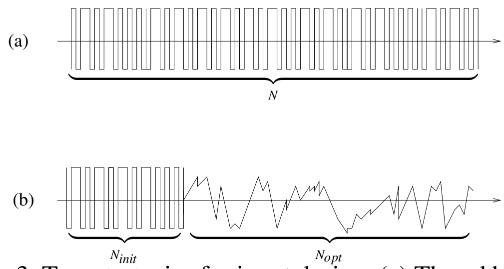

Fig. 3. Two strategies for input design. (a) The ad hoc solution. A PRBS is used for the entire identification experiment as input signal. (b) Adaptive input design. The input data set is split in two parts. The first part, a PRBS, is used for identification of an initial model estimate. The second part is an, *w.r.t* the initial model, optimally designed input.

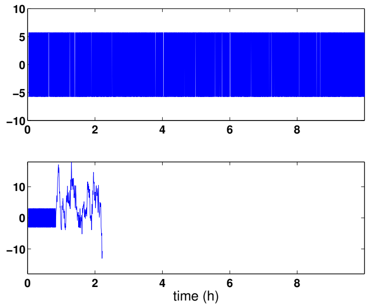

Fig. 4. The process plant. Above: the input sequence not involving optimal input design. Below: the input sequence when involving optimal input design. The first part of the signal is used to identify an initial model estimate.

First consider the two-step adaptive input design approach. We use a PRBS with length $N_{init} = 300$ and amplitude 3 to estimate an initial model estimate G_m of the true system. This model is used for input design based on (6) with no upper bound on the input spectrum and experiment length $N_{opt} = 500$. This strategy is compared to the approach where a single set of PRBS is used in each Monte-Carlo run. For the comparison's sake the amplitude of the the PRBS is tuned so that the signal has the same input power as the average power of the input in the two-step approach. After 1000 Monte-Carlo simulations with different noise realizations, 98.3% of the models with the two-step procedure satisfy $\|\Delta\|_\infty \leq 0.1$. With an experiment length of $N = 3600$, 96.4% of the models satisfy the constraint for the PRBS approach.

One realization of the input sequences for both strategies are plotted versus time in hours in Figure 4. We clearly see that the experiment time when input design is involved is less than 2 hours and 15 minutes, but more than 10 hours for the PRBS input. We conclude that for the considered quality constraint, the experiment time can be shortened substantially when the sub-optimal design is used.

4. A MECHANICAL SYSTEM APPLICATION

In this section, input design is applied to a resonant mechanical system. The system is represented by a slightly modified version of the half-load flexible transmission system proposed in (Landau *et al.*, 1995) as a benchmark problem for control design. It has been used for input design illustrations in (Bombois *et al.*, 2004*a*). The system is defined by the ARX structure (9) with $B(q) = 0.10276q^{-3} + 0.18123z^{-4}$ and $A(q) = 1 - 1.99185q^{-1} + 2.20265q^{-2} - 1.84083q^{-3} + 0.89413q^{-4}$ and $e(t)$ is white noise with variance 0.01. The sampling time is $T_s = 0.05$ seconds.

The experiment time is not an issue for this system. Therefore, the objective of the design is to obtain an input that, for a given data length, has as low excitation level as possible.

4.1 Optimal design compared to white input signals

The optimal design will be based on the true system, as the example in Section 3.1. Here T is a discretization of $\tilde{T}$ and the data length is 500 for the optimal design as well as for the white input. The reason is that we will compare excitation levels rather than experiment times. The order of the input spectrum is $M = 30$.

First, consider the optimal input design when there is no upper bound imposed on the input spectrum. The input power α_{opt} for the optimal input is plotted versus the bandwidth of T in Figure 5. It is clear that the input excitation increases with increasing bandwidth of T. This has to do with the definition of Δ. When the bandwidth increases the relative error around the first resonance peak starts to dominate in $\|\Delta\|_\infty$ and more input power has to be injected. The optimal spectra for low and high bandwidth, respectively, are plotted in Figure 6 and Figure 7. Here we see that the input power is concentrated around the first resonance peak for high bandwidths.

Let α_{white} be the required input power for a white input to achieve the specified model quality. The ratio $\alpha_{white}/\alpha_{opt}$ is plotted versus the bandwidth of T in Figure 5. We can conclude that when the total power is compared there are certainly benefits obtained using an optimal strategy compared to the white input, especially for high bandwidths where the white input requires about ten times the energy required for the optimal design. This is due to the large impact of the first resonance peak and the capability of the optimal design to distribute much energy around this peak and less for other frequencies.

The optimal input design method that is presented can handle frequency-wise conditions on the input spectrum, see $\beta(\omega)$ in (6). This possibility is now used. A frequency-wise upper bound on the input spectrum is included in the design problem (6) that restricts the possibility to inject energy around the first resonance peak. The upper bound and the obtained spectrum is shown in Figure 8. This is a good illustration of the impact of the first resonance peak on the required input power. With this restriction we need about 14 times more energy than without the bound. So there is a delicate trade-off between demanding a small relative model error around the resonance peak, the possibility to excite this frequency band and required total input power.

4.2 Input design in a practical situation

In this section, the parameters of the true system are assumed to be unknown. As for the process ap-

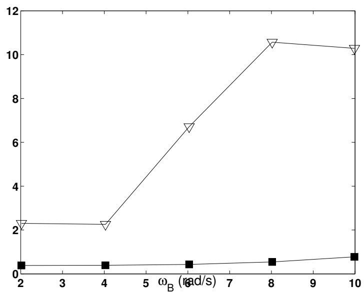

Fig. 5. The mechanical system. The input power α_{opt} (■) and the ratio $\alpha_{white}/\alpha_{opt}$ (▽) as functions of the bandwidth of T.

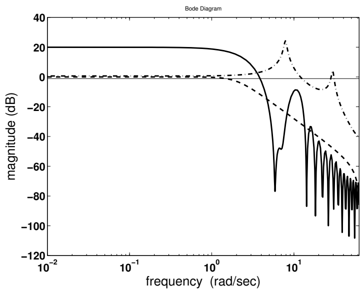

Fig. 6. The mechanical system, low bandwidth of T. Thick solid line: optimal input spectrum. Dashed line: transfer function T. Dash-dotted line: open loop system. Thin solid line: white input spectrum.

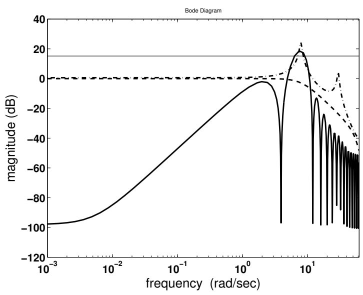

Fig. 7. The mechanical system, high bandwidth of T. Thick solid line: optimal input spectrum. Dashed line: transfer function T. Dash-dotted line: open loop system. Thin solid line: white input spectrum.

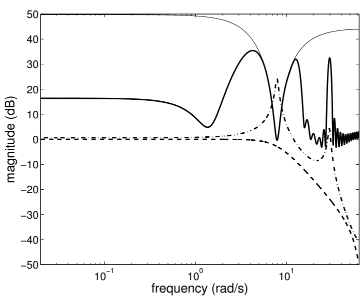

Fig. 8. The mechanical system, high bandwidth of T. Thick solid line: optimal input spectrum. Dashed line: transfer function T. Dash-dotted line: open loop system. Thin solid line: upper bound on Φ_u.

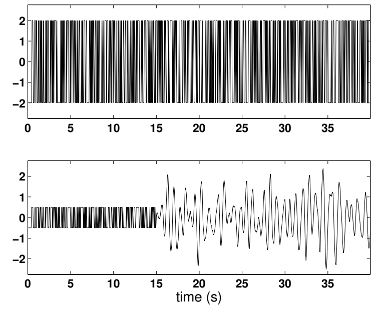

Fig. 9. The mechanical system. Above: the input sequence not involving optimal input design. Below: the input sequence when involving optimal input design. The first part of the signal is used to identify an initial model estimate. The second part is the optimal input signal.

plication we will handle this situation by replacing the optimal design procedure by an adaptive two-step approach. For the mechanical system, the two-step adaptive approach goes as follows. A PRBS with length $N_{init} = 300$ and amplitude 0.5 is used for the estimation of the initial model G_m. An input is designed based on G_m using (6) with $N_{opt} = 500$. This strategy is compared with the approach using a single set of PRBS data of length 800, *i.e.* the data lengths are equalized. The signal power of the PRBS is set to 6 times that of the two-stage sub-optimal input in each Monte-Carlo run. One realization of the input sequences for both strategies are plotted versus time in Figure 9. For 1000 Monte-Carlo simulations, 92.3% of the obtained models from the two-step procedure passed the constraint $\|\Delta\|_\infty \leq 0.1$. The corresponding figure for the PRBS approach was 91.8%. Thus the input excitation for the PRBS approach needs to be about *6 times* the optimal input power to produce equally good models. We conclude that for the given quality constraint, the excitation level of the input signal can be reduced significantly using the illustrated sub-optimal input design.

4.3 Sensitivity of the optimal design

It was illustrated in the previous section that the input design performed well even when the true system was replaced by an estimated model. However, some caution must be taken. In this section we will illustrate, by means of an example, that the optimal input design method may be sensitive to the quality of the initial model.

Let us again consider the mechanical system. Assume that the true system is unknown, but that we have a preliminary model G_m that deviates from the true system G_o. The magnitude plots of G_m and G_o are shown in Figure 10. Now assume that the true system is replaced with G_m in the design problem (6). The resulting sub-optimal input spectrum is plotted in Figure 10 together with the optimal input spectrum based on G_o. The bandwidth of T is 8 rad/s.

We see from Figure 10 that the input power is differently distributed for the sub-optimal design compared to the optimal one. However, the energy is in both cases concentrated around the first resonance peak. This is completely in line with the observations in

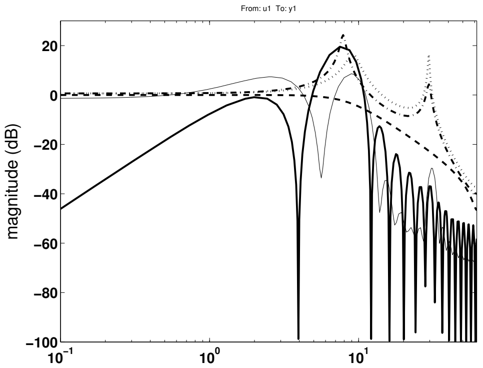

Fig. 10. The mechanical system. Thick solid line: the optimal input spectrum. Thin solid line: the sub-optimal input spectrum (see text). Dashed line: weight function T. Dotted line: the model. Dash dotted line: the true system.

Section 4.1 where it was recognized that it is effective to inject energy around the first resonance peak for high bandwidths of T. However, an input design that concentrates the power around a resonance peak may be very vulnerable with respect to bad model estimates of this peak. The model G_m is one such example. For example, out of 100 Monte-Carlo identification experiments, only 23 of the obtained models with the sub-optimal design achieve the quality constraint $\|\Delta\|_\infty < 0.1$. Optimally it should be at least 95%. We conclude that it is important that the initial experiment captures the resonance peaks of importance. The reason why the sub-optimal method in Section 4.2 performed well is probably that the initial experiment with the PRBS signal excited these peaks yielding proper initial model estimates.

5. CONCLUSIONS

In this paper we have illustrated and quantified possible benefits with optimal input design in identification for two applications. We have compared optimally designed input signals with white input signals. The results show significant benefits with optimal input design. Either the experiment time can be shortened or the input power can be reduced. Through Monte-Carlo simulations it is illustrated that there are advantages also in the case where the true system is replaced by a model estimate in the design.

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