STATE FEEDBACK STABILIZATION OF A CLASS OF NONLINEAR DISCRETE-TIME DELAY SYSTEMS

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Abstract:
In this note we consider the problem of state feedback stabilization of a class of nonlinear discrete-time delay systems. From an appropriate Lyapunov function and judicious mathematical manipulations, we deduce a simple LMI condition (that may be checked easily) to ensure asymptotic stabilization. On the other hand, we provide two explicit state feedback laws that may be seen as a generalization of the existing results on the stabilization of nonlinear systems. The approach developed in this note is simple (without state augmentation) and efficient.

Keywords: Nonlinear systems, time-delay systems, discrete-time systems, stabilization, state feedback, bounded state feedback, LMI.

1. INTRODUCTION

Over the last two decades, tremendous research activities were focused on analysis and synthesis of control design for time delay systems, we may refer the reader to (Mahmoud, 1999), (Fridman, 2001), (Fridman and Shaked, 2002), (Li and de Souza, 1997), (Dugard and Verriest, 1997), (Richard, 2003), (Trinh and Aldeen, 1997), (Boutayeb and Darouach, 2001) and the references therein.

This interest is due to the great practical and theoretical importance of such kind of systems. Indeed, delays are typical in a large range of industrial processes like chemical or teleoperation systems. We notice however that, since most physical processes evolve naturally in continuous-time, the major results were developed for continuous-time systems. Little attention has been drawn to the discrete-time case and even less to nonlinear models.

For linear discrete-time delay systems, one of the pioneering work on control design has been established by Aström and Wittenmark (Aström and Wittenmark, 1984). This approach transforms the delay difference system to a higher-order one without delays. However for systems with large delays, the proposed scheme will invariably lead to large scale systems. Furthermore, for systems with unknown or time-varying delays the proposed technique can not be applied.

An alternative approach was proposed in (Verriest and Ivanov, 1995), where a Riccati equation is developed, for analyzing the stability of discrete-time systems with arbitrary unknown delay. An extention of these results was proposed in (Kapila and Haddad, 1998), where sufficient conditions for H_∞ state feedback control of discrete-time systems with state delay was provided.

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Several Riccati equations giving sufficient stability conditions have been formulated in (Kolmanovski *et al.*, 1999). The various matrix Riccati equations have the same dimension as the state vector. This is an advantage for high order systems.

In (Fridman and Shaked, 2003) a delay-dependent and independent conditions have been derived for determining the asymptotic stability of discrete-time systems with uncertain delay, time-varying delay and norm-bounded uncertainties.

Most of the works presented below are for linear systems. In the case of nonlinear systems, the problem of state feedback stabilization for delay systems remains poorly studied and challenging (e.g. (Mao, 1996), (Guay and Li, 2002), (Jankovic, 2001), (Fu *et al.*, 2003), (Fridman, 2003) and the references therein).

This paper is devoted to the problem of state feedback and bounded state feedback stabilization of a class of nonlinear discrete-time systems with delays. We provide two explicit state feedback control laws that may be seen as a generalization of the Jurdjevic-Quinn controller and passive theory results in the nonlinear field ((Lin, 1995), (Lin and Byrnes, 1994), (Boutayeb *et al.*, 2002), (Bouazza *et al.*, 2004*b*) and (Bouazza *et al.*, 2004*a*)).

From an appropriate Lyapunov function and sufficient conditions, expressed in terms of a simple LMI, an asymptotic stabilization of a class of nonlinear discrete-time delay systems was achieved through two different controllers; a state feedback controller and a bounded state feedback controller.

2. PROBLEM FORMULATION

Consider the following discrete-time delay system

$$x_{k+1} = Ax_k + A_d x_{k-h} + g(x_k, x_{k-h})u_k \quad (1)$$

where $x_k \in \mathbf{R}^n$ and $u_k \in \mathbf{R}^m$ denote the state and input vectors respectively at time instant k. A and A_d are constant matrices of appropriate dimensions. $g(x_k, x_{k-h})$ is a nonlinear map of appropriate dimension and h is a known positive number representing the delay. For simplicity of notations, we replace $g(x_k, x_{k-h})$ by g_k in the sequel.

Our goal in this paper is to provide a control law that ensures the asymptotic stabilization of the system (1).

The unforced dynamics are governed by

$$x_{k+1} = Ax_k + A_d x_{k-h} \quad (2)$$

We assume that, possibly after using a smooth feedback, the unforced dynamic system (2) is Lyapunov stable.

Lemma 1. (Bouazza *et al.*, 2004*a*) A sufficient condition for (2) to be Lyapunov stable is that there exists an $n \times n$ positive-definite matrix P and an $n \times n$ nonnegative-definite matrix Q, such that

$$\text{H1)} \begin{bmatrix} P - A^T PA - Q & A^T PA_d \\ A_d^T PA & M \end{bmatrix} \geq 0$$

where

$$M = Q - A_d^T PA_d > 0.$$

□

Before proceeding, let us define the sets

$$\Omega = \{x_k \in \mathbb{R}^n : x_k^T \left(A^T PA - P + Q + A^T P \times A_d M^{-1} A_d^T PA\right) x_k = 0,\ k = 0, 1, \ldots\}$$

$$S1 = \{x_k \in \mathbb{R}^n : g^T(x_k, x_{k-h}) \times PAx_k = 0,\ k = 0, 1, \ldots\}$$

$$S2 = \{x_k \in \mathbb{R}^n : g^T(x_k, x_{k-h}) PA_d x_{k-h} = 0,\ k = h, h{+}1, \ldots\}$$

$$H = \{x_k \in \mathbb{R}^n : A_d^T PAx_k - (Q - A_d^T PA_d)x_{k-h} = 0,\ k = h, h{+}1, \ldots\}$$

3. STATE FEEDBACK STABILISATION

Theorem 2. Suppose that there exists an $n \times n$ positive-definite matrix P and an $n \times n$ nonnegative-definite matrix Q, such that H1) holds.

If $\Omega \cap S1 \cap S2 \cap H = \{0\}$, then the nonlinear discrete-time delay system (1) is globally asymptotically stabilized by the following state feedback

$$\begin{aligned} u(x_k, x_{k-h}) &= -K_1 x_k - K_2 x_{k-h} \\ &= -\left[I + g_k^T P g_k\right]^{-1} g_k^T PAx_k \\ &\quad - \left[I + g_k^T P g_k\right]^{-1} g_k^T PA_d x_{k-h} \end{aligned} \quad (3)$$

□

Proof

To show the stability of the closed-loop system (1)-(3), we consider the following Lyapunov function

$$V_k = x_k^T P x_k + \sum_{i=k-h}^{k-1} x_i^T Q x_i \qquad (4)$$

Notice that, since P is positive definite and Q nonnegative definite, V_k is then positive definite.

The difference of this Lyapunov function along the trajectory of the closed-loop (1)-(3) is given by

$$\begin{aligned}\Delta V_k &= V_{k+1} - V_k \\ &= x_{k+1}^T P x_{k+1} + \sum_{i=k+1-h}^{k} x_i^T Q x_i \\ &\quad - x_k^T P x_k - \sum_{i=k-h}^{k-1} x_i^T Q x_i \qquad (5)\end{aligned}$$

or equivalently,

$$\begin{aligned}\Delta V_k &= x_{k+1}^T P x_{k+1} + x_k^T Q x_k \\ &\quad - x_k^T P x_k - x_{k-h}^T Q x_{k-h} \qquad (6)\end{aligned}$$

Now, using the equation (1), we have

$$\begin{aligned}\Delta V_k &= [A x_k + A_d x_{k-h} + g_k u_k]^T P \\ &\quad \times [A x_k + A_d x_{k-h} + g_k u_k] + x_k^T Q x_k \\ &\quad - x_k^T P x_k - x_{k-h}^T Q x_{k-h} \qquad (7)\end{aligned}$$

which is also

$$\begin{aligned}\Delta V_k &= x_k^T A^T P A x_k + 2 x_k^T A^T P g_k u_k \\ &\quad + u_k^T g_k^T P g_k u_k - x_k^T P x_k + x_k^T Q x_k \\ &\quad + 2 x_k^T A^T P A_d x_{k-h} + x_{k-h}^T A_d^T P A_d x_{k-h} \\ &\quad + 2 x_{k-h}^T A_d P g_k u_k - x_{k-h}^T Q x_{k-h} \qquad (8)\end{aligned}$$

then

$$\begin{aligned}\Delta V_k &= x_k^T [A^T P A - P + Q] x_k + 2 x_k^T A^T P A_d x_{k-h} \\ &\quad + 2 x_k^T A^T P g_k u_k + 2 x_{k-h}^T A_d^T P g_k u_k \\ &\quad + u_k^T g_k^T P g_k u_k - x_{k-h} M x_{k-h} \qquad (9)\end{aligned}$$

where $M = Q - A_d^T P A_d$.

Since

$$g_k^T P g_k = (I + g_k^T P g_k) - I$$

Then, equation (9) can be written in the following form

$$\begin{aligned}\Delta V_k &= x_k^T [A^T P A - P + Q] x_k \\ &\quad + 2 x_k^T A^T P A_d x_{k-h} - x_{k-h} M x_{k-h} \\ &\quad + 2 x_k^T A^T P g_k u_k + 2 x_{k-h}^T A_d^T P g_k u_k \\ &\quad + u_k^T (I + g_k^T P g_k) u_k - u_k^T u_k \qquad (10)\end{aligned}$$

Using the control law (3), we get

$$\begin{aligned}\Delta V_k &= x_k^T [A^T P A - P + Q] x_k \\ &\quad + 2 x_k^T A^T P A_d x_{k-h} - x_{k-h} M x_{k-h} \\ &\quad + 2 x_k^T A^T P g_k u_k + 2 x_{k-h}^T A_d^T P g_k u_k \\ &\quad + x_k^T A^T P g_k (I + g_k^T P g k)^{-1} g_k^T P A x_k \\ &\quad + 2 x_k^T A^T P g_k (I + g_k^T P g k)^{-1} g_k^T P A_d x_{k-h} \\ &\quad + x_{k-h}^T A_d^T P g_k (I + g_k^T P g_k)^{-1} \\ &\quad \times g_k^T P A_d x_{k-h} - u_k^T u_k \qquad (11)\end{aligned}$$

which is equivalent to

$$\begin{aligned}\Delta V_k &= x_k^T [A^T P A - P + Q] x_k \\ &\quad + 2 x_k^T A^T P A_d x_{k-h} - x_{k-h} M x_{k-h} \\ &\quad - 2 x_k^T A^T P g_k (I + g_k^T P g_k)^{-1} g_k^T P A x_k \\ &\quad - 4 x_k^T A^T P g_k (I + g_k^T P g_k)^{-1} g_k^T P A_d x_{k-h} \\ &\quad - 2 x_{k-h}^T A_d^T P g_k (I + g_k^T P g_k)^{-1} g_k^T P A_d x_{k-h} \\ &\quad + x_k^T A^T P g_k (I + g_k^T P g k)^{-1} g_k^T P A x_k \\ &\quad + 2 x_k^T A^T P g_k (I + g_k^T P g k)^{-1} g_k^T P A_d x_{k-h} \\ &\quad + x_{k-h}^T A_d^T P g_k (I + g_k^T P g k)^{-1} g_k^T P A_d x_{k-h} \\ &\quad - u_k^T u_k \qquad (12)\end{aligned}$$

This, in turn, implies

$$\begin{aligned}\Delta V_k &= x_k^T [A^T P A - P + Q] x_k \\ &\quad + 2 x_k^T A^T P A_d x_{k-h} - x_{k-h} M x_{k-h} \\ &\quad - x_k^T A^T P g_k (I + g_k^T P g_k)^{-1} g_k^T P A x_k \\ &\quad - 2 x_k^T A^T P g_k (I + g_k^T P g_k)^{-1} g_k^T P A_d x_{k-h} \\ &\quad - x_{k-h}^T A_d^T P g_k (I + g_k^T P g_k)^{-1} g_k^P A_d x_{k-h} \\ &\quad - u_k^T u_k \qquad (13)\end{aligned}$$

or equivalently

$$\begin{aligned}\Delta V_k &= x_k^T [A^T P A - P + Q] x_k \\ &\quad + 2 x_k^T A^T P [A_d - g_k K_2] x_{k-h} - x_{k-h} M x_{k-h} \\ &\quad - x_k^T A^T P g_k (I + g_k^T P g_k)^{-1} g_k^T P A x_k \\ &\quad - x_{k-h}^T A_d^T P g_k (I + g_k^T P g_k)^{-1} g_k^T P A_d x_{k-h} \\ &\quad - u_k^T u_k \qquad (14)\end{aligned}$$

Let $\tilde{A} = A_d - g_k K_2$, then equation (14) becomes

$$\begin{aligned}\Delta V_k &= x_k^T [A^T P A - P + Q] x_k \\ &\quad + 2 x_k^T A^T P \tilde{A} x_{k-h} - x_{k-h} M x_{k-h} \\ &\quad - x_k^T A^T P g_k (I + g_k^T P g_k)^{-1} g_k^T P A x_k \\ &\quad - x_{k-h}^T A_d^T P g_k (I + g_k^T P g_k)^{-1} g_k^T P A_d x_{k-h} \\ &\quad - u_k^T u_k \qquad (15)\end{aligned}$$

Next, adding and subtracting $x_k^T A^T P \tilde{A} M^{-1} \tilde{A}^T P A x_k$ to and from the equation (15), we have

$$\Delta V_k = x_k^T[A^TPA - P + Q + A^TP\tilde{A} \\ \times M^{-1}\tilde{A}^TPA]x_k + 2x_k^TA^TP\tilde{A}x_{k-h} \\ - x_kA^TP\tilde{A}M^{-1}\tilde{A}^TPAx_k - x_{k-h}Mx_{k-h} \\ - x_k^TA^TPg_k(I + g_k^TPg_k)^{-1}g_k^TPAx_k \\ - x_{k-h}^TA_d^TPg_k(I + g_k^TPg_k)^{-1}g_k^TPA_dx_{k-h} \\ - u_k^Tu_k \quad (16)$$

Thus, a simple manipulation yields

$$\Delta V_k = x_k^T[A^TPA - P + Q + A^TP\tilde{A}M^{-1}\tilde{A}^T \\ \times PA]x_k - [M^{-\frac{1}{2}}\tilde{A}^TPAx_k - M^{\frac{1}{2}}x_{k-h}]^T \\ \times [M^{-\frac{1}{2}}\tilde{A}^TPAx_k - M^{\frac{1}{2}}x_{k-h}] \\ - x_k^TA^TPg_k(I + g_k^TPg_k)^{-1}g_k^TPAx_k \\ - x_{k-h}^TA_d^TPg_k(I + g_k^TPg_k)^{-1}g_k^TPA_dx_{k-h} \\ - u_k^Tu_k \quad (17)$$

A sufficient condition to have $\Delta V_k \leq 0$, is that

$$A^TPA - P + Q + A^TP\tilde{A}M^{-1}\tilde{A}^TPA \leq 0 \quad (18)$$

Note that

$$\tilde{A} = A_d - g_kK_2$$
$$\tilde{A} = A_d - g_k(I + g_k^TPg_k)^{-1}g_k^TPA_d$$
$$\tilde{A} = \left(I - g_k(I + g_k^TPg_k)^{-1}g_k^TP\right)A_d$$
$$\tilde{A} = P^{-1}\left(P - Pg_k(I + g_k^TPg_k)^{-1}g_k^TP\right)A_d$$

Since $Pg_k(I + g_k^TPg_k)^{-1}g_k^TP$ is positive definite, we have

$$P - Pg_k(I + g_k^TPg_k)^{-1}g_k^TP \leq P$$

and then, we can deduce that

$$A^TPA - P + Q + A^TP\tilde{A}M^{-1}\tilde{A}^TPA \\ \leq A^TPA - P + Q + A^TPA_dM^{-1}A_d^TPA \quad (19)$$

Trough the hypothesis H1) we have, using the Schur complement,

$$P - A^TPA - Q - A^TPA_dM^{-1}A_d^TPA \geq 0$$

or equivalently,

$$A^TPA - P + Q + A^TPA_dM^{-1}A_d^TPA \leq 0$$

and therefore

$$\Delta V_k \leq 0$$

Thus, the Lyapunov stability of the closed-loop (1)-(3) is proved.

To show the asymptotic stability of the origin, it suffices to show that the largest subset of $\Delta V_k = 0$ invariant under closed-loop dynamics is $\{0\}$.

Setting $\Delta V_k = 0$, it follows from (17) that

$$x_k^T[A^TPA - P + Q + \tilde{A}^TPA_dM^{-1}A_d^TP\tilde{A}]x_k = 0 \quad (20)$$

$$g^T(x_k, x_{k-h})PAx_k = 0 \quad (21)$$

$$g^T(x_k, x_{k-h})PA_dx_{k-h} = 0 \quad (22)$$

$$M^{-1/2}A_d^TP\tilde{A}x_k - M^{1/2}x_{k-h} = 0 \quad (23)$$

$$u(x_k) = 0 \quad (24)$$

using (24), equations (20), (21), (22) and (23) becomes

$$x_k^T[A^TPA - P + Q + A^TPA_dM^{-1}A_d^TPA]x_k = 0 \quad (25)$$

$$g^T(x_k, x_{k-h})PAx_k = 0 \quad (26)$$

$$g^T(x_k, x_{k-h})PA_dx_{k-h} = 0 \quad (27)$$

$$A_d^TPAx_k - (Q - A_d^TPA_d)x_{k-h} = 0 \quad (28)$$

Thus, we can conclude from the assumption $\Omega \cap S1 \cap S2 \cap H = \{0\}$ that $\Delta V(x_k) = 0$, for $k = 0, 1, \ldots$ implies $x_k \equiv 0$. We can conclude that the asymptotic stability is proved because all the conditions of LaSalle's invariance principle are verified.

Therefore, the origin is an asymptotically stable equilibrium of the closed loop-system (1)-(3) since $V(x_k) \to \infty$ as $\|x_k\| \to \infty$. ■

4. BOUNDED STATE FEEDBACK STABILISATION

Now we present the second result of this paper; it concerns the stabilization of systems of the form of (1) using a bounded state feedback.

The interest of this second result is that, for this class of systems, it is useful to have a bounded control to design observers or observer-based controllers (Lin, 1995; Bouazza *et al.*, 2004*a*).

Theorem 3. Suppose that there exists an $n \times n$ positive-definite matrix P and an $n \times n$ nonnegative-definite matrix Q, such that H1) holds.

If $\Omega \cap S1 \cap S2 \cap H = \{0\}$, then the nonlinear discrete-time delay system (1) is globally asymptotically stabilized by the following bounded state feedback

$$\begin{aligned} u_\alpha(x_k, x_{k-h}) &= -L_1 x_k - L_2 x_{k-h} \\ &= -\alpha_1 \left[I + g_k^T P g_k\right]^{-1} \frac{g_k^T P A x_k}{1 + \|g_k^T P A x_k\|} \\ &\quad -\alpha_2 \left[I + g_k^T P g_k\right]^{-1} \frac{g_k^T P A_d x_{k-h}}{1 + \|g_k^T P A_d x_{k-h}\|} \quad (29) \end{aligned}$$

(for any $0 < \alpha_1 < 1$ and $0 < \alpha_2 < 1$)

□

Proof

Set

$$\gamma_1 = \frac{\alpha_1}{1 + \|g_k^T P A x_k\|}$$

and

$$\gamma_2 = \frac{\alpha_2}{1 + \|g_k^T P A_d x_{k-h}\|},$$

then the control bounded state feedback can also be written

$$u_\alpha(x_k) = -\gamma_1 K_1 x_k - \gamma_2 K_2 x_{k-h} \quad (30)$$

where K_1 and K_2 are defined as in (3).

Using the same Lyapunov-Krasovskii function (4), the control law (29) and after some matrix manipulations, we get

$$\begin{aligned} \Delta V_k &= x_k^T [A^T P A - P + Q] x_k \\ &+ 2 x_k^T A^T P [A_d - \gamma_2 g_k K_2] x_{k-h} - x_{k-h} M x_{k-h} \\ &- 2\gamma_1 x_k^T A^T P g_k K_1 x_k + \gamma_1^2 x_k^T A P g_k K_1 x_k \\ &- 2\gamma_1 x_{k-h}^T A_d^T P g K_1 x_k + \gamma_1 \gamma_2 x_{k-h}^T A_d^T P g_k K_1 x_k \\ &- 2\gamma_2 x_{k-h}^T A_d^T P g K_2 x_{k-h} + \gamma_2^2 x_{k-h}^T A_d \\ &\times P g_k K_2 x_{k-h} - u_k^T u_k \quad (31) \end{aligned}$$

with $M = Q - A_d^T P A_d$.

From the equation (31), since $0 < \alpha_1 < 1$ and $0 < \alpha_2 < 1$, we obtain the following inequality

$$\begin{aligned} \Delta V_k &\le x_k^T [A^T P A - P + Q] x_k \\ &+ 2 x_k^T A^T P \hat{A} x_{k-h} - x_{k-h} M x_{k-h} \quad (32) \end{aligned}$$

where $\hat{A} = A_d - \gamma_2 g_k K_2$.

Adding and subtracting $x_k^T A^T P \hat{A} M^{-1} \hat{A}^T P A x_k$ to and from the inequality (32), we have

$$\begin{aligned} \Delta V_k &\le x_k^T [A^T P A - P + Q + A^T P \hat{A} M^{-1} \hat{A}^T P A] x_k \\ &+ 2 x_k^T A^T P \hat{A} x_{k-h} - x_{k-h} M x_{k-h} \\ &- x_k^T A^T P \hat{A} M^{-1} \hat{A}^T P A x_k \quad (33) \end{aligned}$$

then

$$\begin{aligned} \Delta V_k &\le x_k^T [A^T P A - P + Q + A^T P \hat{A} M^{-1} \hat{A}^T P A] x_k \\ &- [M^{-\frac{1}{2}} \hat{A}^T P A x_k - M^{\frac{1}{2}} x_{k-h}]^T \\ &\times [M^{-\frac{1}{2}} \hat{A}^T P A x_k - M^{\frac{1}{2}} x_{k-h}] \quad (34) \end{aligned}$$

We deduce that

$$\begin{aligned} \Delta V_k &\le x_k^T [A^T P A - P + Q \\ &+ A^T P \hat{A} M^{-1} \hat{A}^T P A] x_k \quad (35) \end{aligned}$$

A sufficient condition to have $\Delta V_k \le 0$ is

$$A^T P A - P + Q + A^T P \hat{A} M^{-1} \hat{A}^T P A \le 0 \quad (36)$$

Let us compute $\hat{A}$.

$$\begin{aligned} \hat{A} &= A_d - \gamma_2 g_k K_2 \\ \hat{A} &= A_d - \gamma_2 g_k (I + g_k^T P g_k)^{-1} g_k^T P A_d \\ \hat{A} &= \left(I - \gamma_2 g_k (I + g_k^T P g_k)^{-1} g_k^T P\right) A_d \\ \hat{A} &= P^{-1} \left(P - P \gamma_2 g_k (I + g_k^T P g_k)^{-1} g_k^T P\right) A_d \end{aligned}$$

Since $P - P\gamma_2 g_k (I + g_k^T P g_k)^{-1} g_k^T P \le P$ we conclude that

$$\begin{aligned} & A^T P A - P + Q + A^T P \hat{A} M^{-1} \hat{A}^T P A \\ &\le A^T P A - P + Q + A^T P A_d M^{-1} A_d^T P A \quad (37) \end{aligned}$$

So, if H1) is verified, then

$$\Delta V_k = V_{k+1} - V_k \le 0$$

This prove that the closed loop system is Lyapunov stable.

We obtain the asymptotic stability using the hypothesis $\Omega \cap S1 \cap S2 \cap H = \{0\}$, as in the proof of Theorem 2.

Therefore, the origin is an asymptotically stable equilibrium of the closed loop-system (1)-(29) since $V(x_k)$ is proper. ■

Remark 4. The condition

$$\Omega \cap S1 \cap S2 \cap H = \{0\}$$

seems to be a controllability-like rank condition in the stabilization of nonlinear systems. For standard systems, this statement is proved using the passivity theory approach (Byrnes *et al.*, 1991)(Byrnes *et al.*, 1993). We have not extend this point to the delay systems yet.

5. CONCLUSION

The problem of state feedback stabilization of a class of nonlinear discrete-time systems with delays has been studied. A state feedback and a bounded state feedback control laws, based on a LMI sufficient conditions, have been developed. This control laws ensure the stability of the resulting closed-loop systems.

These results are an extension to delay systems of the well known Jurdjevic-Quinn approach and the passivity theory approach established for nonlinear systems.

The approach developed in this note is simple (without state augmentation) and efficient. The use of this approach in the design of an observer based controller for such kind of systems is under study.

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