BICRITERIAL DUAL CONTROL WITH MULTIPLE LINEARIZATION

Miroslav Flídr and Miroslav Šimandl

Department of Cybernetics and
Research Centre: Data - Algorithms - Decision
University of West Bohemia in Pilsen
Univerzitní 8, 30614 Plzeň, Czech Republic
e-mail: flidr@kky.zcu.cz, simandl@kky.zcu.cz

Abstract: A suboptimal dual controller for discrete stochastic systems with unknown parameters based on the bicriterial approach is proposed and discussed. It is supposed that all the random quantities are non-Gaussian. This assumption induces that a global estimation method has to be used. The Gaussian sum method with multiple linearization technique was chosen and applied in the bicriterial control approach. The probing part of the control law is determined for each local node of estimated probability density function separately and respects accuracy of each local estimate inherent in the estimated probability density function. A comparison of the proposed modified bicriterial controller and the bicriterial controller which uses global point estimate only is shown in some numerical examples.

Keywords: adaptive control, dual control, nonlinear filtering, stochastic systems, state and parameter estimation

1. INTRODUCTION

In many control problems it is necessary to cope with insufficient knowledge of some of the system parameters. Such a problem leads to adaptive control which provides instruments for simultaneous control of the system and estimation of unknown quantities. The simplest approaches use the separation principle with imposing the certainty equivalence property, and thus omitting the stochastic character of the problem, or lead to cautious control. However, most of the problems lead to the controller with inherent dual properties. Such a controller has to meet two conflicting goals. It should meet the control objective and at the same time to improve knowledge about unknown parameters (Feldbaum, 1965).

Unfortunately for such problem closed form solution cannot be mostly analytically found. Many suboptimal solutions providing dual control properties were proposed. However, they don't provide universal tools for solving the problem. They are valid for limited class of considered systems only or impose some restrictions. The necessity of lowering the computational burden leads mostly to one step ahead minimization accompanied by enhancement of the criteria (Wittenmark, 1975; Milito *et al.*, 1982) or by augmenting the resulting control with some arbitrary probing signal (Alster and Bélanger, 1974). Other dual control approaches are based upon application of suitable approximation (Sternby, 1976; Maitelli and Yoneyama, 1994; Pronzato *et al.*, 1996; Lindoff *et al.*, 1999) and can be mostly very computationally intensive (Birmiwal and Bar-Shalom, 1984).

A promising approach seems to be the bicriterial controller based on application of two criteria each reflecting one of the conflicting goals of the dual control (Filatov *et al.*, 1997). This controller is not only computationally moderate but also practically realizable

(Filatov *et al.*, 1996; Filatov, 1998). As it was shown in Šimandl and Flídr (2001) it is possible to use this controller for state space systems with time varying parameters where all the stochastic quantities can be suggested as non-Gaussian. In such a case it is desirable to employ a suitable global estimation method such as the Gaussian sum method (GSM) (Šimandl and Flídr, 1997; Alspach and Sorenson, 1971) that can ensure better quality of the state and parameter estimates by respecting the nonlinear nature of the identification problem.

Goal of this paper is to design an alternative bicriterial controller fully utilizing the character of the GSM. The intention is to change the control law in such a way that the controller makes use of the whole information provided by the estimated probability density function (pdf) and not just a point estimate and to analyze probing signal and quality of the proposed bicriterial dual controller.

The following section deals with the formulation of the problem and description of a suboptimal dual controller derived using the bicriterial approach. Section 3 is then dedicated to description of the bicriterial controller with multiple linearization and Section 4 to numerical illustrations of its properties and for comparison with the bicriterial controller which utilizes global estimates only. Eventually, the results are summarized in Section 5.

2. PROBLEM STATEMENT

Consider the discrete time stochastic system

$$s_{k+1} = A(\theta_k)s_k + B(\theta_k)u_k + w_k, \quad (1)$$

$$\theta_{k+1} = \Phi_k\theta_k + \epsilon_k, \quad (2)$$

$$y_k = Cs_k + v_k, \quad (3)$$

where $k = 0, 1, \dots, N-1$ denotes time instants, $s_k \in \mathbb{R}^n$ represents the state vector of the controlled system, $\theta_k \in \mathbb{R}^p$ the vector of parameters, $y_k \in \mathbb{R}^m$ the measurement vector of the controlled system, $u_k \in \mathbb{R}^r$ is the control input vector and $w_k \in \mathbb{R}^n$, $\epsilon_k \in \mathbb{R}^p$, $v_k \in \mathbb{R}^m$ are the vectors of independent random quantities. The elements of the matrices $A(\theta_k)$ and $B(\theta_k)$ are known linear functions of the unknown random parameters θ_k. Dimensions of the matrices $A(\theta_k)$ and $B(\theta_k)$ correspond with dimensions of the state vector s_k and the control input u_k, respectively. The matrices C and Φ_k are know and have appropriate dimensions. The random processes $\{w_k\}$, $\{\epsilon_k\}$ and $\{v_k\}$ are white mutually independent sequences and independent of the random quantities s_0 and θ_0 as well. The probability density function (pdf) of the random quantities s_0, θ_0, w_k, ϵ_k, v_k $\forall k$ are supposed to be known.

As it was mentioned earlier, each objective of the controller will be reflected by a separate criterion. The first used criterion, concerning the control objective, is suggested in the following form

$$J_k^c(u_k) = E\left\{(y_{k+1} - \bar{y}_{k+1})^T V_{k+1}(y_{k+1} - \bar{y}_{k+1}) + u_k^T W_k u_k \Big| \Im_k\right\}, \quad (4)$$

where $\bar{y}_{k+1} \in \mathbb{R}^m$ is the setpoint vector. The positive semidefinite matrix V_{k+1} and the positive definite matrix W_k are suitably selected symmetric matrices. $\Im_k = (u_0^{k-1}, y_0^k)$ represents information available up to the time instant k. Besides having to rate the control quality, the criterion (4) is designed to enable analytical derivation of the control law. The control found using (4) is so called *caution control* which can consider, to a certain degree, uncertainty in knowledge of the nonmeasurable parameters and states of the controlled system. Uncertainty is respected only partially due to the fact that the controller can take into account, in probabilistic sense, only the next time instant.

The control gained using (4) reflects only one aspect of dual control. In order to obtain the second one which would embody the so-called *active learning* property, the second criterion will be used. It rates quality of estimation process. This criterion is chosen as

$$J_k^e(u_k) = E\left\{(y_{k+1} - \hat{y}_{k+1})^T \mathcal{V}_{k+1} \times (y_{k+1} - \hat{y}_{k+1}) \Big| \Im_k\right\}, \quad (5)$$

where $\hat{y}_{k+1} \in \mathbb{R}^m$ is the one step ahead prediction vector of the controlled system output and the matrix $\mathcal{V}_{k+1}$ is positively definite. The sought control input u_k is such as to maximize criterion (5) on a specified domain. This domain is set by choosing convenient coupling condition between both criteria so that feasible compromise between these two criteria is ensured without necessity to employ multicriterial optimization methods which are usually computationally demanding.

Outgoing from the character of dual control that consists of two components, caution and active learning, it is reasonable to search the extreme of the criterion (5) in suitably selected domain Ω_k

$$\Omega_k = [u_k^c - \delta_k, u_k^c + \delta_k], \quad (6)$$

of the caution control u_k^c

$$u_k^c = \underset{u_k}{\operatorname{argmin}}\, J_k^c(u_k) \quad (7)$$

gained by minimization of the criterion (4)

The choice of the vector δ_k stems from reasoning that it is necessary to enrich the caution control with probing in proportion to uncertainty of the nonmeasurable quantities of the controlled system. This uncertainty is naturally represented by conditional covariance matrix P_k of the filtering pdf of these quantities. The pdf is obtained by nonlinear filtering methods. The vector δ_k depends on P_k thus

$$\delta_k = f(P_k), \quad (8)$$

where the function $f\,(\boldsymbol{P}_k)$ can be chosen as

$$f\,(\boldsymbol{P}_k) = \eta \text{tr} \boldsymbol{P}_k, \qquad \eta \geq 0. \tag{9}$$

The bicriterial control $\boldsymbol{u}_k^*$ is then searched as

$$\boldsymbol{u}_k^* = \underset{\boldsymbol{u}_k \in \Omega_k}{\text{argmax}}\, J_k^e\,(\boldsymbol{u}_k)\,. \tag{10}$$

The cautious control u_k^c is then obtained from the condition (4) as follows

$$\begin{aligned} u_k^c = &- \left[\boldsymbol{W}_k + E\left\{ \boldsymbol{B}^T(\boldsymbol{\theta}_k)\boldsymbol{C}^T\boldsymbol{V}_{k+1}\boldsymbol{C}\boldsymbol{B}(\boldsymbol{\theta}_k) \middle| \Im_k \right\} \right]^{-1} \times \\ &\times \left(E\left\{ \boldsymbol{B}^T(\boldsymbol{\theta}_k)\boldsymbol{C}^T\boldsymbol{V}_{k+1}\boldsymbol{C}\boldsymbol{A}(\boldsymbol{\theta}_k)\boldsymbol{s}_k \middle| \Im_k \right\} + \right. \\ &+ E\left\{ \boldsymbol{B}^T(\boldsymbol{\theta}_k)\boldsymbol{C}^T\boldsymbol{V}_{k+1}\boldsymbol{C}\boldsymbol{w}_k \middle| \Im_k \right\} + \\ &+ E\left\{ \boldsymbol{B}^T(\boldsymbol{\theta}_k)\boldsymbol{C}^T\boldsymbol{V}_{k+1}\boldsymbol{v}_{k+1} \middle| \Im_k \right\} - \\ &\left. - \; E\left\{ \boldsymbol{B}^T(\boldsymbol{\theta}_k)\boldsymbol{C}^T\boldsymbol{V}_{k+1} \middle| \Im_k \right\} \bar{\boldsymbol{y}}_{k+1} \right). \end{aligned} \tag{11}$$

the bicriterial control is given as

$$\boldsymbol{u}_k^* = \boldsymbol{u}_k^c + \boldsymbol{\delta}_k \,\text{sign}\,(\boldsymbol{\omega}_k)\,, \tag{12}$$

where

$$\begin{aligned} \boldsymbol{\omega}_k =& J_k^e\left(\boldsymbol{u}_k^c + \boldsymbol{\delta}_k\right) - J_k^e\left(\boldsymbol{u}_k^c - \boldsymbol{\delta}_k\right) = \\ =& 4\boldsymbol{\delta}_k^T E\left\{ [\boldsymbol{B}(\boldsymbol{\theta}_k) - \boldsymbol{B}(\hat{\boldsymbol{\theta}}_k)]^T \boldsymbol{C}^T \boldsymbol{\mathcal{V}}_{k+1} \times \right. \\ &\left. \times \boldsymbol{C}[\boldsymbol{A}(\boldsymbol{\theta}_k)\boldsymbol{s}_k - \boldsymbol{A}(\hat{\boldsymbol{\theta}}_k)\hat{\boldsymbol{s}}_k] \middle| \Im_k \right\} + \\ &+ 4\boldsymbol{\delta}_k^T E\left\{ [\boldsymbol{B}(\boldsymbol{\theta}_k) - \boldsymbol{B}(\hat{\boldsymbol{\theta}}_k)]^T \boldsymbol{C}^T \boldsymbol{\mathcal{V}}_{k+1} \times \right. \\ &\left. \times \boldsymbol{C}[\boldsymbol{B}(\boldsymbol{\theta}_k) - \boldsymbol{B}(\hat{\boldsymbol{\theta}}_k)] \middle| \Im_k \right\} \boldsymbol{u}_k^c. \end{aligned} \tag{13}$$

The sign of the intentional probing $\boldsymbol{\delta}_k\,\text{sign}\,(\boldsymbol{\omega}_k)$ depends on the values of $\hat{\boldsymbol{s}}_k$, $\boldsymbol{u}_k^c$ and on the part of $\boldsymbol{P}_k$ corresponding to the system parameters $\boldsymbol{\theta}_k$. In case of independency of the known matrix $\boldsymbol{A}$ on the parameters $\boldsymbol{\theta}_k$ the intentional probing retains the sign of the caution control. In case that also the known matrix $\boldsymbol{B}$ is independent of parameters $\boldsymbol{\theta}_k$ the variable $\boldsymbol{\omega}_k$ will be equal to zero and hence there will be no probing at all.

In order to evaluate the relations (11), (13) it is necessary to perform state and parameter estimation. The evaluation of the mean values requires knowledge state and parameter joint pdf. This pdf has to be obtained using a suitable nonlinear filtering method.

Heretofore the type of pdf's of all the random quantities $\boldsymbol{s}_0$, $\boldsymbol{\theta}_0$, $\boldsymbol{w}_k$, $\boldsymbol{\epsilon}_k$, $\boldsymbol{v}_k$ $\forall k$ has not been intentionally specified. The derivation of the bicriterial control is not affected by the type of pdf's of these quantities. If non-Gaussian pdf's are used for description of the disturbances and the initial augmented state, then it is inevitable to employ a global filtering method in order to maintain sufficient quality of estimates.

Let's suppose that all the non-Gaussian pdf's are in the form of the Gaussian mixture

$$p(\boldsymbol{\xi}) \triangleq \sum_{i=1}^{\ell} \alpha_i \mathcal{N}(\hat{\boldsymbol{\xi}}_i, \boldsymbol{\Xi}_i), \tag{14}$$

where $\alpha_i \geq 0$, $\sum_{i=1}^{\ell} \alpha_i = 1$, $\hat{\boldsymbol{\xi}}_i$ denotes the mean value and $\boldsymbol{\Xi}_i$ the covariance matrix of $\boldsymbol{\xi}$. Such representation makes an approximation of virtually any pdf possible. Moreover it can be employed for modeling e.g. of abrupt state or parameter changes (Šimandl, 1996) and/or outliers in measurements (Šimandl, 1997), respectively.

A natural choice of global nonlinear estimation method in case where the pdf's are supposed in form (14) seems to be the GSM. The basic idea behind the GSM is deploying multiple Extended Kalman filters (EKF's) providing local estimates in form of Gaussian pdf parameters. The EKF's are accompanied with an algorithm for weight evaluation and if necessary with suitable algorithm that reduces fast growing number of EKF's and thus reduces computing demands of the algorithm.

Ordinary only the global point estimate provided by the GSM would be used in order to determine the mean values in relations (11), (13). This approach provides reasonable control quality (Šimandl and Flídr, 2001). However, such control scheme does not make full use of the information provided by the GSM. The next section will thus cope with an alternative controller scheme respecting character of the GSM producing approximate pdf in the form of a Gaussian mixture.

3. BICRITERIAL DUAL CONTROL WITH MULTIPLE LINEARIZATION

As it was mentioned in previous section the simplest technique to bicriterial control evaluation is to use a point estimate. However the GSM producing filtering pdf given by a set of local estimates will be used in this section as a convenient tool for estimation of unknown state and parameters. From the estimation point of view the system is described by relations

$$\boldsymbol{x}_{k+1} = \boldsymbol{f}_k(\boldsymbol{u}_k, \boldsymbol{x}_k) + (\boldsymbol{w}_k, \boldsymbol{\epsilon}_k)^T \tag{15}$$

$$\boldsymbol{y}_k = \boldsymbol{h}_k(\boldsymbol{x}_k) + \boldsymbol{v}_k, \tag{16}$$

where the augmented state vector $\boldsymbol{x}_k$ is defined as

$$\boldsymbol{x}_k = \begin{pmatrix} \boldsymbol{s}_k \\ \boldsymbol{\theta}_k \end{pmatrix} \tag{17}$$

and the nonlinear vector functions $\boldsymbol{f}(\boldsymbol{u}_k, \boldsymbol{x}_k)$, $\boldsymbol{h}(\boldsymbol{x}_k)$ are specified as

$$\boldsymbol{f}_k(\boldsymbol{u}_k, \boldsymbol{x}_k) = \begin{pmatrix} \boldsymbol{\Phi}_k \boldsymbol{\theta}_k \\ \boldsymbol{A}(\boldsymbol{\theta}_k)\boldsymbol{s}_k + \boldsymbol{B}(\boldsymbol{\theta}_k)\boldsymbol{u}_k \end{pmatrix} \tag{18}$$

$$\boldsymbol{h}_k(\boldsymbol{x}_k) = (\boldsymbol{\mathcal{O}} | \boldsymbol{C})\, \boldsymbol{x}_k, \tag{19}$$

where $\boldsymbol{\mathcal{O}}$ is zero matrix.

The nonlinear vector functions (15) and (16) are linearized (Šimandl and Flídr, 2001) at each time instant in multiple points specified by Gaussian mixture terms describing the predictive and the filtering pdf respectively. The estimation is thus divided among multiple EKF's. The GSM enhances the local estimates provided by the EKF's with an algorithm for appropriate mixture weights evaluation. It is necessary to cope with possibly growing mixture terms number e.g. with specifying fixed maximum mixture terms number.

In Figure 1 a typical structure of the bicriterial controller employing the GSM for state estimation is depicted. In such a case the bicriterial controller employs the global point estimate. For evaluation of equations (11) and (13) it is sufficient to determine only the first three moments of the filtering pdf given by weighted mixture of Gaussians. Calculation of the third moment can be computationally very demanding. It is possible to simplify calculation assuming mutual independence of the unknown parameters $\boldsymbol{\theta}_{k+1}$ and the state $\boldsymbol{s}_{k+1}$. In such a case it is necessary to know only the first two moments of the filtering pdf given as a Gaussian mixture (14) in the following form

$$\hat{\boldsymbol{x}}_k = E\{\boldsymbol{x}_k|\Im_k\} = \sum_{i=1}^{\ell} \alpha_{ki} E\{\boldsymbol{x}_k|\Im_k, i\} = \sum_{i=1}^{\ell} \alpha_{ki}\hat{\boldsymbol{x}}_{ki} \tag{20}$$

$$P_k = \mathrm{cov}(\boldsymbol{x}_k|\Im_k) = \sum_{i=1}^{\ell} \alpha_{ki}\left(P_{ki} + \hat{\boldsymbol{x}}_{ki}\hat{\boldsymbol{x}}_{ki}^T\right) \tag{21}$$

where $\hat{\boldsymbol{x}}_{ki}$ and P_{ki} are mean and covariance matrix of the filtering pdf provided by i-th EKF, respectively.

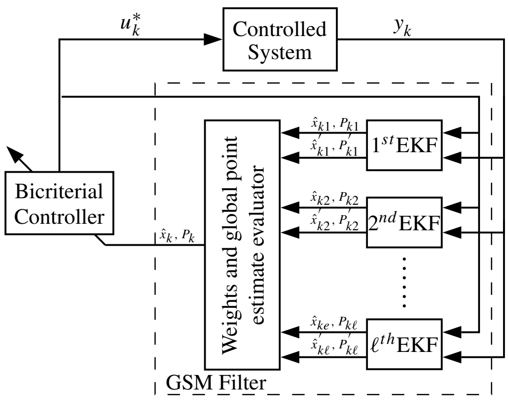

Fig. 1. Scheme of bicriterial controller where $\hat{\boldsymbol{x}}_{ki}^{'} = E\{\boldsymbol{x}_k|\Im_{k-1}, i\}$, $P_{ki}^{'} = cov(x_k|\Im_{k-1}, i)$

This bictriterial controller is straightforward and provides plausible control performance which is better than caution control. However, taking into account the nonlinear character of the control law (11) it is natural to assume, it could provide different performance by coupling the bicriterial controller directly with local estimates provided by the EKF's. This means that the bicriterial controller will be linearized in multiple points as well. The points are identical to those used within the EKF's. This controller scheme is depicted in Figure 2 and can be described by the following equations

$$\boldsymbol{u}_k^* = \sum_{i=1}^{\ell} \alpha_i \boldsymbol{u}_{ki}^* \tag{22}$$

where

$$\boldsymbol{u}_{ki}^* = \boldsymbol{u}_{ki}^c + \delta_k \,\mathrm{sign}\,(\boldsymbol{\omega}_{ki}). \tag{23}$$

$\boldsymbol{u}_{ki}^c$ and $\boldsymbol{\omega}_{ki}$ are derived from the equations (11) and (13) under the assumption that for evaluation of all the means the pdf provided by i-th EKF is used. The equation (22) can be rewritten as

$$\boldsymbol{u}_k^* = \boldsymbol{u}_k^c + \delta_k \sum_{i=1}^{\ell} \alpha_i \,\mathrm{sign}\,(\boldsymbol{\omega}_{ki}). \tag{24}$$

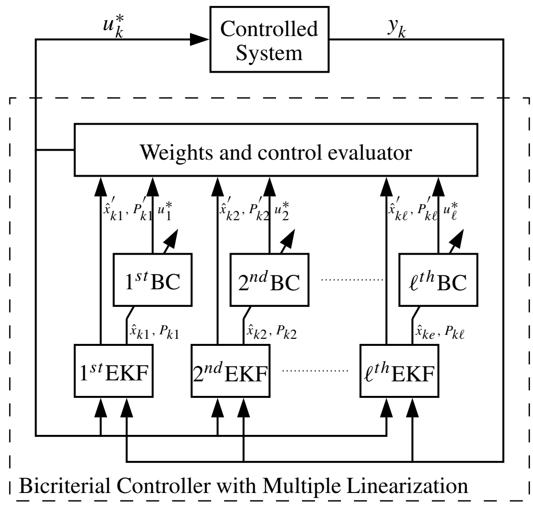

Fig. 2. Scheme of bicriterial controller with multiple linearization

These two controller schemes should generally provide different control sequence. It stems from the fact that in general case the next relation concerning probing signals is fulfilled

$$\delta_k \,\mathrm{sign}\,(\boldsymbol{\omega}_k) \neq \delta_k \sum_{i=1}^{\ell} \alpha_i \,\mathrm{sign}\,(\boldsymbol{\omega}_{ki}). \tag{25}$$

It means the equality between probing evaluated upon knowledge of global point estimate and the sum of probing depending on local estimates given by separate EKF's is not guaranteed. It follows from the fact that the probing in case of multiple linearization is evaluated on different domains

$$\Omega_{ki} = [\boldsymbol{u}_{ki}^c - \delta_k, \boldsymbol{u}_{ki}^c + \delta_k], \qquad i = 1, \ldots, \ell. \tag{26}$$

It is possible to assume that the difference in retrieval of the probing signal may bring some benefit regarding the overall control quality. This assumption stem from the fact that the probing signal is composed from ℓ probing signals designed for each node represented by one EKF. That means that the probing signal is designed to improve the quality of the relevant estimate. Unfortunately this assumption cannot be analytically proven because of the inherent nonlinearities and uncertainties. Validity of the relation (25) will be demonstrated by comparison of probing signals of both bicriterial controllers and their impact on control quality will shown in some numerical examples.

4. NUMERICAL ILLUSTRATIONS

To compare bicriterial controller with multiple linearization and bicriterial controller utilizing a global point estimate only, the following index is chosen as a measure of the control performance

$$\mathcal{M} = \sqrt{\frac{1}{N}\sum_{k=1}^{N}(y_k - \bar{y}_k)^2}, \tag{27}$$

where y_k represents the measurement of the controlled SISO system, $\bar{y}_k$ is the reference value and N determines the length of each simulation. The expected value $\hat{\mathcal{M}} = E\{\mathcal{M}\}$ is estimated using 5000 Monte Carlo simulations.

The comparison of behavior of both controller designs will be demonstrated in two examples. In both examples the parameters of the criterion (4) are given as $V_{k+1} = 1$ and $W_{k+1} = 0.001$ and the parameter in criterion (5) is given as $\mathcal{V}_{k+1} = 1$.

Example 4.1. The first comparison will be performed for first order SISO system with time varying parameters $\boldsymbol{\theta}_k = (\theta_{1k}, \theta_{2k})^T$ described by the following relations

$$s_{k+1} = \theta_{1k}s_k + \theta_{2k}u_k + w_k, \tag{28}$$
$$\boldsymbol{\theta}_{k+1} = \boldsymbol{\theta}_k + \boldsymbol{\epsilon}_k, \tag{29}$$
$$y_k = s_k + v_k. \tag{30}$$

The prior pdf of the parameters is Gaussian

$$p(\boldsymbol{\theta}_0) = \mathcal{N}\left((0.8,\ 1)^T, \boldsymbol{I}\right) \tag{31}$$

and the parameter noise is supposed to be Gaussian mixture

$$p(\boldsymbol{\epsilon}_k) = 0.98\mathcal{N}\left((0,\ 0)^T, \begin{pmatrix}10^{-4} & 0\\ 0 & 10^{-4}\end{pmatrix}\right) + +0.02\mathcal{N}\left((0,\ 0)^T, \begin{pmatrix}0.04 & 0\\ 0 & 0.04\end{pmatrix}\right). \tag{32}$$

This mixture can be seen as a model of abrupt parameter changes (Šimandl, 1996). All the remaining random quantities $\boldsymbol{w}_k$, v_k and $\boldsymbol{s}_0$ are described by

$$p(\boldsymbol{w}_k) = \mathcal{N}(0, 0.5) \tag{33}$$
$$p(v_k) = \mathcal{N}(0, 0.09) \tag{34}$$
$$p(\boldsymbol{s}_0) = \mathcal{N}(0.13, 0.25). \tag{35}$$

The parameter η in (9) is chosen as $\eta = 5.7$.

Example 4.2. The following stochastic system

$$\boldsymbol{s}_{k+1} = \begin{pmatrix}0 & 1\\ \theta_{1k} & \theta_{2k}\end{pmatrix}\boldsymbol{s}_k + \begin{pmatrix}\theta_{3k}\\ \theta_{4k}\end{pmatrix}u_k + \boldsymbol{w}_k, \tag{36}$$
$$y_k = (1,\ 1)\boldsymbol{s}_k + v_k, \tag{37}$$

is used for the second comparison. The random quantities $\boldsymbol{w}_k$, v_k and $\boldsymbol{s}_0$ are described by

$$p(\boldsymbol{w}_k) = \mathcal{N}\left((0,\ 0)^T, 10^{-4}\boldsymbol{I}\right) \tag{38}$$
$$p(v_k) = \mathcal{N}\left(0,\ 10^{-3}\right) \tag{39}$$
$$p(\boldsymbol{s}_0) = \mathcal{N}\left((0,\ 0)^T,\ 5\boldsymbol{I}\right). \tag{40}$$

In this example only one parameter is considered as time varying, the parameter vector is given as

$$\boldsymbol{\theta}_k = (-2.0427,\ 0.3427,\ 0,\ \theta_{4k})^T \tag{41}$$

where the parameter θ_{4k} evolves according to the relation

$$\theta_{4k+1} = \theta_{4k} + \epsilon_k, \tag{42}$$

with a priori pdf $p(\theta_{4,0}) = \mathcal{N}(1, 0.01)$. The pdf of this parameter noise is represented by uniform random quantity as $\epsilon_k \sim \mathcal{U}(-0.01, 0.01)$. This distribution will be approximated using a Gaussian mixture

$$p(\epsilon_k) = \sum_{i=1}^{\ell}\alpha_i\mathcal{N}(\hat{\epsilon}_{ki}, \operatorname{cov}\epsilon_{ki}) \tag{43}$$

where weights, means and covariance matrices are given in Table 1

Table 1. Parameter of Gaussian mixture approximating uniform distribution $\mathcal{U}(-0.01, 0.01)$

i	α_i	ϵ_{ki}	$\operatorname{cov}\epsilon_{ki}$
1	0.1656	-0.0080	1.65e-06
2	0.2427	-0.0034	6.25e-06
3	0.1973	-0.0004	1.163e-06
4	0.2471	0.0041	4.90e-06
5	0.1473	0.0083	1.20e-06

The prior pdf of parameters $\boldsymbol{\theta}_0$ is Gaussian

$$p(\boldsymbol{\theta}_0) = \mathcal{N}\left(\hat{\boldsymbol{\theta}}_0, \operatorname{diag}(1, 1, 1, 0.01)\right), \tag{44}$$

where

$$\hat{\boldsymbol{\theta}}_0 = (-2.0427,\ 0.3427,\ 0,\ 1)^T \tag{45}$$

The function diag$(\cdot)$ denotes diagonal matrix. The parameter η in (9) is chosen as $\eta = 1.5$.

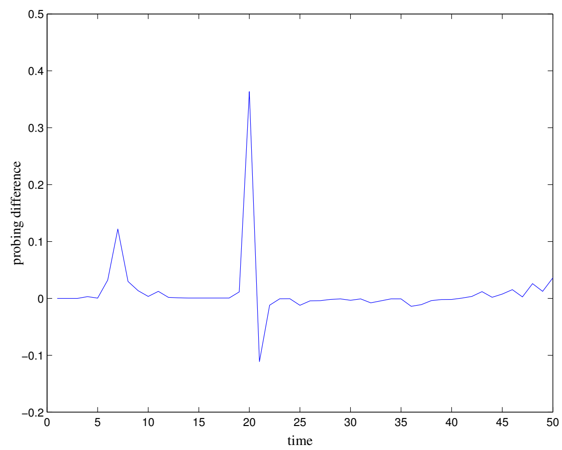

Fig. 3. The difference between probing signals generated by the Bicriterial controller (BC) and the multiple linearized BC from example 4.1.

The comparison of probing signals for both examples is depicted in Figures 3, 4. The Figures show the difference between probing generated by bicriterial controller which used global point estimate only and the multiple linearized bicriterial controller.

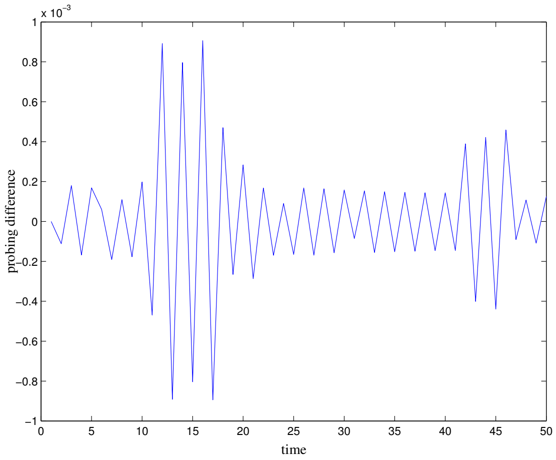

Fig. 4. The The difference between probing signals generated by the Bicriterial controller (BC) and the multiple linearized BC from example 4.2.

As it was expected the probing signals indeed differ. However the numerical examples also showed that application of the BC with multiple linearization does not bring substantial improvement of control quality as it is shown in Table 2.

Table 2. Control quality comparison using index $\hat{\mathcal{M}}$

	Example 4.1	Example 4.2
Bicriterial control	4.8174	3.8841
BC with multiple linearization	4.8146	3.8829

5. CONCLUSION

The bicriterial controller with multiple linearization was proposed for the discrete-time non-Gaussian stochastic system with unknown parameters. Structure of the proposed bicriterial controller corresponds to structure of the estimator and consists of a set of local bicriterial controllers connected with corresponding local estimators. The probing signal generated by this multiple linearized controller differs from the probing of the controller that uses the global point estimate only and induces slightly better control quality. Both considered bicriterial dual controllers give substantial quality improvement comparing to caution control.

6. ACKNOWLEDGEMENT

The work was supported by the Ministry of Education, Youth and Sports of the Czech Republic, projects MSM 235200004 and 1M6798555601.

REFERENCES

Alspach, D. L. and H. W. Sorenson (1971). Recursive Bayesian estimation using Gaussian sums. *Automatica* **7**, 465–479.

Alster, J. and P. Bélanger (1974). A technique for dual adaptive control. *Automatica* **10**, 627–634.

Birmiwal, K. and Y. Bar-Shalom (1984). Dual control for identification and guidance. In: *Proceeding of 9th IFAC World Congress*. Vol. VII. Budapest. pp. 65–70.

Feldbaum, A.A. (1965). *Optimal Control Systems*. Academic Press. New York.

Filatov, N. M. (1998). Application of adaptive dual controllers to a dc-motor. In: *IEE Proceedings On Control Theory and Applications*. Vol. 145. pp. 299–305.

Filatov, N. M., H. Unbehauen and U. Keuchel (1997). Dual pole-placement controller with direct adaptation. *Automatica* **33**(1), 113–117.

Filatov, N. M., U. Keuchel and H. Unbehauen (1996). Dual control for an unstable mechanical plant. *IEEE Control Systems Magazine* **16**(4), 31–37.

Lindoff, B., J. Holst and B. Wittenmark (1999). Analysis of approximations of dual control. *Int. J. Adapt. Control Signal Process.* **13**, 593–620.

Maitelli, A. L. and T. Yoneyama (1994). A two-stage dual suboptimal controller for stochastic systems using approximate moments. *Automatica* **30**(12), 1949–1954.

Milito, R., C.S. Padilla, R.A. Padilla and D. Cadorin (1982). An innovations approach to dual control. *IEEE Trans. Automat. Contr.* **AC-27**(1), 132–137.

Pronzato, L., C. Kulcsár and E. Walter (1996). An actively adaptive control policy for linear models. *IEEE Trans. Automat. Control* **41**(6), 855–858.

Sternby, J. (1976). A simple dual control problem with an analytical solution. *IEEE Trans. Automat. Control* **AC-21**, 841–844.

Šimandl, M. (1996). State estimation for non-Gaussian models. In: *Preprints of 13th IFAC World Congress*. IFAC. San Francisco. pp. 463–468. June 30–July 5.

Šimandl, M. (1997). Multi-process models and outliers elimination. In: *Proceedings of The IASTED International Conference on Signal and Image Processing (SIP'97)*. IASTED/ACTA Press. New Orleans, Louisiana. pp. 320–325. December 4–6.

Šimandl, M. and M. Flídr (1997). Nonlinear non-normal dynamic models: State estimation and software. In: *Computer-Intensive Methods in Control and Signal Processing* (K. Warwick and M. Kárný, Eds.). pp. 195–208. Birkhäuser. Boston.

Šimandl, M. and M. Flídr (2001). Bicriterial dual control for stochastic systems with unknown variable parameters. In: *Proceeding of the Fifth IFAC Symposium on Nonlinear Control Systems*. IFAC. Saint-Petersburg, Russia. pp. 1069–1074.

Wittenmark, B. (1975). An active suboptimal dual controller for systems with stochastic parameters. *Automatic Control Theory and Application* **3**(1), 13–19.