CONTROL OF A HEAT EXCHANGER USING AN ITERATIVE DESIGN APPROACH

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Abstract
This paper deals with the control of the heat exchangers. Due to the resonant effect on this systems, simplified lumped parameters models obtained from the step response can be not enough appropriate to controller design if high performance is required. To design the control, instead of looking for a precise heat exchanger model obtained through complex experiments and sophisticated parameter estimation methods, the use of an iterative approach is suggested.

Keywords: heat exchanger, resonance, iterative design, predictive control

1. INTRODUCTION

In the process industries as well as in some non-industrial applications the use of heat exchangers is very extensive. Several studies have been carried out dealing with the modelling of these systems: (Abdelghani-Idrissi *et al.*, 2001),(Lakshmanan and Potter, 1994),(Lachi *et al.*, 1997), (Romie, 1984), (Romie, 1985), (Romie, 1999) , (Roetzel and Xuan, 1992), (Shah, 1981), (Tan and Spinner, 1991), (Xuan and Roetzel, 1993), (Yin and Jensen, 2003). Owing that the heat exchanger parameters are distributed and interacting, the exact dynamic equations for ordinary cocurrent exchanger are quite complex, and lengthy calculations are required just to determine the open-loop frequency response. Beside, simplified lumped parameters models of transient response are mostly used for control purposes when the control focuses only on the system outlets without paying attention to its spatial profiles. In most cases this kind of model is developed after a step change in the input variables takes place: mass flow rate or temperature. However, since the models do not take into account relevant issues of the heat exchanger behavior (e.g., distributed parameters) their approximation of the transient response can be not enough to controller design.

This paper has two objectives: first it will be show that the step response models of heat-exchanger have some limitations for the design of the controller when high performance is required. This models explains the low frequency behavior but they do not cover significant aspects of behavior of the process such as resonant effect due to distributed operation.

The second objective of the paper is to provide a solution to overcome the previous difficulty. To design the control, instead of looking for a precise heat exchanger model obtained through complex experiments and sophisticated parameter estimation methods, the use of an iterative approach is suggested. This iterative approach has been proved to work well for systems with high frequency almost resonant modes, (Albertos *et al.*, 2002; Albertos *et al.*, 2004). Although the approach can be used with a variety of control design and/or identification techniques, in this paper the iterative method is used to design a GPC controller, a model-based optimal control strategy.

The paper is organized as follows: first, the structural model of the heat exchanger is presented. For the sake of clarity, a simple model that take into account the resonant effect in the heat exchanger is considered. The GPC controller algorithm is treated in section 3. The iterative control design methodology is addressed in section 4. Section 5 deals with the application of the iterative design of GPC controller to heat exchanger. A simulation study is presented. Finally, section 6 summarizes the conclusions derived from this work.

2. STRUCTURAL MODEL

In this section, a theoretical model of cocurrent tubular heat exchanger is presented.

A relatively simple type of exchanger is one in which the temperature on one side of the wall is constant, as when a pure vapor is condensing on the outside of the tube. In this study it is considered that the exit temperature (T_f) of one cold fluid is controlled by changing the temperature of the vapor (T_v).

For the sake of simplicity the steady-state assumption of no backmixing, negligible axial conduction and constant fluid properties are undertaken. Furthermore, in order to limit the structural model to comply with the second order one, we assumes no wall resistance.

The energy balance for the fluid stream in a tube is written as:

$$M_f C_f \frac{\partial T_f}{\partial t} dx + F C_f \frac{\partial T_f}{\partial x} = h_1 A_1 dx (T_w - T_f) \quad (1)$$

or in dimensionless form:

$$\tau_1 \frac{\partial T_f}{\partial t} + v \tau_1 \frac{\partial T_f}{\partial x} = T_w - T_f \quad (2)$$

where:

$$\tau_1 = \frac{M_f C_f}{h_1 A_1} \quad (3)$$

M_f: mass flow rate.

T_w: wall temperature.

C_f: heat capacity of fluid.

h_1: internal convection coefficient.

A_1: internal area.

$v = \dfrac{F}{M_f}$: mean velocity

The energy balance for the wall is given by:

$$M_w c_w \frac{\partial T_w}{\partial t} = h_2 A_2 (T_v - T_w) - h_1 A_1 (T_w - T_f) \quad (4)$$

or in dimensionless form:

$$\tau_2 \frac{\partial T_w}{\partial t} dx = (T_v - T_w) - \frac{\tau_2}{\tau_{12}} (T_w - T_f) \quad (5)$$

where:

$$\tau_2 = \frac{M_w c_w}{h_2 A_2} \quad (6)$$

$$\tau_{12} = \frac{M_w c_w}{h_1 A_1} \quad (7)$$

$M_w c_w$: wall capacity.

h_2: external convectin coefficient.

A_2: external area.

The preceding partial differential equation are converted into ordinary differential equation by taking the Laplace transformation with respect to time. The dependent variables T_f and T_w in the following transformed equations represent deviations from the normal values at any station along of the heat exchanger:

$$\tau_1 s T_f + v \tau_1 \frac{\partial T_f}{\partial x} = T_w - T_f \quad (8)$$

$$\tau_2 s T_w dx = (T_v - T_w) - \frac{\tau_2}{\tau_{12}} (T_w - T_f) \quad (9)$$

Combining equations 8 and 9 in a suitable form results in:

$$\frac{v}{a} \frac{dT_f}{dx} + T_f = \frac{b}{a} T_v \quad (10)$$

where:

$$a = \frac{(\tau_1 s + 1)(\tau_{12} \tau_2 s + \tau_{12} + \tau_2) - \tau_2}{\tau_1 (\tau_{12} \tau_2 s + \tau_{12} + \tau_2)} \quad (11)$$

$$\frac{b}{a} = \frac{1}{\tau_1 \tau_2 s^2 + (\tau_1 + \tau_2 + \tau_1 \tau_2 / \tau_{12}) s + 1} \quad (12)$$

The solution of equation 10 satisfying the boundary condition $T_f = 0$ and $x = 0$ yields:

$$\frac{T_f(s)}{T_v(s)} = \frac{b}{a}(1 - e^{-aL}) \qquad (13)$$

where $L = x/v$ is the time for fluid to flow through the tube, that is the residence time.

2.1 Resonance effect

Since the term a has s^2 in the numerator and s in the denominator, e^{-aL} is a vector with ever increasing phase lag and a length less than 1. The term $1-e^{-aL}$ therefore shows regular fluctuations in amplitude and phase lag with the frequency, which lead to resonant peaks in the frequency response, figure 1. The resonance arises because the heat exchanger is forced in a distributive manner, i,e., the vapor temperature is changed along the entire length of the exchanger and not just at the one end.

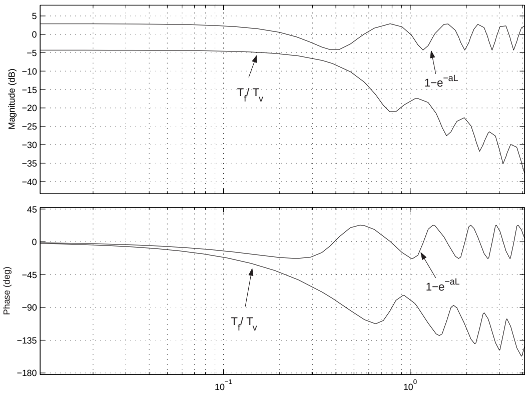

Figure 1. Bode diagram for a heat exchanger.

3. GENERALIZED PREDICTIVE CONTROL

Generalized Predictive Control (GPC) (Clarck *et al.*, 1987), resulting in a control scheme as shown in the figure 2, is a model-based optimal control strategy where uncertainties and disturbances are handled by just applying the first part of the computed control sequence and recomputing again the optimal control over a finite optimization horizon. There are many different settings for this approach. The basic cost index is assumed as:

$$J_{\lambda_j} = \sum_{i=N_1}^{N_2} \alpha_i [y(k+i|k) - w(k+i)]^2 + \sum_{j=N_1}^{N_u} \lambda_j [\Delta u(k+j-1)]^2 \qquad (14)$$

where: $y(k+i|k)$: estimated output at $(k+i)$ based on data at k; $w(k+i)$: reference at $(k+i)$; Δu: increment in the control action; N_1: minimum prediction horizon; N_2: maximum prediction horizon, large enough to cover the system dynamic response; N_u: control horizon; α_i: error weighting factor; λ_j: control action weighting factor.

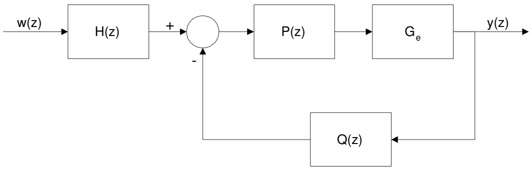

Figure 2. GPC controller.

The last parameter is directly connected to the control effort and system response "vivacity": the smaller the parameter λ_j is the stronger the controller actions are and the faster the system response is. That is, the smaller this parameter is the greater the bandwidth of the controlled system is. But, as for any model-based control design technique, the GPC resulting controller is only good for a given range of processes, not far from the model used for the computation. If strong control actions are applied, non-expected high frequency process modes, such as resonant effect in heat exchangers, can be excited and the behavior will greatly differ from the one designed.

4. ITERATIVE CONTROL DESIGN

A detailed explanation of the iterative control design is described in (Albertos *et al.*, 2000). The general control design methodology is described as follows: given a process, several control goals and constraints, the idea is to design by an iterative approach, the most effective control system without violating the constraints. The general procedure is:

1. Postulate a raw model of the process to be controlled. Usually, this model explains its low frequency behavior.
2. For the available model, design a controller. Any model based design approach can be used.
3. Apply the designed controller to the actual process and check the constraint's fulfillment. Due to the mismatch between the real process and its model, conservative performances are required.
4. If the constraints are fulfilled: a) stop if a satisfactory control is achieved, or b) redesign or retune the controller to enhance the controlled process performances, and go to step 3 otherwise, if there is a constraint violation, used the last controller and:
5. Carry on an identification run a) if a better model is obtained, go to step 2, otherwise b) stop and keep the achieved controller.

This approach seems to be appropriated for the control design of heat exchangers as long as: *a)* A rough mode can be obtained based on step response (As it is shown in next section, it could be a first order model). *b)* The model mismatch is due to high frequency resonant effect. *c)* It is important to achieve a fast response. *d)* There is always some high frequency noise in the measurements. *f)* The constraints, such as a maximum overshoot or an excessive ripple, can be checked.

5. ITERATIVE DESIGN OF GPC. APPLICATION TO HEAT EXCHANGER.

The requirement considered for the operation of heat exchanger is to control the temperature T_f as quickly as possible. This requirement can be evaluated by mean of the following cost index:

$$J_e = \sum_{i=1}^{N} t(k+i) \| y(k+i|k) - w(k+i) \| \qquad (15)$$

wish is discrete version of ITAE index. The constraints are reduced to avoid the presence of oscillations in the output variable.

The iterative design of GPC for the heat exchanger will be as follows:

1. Based on the step response of the plant obtain a simple model.
2. For this model, a "passive" GPC controller is designed. By passive GPC, it is denoted a controller where the weight of the control actions (λ_j) in the cost index 14 is high.
3. The resulting controller is applied to the real process. Evaluate the controlled system response by mean of index based on the system error, i.e., equation 15. Check the fulfillment of the restrictions also.
4. If the restrictions are fulfilled, design for the same model a more "active" controller, that is, reducing λ_j. The index J_e is expected to decrease. The coefficient λ_j can be reduced until an increment in the index is detected. This is in contradiction of the design criterion, where the errors are becoming more weighted than the control actions. This statement should be related to the appearance of oscillations due to the resonant effect.
5. Use the last controller and proceed to a new identification procedure. For the model obtained a new GPC controller is designed.
6. Repeat steps 3-5, until the control is satisfactory.

Figure 3 synthesizes the design procedure.

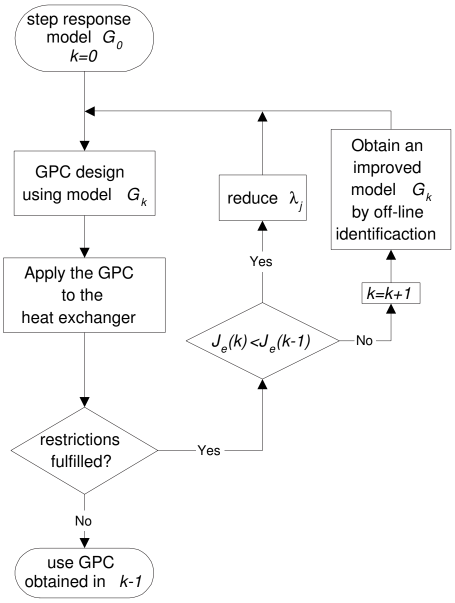

Figure 3. Iterative design of GPC.

5.1 Simulation

In the simulation study a structural model (equation 13) is used as the heat exchanger to be controlled. Specifically, the following structural model is considered:

$$G_e: \quad \frac{T_f(s)}{T_v(s)} = \frac{1 - 0.8e^{-2s}}{(20s+1)(0.26s+1)} \qquad (16)$$

It is assumed an output measurement noise that is normally (gaussian) distributed, with zero mean and a variance of 0,0001.

Based on the step response, (figure 4), a raw first order model is proposed to approximate the transient response:

$$G_0(s): \quad \frac{T_f(s)}{T_v(s)} = \frac{0.2}{10s+1} \qquad (17)$$

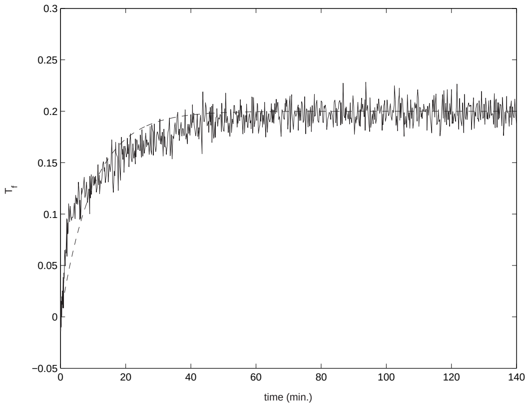

Figure 4. Step response. Solid line: heat exchanger, G_e. Dashed line: initial model, $G_0(s)$.

For the design of GPC the model is discretised with a sample period $t_s = 0.5\ min.$ leading to:

$$G_0(z): \quad \frac{T_f(z)}{T_v(z)} = \frac{0.009754z^{-1}}{1-0.9512z^{-1}} \tag{18}$$

Initially, the design of GPC controller is very conservative, since no satisfactory as no good knowledge of the process is obtained from just a step response. The GPC design parameters are taken as: $\lambda_j = 5$, $N_1 = 1$, $N_2 = 20$, $N_u = 4$, with polynomial $T = 1$.

Once the error index is evaluated and the design is validated, (that is, there are not oscillations), lower values are assigned to the weighting factor λ_j with the objective of obtaining quicker response of the exchanger.

The simulated step response, for different values of λ_j, are plotted in figure 5. The controlled heat exchanger response is improved as far as the control weight is reduced, points A, B, C y D in Figure 7. As expected, for low values of λ_j ($= 0.005$), the heat exchanger response becomes oscillatory due to the resonant effect. Because of this oscillations the value of index J_e increases from 43 (point D in Figure 7) to 82 (point E).

At this point it is necessary to developed a new identification. With the aide of System Identification Toolbox of Matlab software (Ljung, 1997), the following model is obtained:

$$G_1(z): \quad \frac{T_f(z)}{T_v(z)} = \frac{\sum_{i=1}^{6} b_i z^{-i}}{\sum_{i=0}^{2} a_i z^{-i}} \tag{19}$$

whose coefficients are listed in table 1.

With this new model a new controller is designed. The controlled system step response is drawn in figure 6. A remarkable improvement is obtained, in addition, the settling time is reduced and the oscillations do not appear, consequently, the values of performance index J_e is reduced from 82 (point E in Figure 7) to 22 (point F).

Further designs was carry out for lower values λ_j using model G_1, but it was not obtained a reduction of J_e, (points G and H in Figure 7), so the procedure finalizes.

Table 1. Coefficients of model $G_1(z)$.

i	a_i	b_i
0	1	-
1	- 1.118	0.01371
2	0.1391	0.007352
3	-	0.0001743
4	-	-0.00001457
5	-	- 0.01107
6	-	0.00592

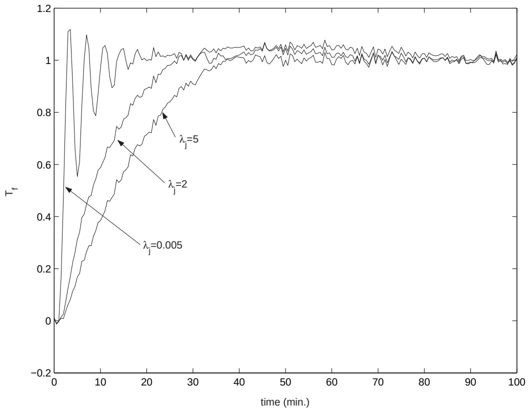

Figure 5. Step response of GPC controlled heat exchanger using G_0 and different weighting factor λ_j on the design of GPC.

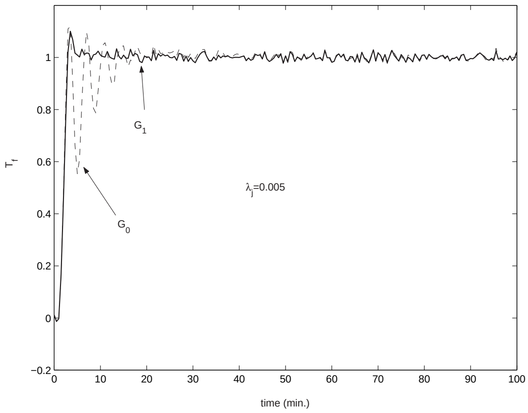

Figure 6. Step response of final GPC controlled heat exchanger.

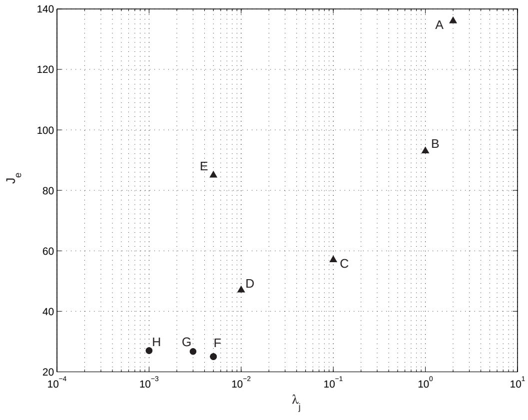

Figure 7. Values of index J_e obtained for different weighting factor λ_j. Triangles: using model G_0 for the design of GPC; circles: using model G_1.

6. CONCLUSIONS

The conclusion of this paper are summarized as follows:

- The step response models of heat exchanger has some limitations for the design of controller when high performance is required. This models explains the low frequency behavior but they don't cover significant aspects of behavior of this process such as resonant effect due to distributed operation.
- The resonant effect on heat exchangers appears under high performing operating conditions, being quite difficult to realize if the system is softly activated or without any control. In these cases, better than carrying out sophisticated identification procedures, an iterative approach has been suggested.
- Although the iterative approach can be used with a variety of control design and/or identification techniques, in this paper the iterative method is used to design a GPC controller, a model-based optimal control strategy.
- For the simulation study an structural model that take into account the resonant effect in the heat exchangers was considered.
- The iterative approach provides good results for the control of heat exchanger: after one iteration the settling time of the fluid temperature is reduced and the oscillations, due to resonant effect, do not appear.

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